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Interaction of magnetic reconnection and Kelvin-Helmholtz modes for large magnetic shear: 1. Kelvin-Helmholtz trigger

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Abstract At the Earth’s magnetopause, both magnetic reconnection and the Kelvin-Helmholtz (KH) instability can operate simultaneously for southward interplanetary magnetic field conditions. The dynamic evolution of such a system can be expected to depend on the importance of KH wave evolution versus reconnection and therefore on the respective initial perturbations. In this study, a series of local three-dimensional MHD and Hall MHD simulations are carried out to investigate the situation where the Kelvin-Helmholtz instability is initially the primary process. It is demonstrated that magnetic reconnection is driven and strongly modified by nonlinear KH waves. The highest reconnection rate is close to the Petschek rate, but the total open flux is limited by the size of the nonlinear KH wave. Most of the total open magnetic flux has no flux rope structure and originates from reconnection at thin current layers which connect adjacent vortices. In contrast, complex flux ropes generated by patchy reconnection within the KH vortices dominate the vicinity of the equatorial plane; however, the associated open flux with flux ropes is a minor contribution to the total open flux. Although the presence of Hall physics leads to a fast early increase of the reconnection rate, the maximum reconnection rate and the total amount of open magnetic flux at saturation are the same as in the MHD case.

1. Introduction

Magnetic reconnection and Kelvin-Helmholtz (KH) instability are often considered as the two most important mechanisms for solar wind-magnetosphere coupling [Axford, 1964; Dungey, 1961]. For southward interplanetary magnetic field (IMF) conditions, both magnetic reconnection and KH modes can operate simultaneously, and there are only a few studies to examine this situation. Recently, Hwang et al. [2011] presented the first in situ observations of nonlinear KH waves during southward IMF conditions. They suggested that for southward IMF, KH vortices become irregular and temporally intermittent. Motivated by the unresolved question of the interaction between KH instability and magnetic reconnection for large magnetic shear, we conducted a series of local MHD simulations to investigate this problem.

Magnetic reconnection occurs in the presence of sufficiently large antiparallel magnetic field components (e.g., in the case of southward IMF close to the equatorial magnetopause). Magnetic reconnection requires a local width of the current layer comparable to the diffusion width (i.e., ion or even electron inertial or gyroscale). A localized diffusion caused by microinstabilities or electron pressure anisotropy breaks the so-called “frozen-in” condition and therefore changes the topology of the magnetic field, allowing plasma transport across the Earth’s magnetospheric boundary. The efficiency of magnetic reconnection is measured by the rate of the magnetic flux transport (also called the “reconnection rate”). Both theory and simulation show that a fast (Petschek type) normalized magnetic reconnection rate is about 0.1 [Petschek, 1964; Birn et al., 2001], which is based on a two-dimensional reconnection geometry assuming a unit distance into the invariant direction.

In comparison, KH modes occur for a sufficiently large shear flow [Chandrasekhar, 1961; Miura and Pritchett, 1982; Miura, 1982, 1984, 1992, 1995a; Otto and Fairfield, 2000; Fairfield et al., 2000; Hasegawa et al., 2004]. At the Earth’s magnetopause, the total velocity difference between the solar wind plasma and the stagnant magnetospheric plasma varies from zero at the subsolar point to values close to the solar wind speed (about 300 ∼ 1000 km/s) near the tailward flank boundary, which determines the KH stability and growth rates. It is known that plasma compressibility, a finite width of the shear flow transition, and a magnetic field component parallel to the shear flow can stabilize the instability [Chandrasekhar, 1961; Miura and Pritchett, 1982].
Since the KH instability is an ideal instability, mass transport across the magnetospheric boundary is not expected in ideal MHD. However, in resistive MHD and two-fluid simulations, nonlinear KH modes can drive magnetic reconnection even for northward IMF conditions [Otto and Fairfield, 2000; Fairfield et al., 2000; Nykyri and Otto, 2001, 2004; Nakamura et al., 2006, 2008]. Recently, Cowee et al. [2010] used hybrid simulation to demonstrate that the KH instability is likely to generate an enhanced diffusive transport. Although KH waves are also observed in some global MHD simulations for both northward [Guo et al., 2010; Li et al., 2012] and southward [Hwang et al., 2011] IMF conditions, their resolution cannot resolve the detailed structure of mesoscale nonlinear KH waves. Therefore, high-resolution three-dimensional local MHD simulations are important tools for a better understanding of this process.

For northward IMF, magnetic reconnection cannot operate in the unperturbed magnetopause geometry because magnetic shear is small. Therefore, KH modes are the primary instability, and reconnection occurs only if the magnetic field has been strongly modified by the KH wave. For such conditions satellite observations provide strong evidence for the presence of nonlinear KH waves [Fairfield et al., 2000; Hasegawa et al., 2004].

A system which is unstable to both KH modes and tearing modes (linear stage of magnetic reconnection) requires that velocity shear and magnetic shear are not aligned, otherwise either KH modes or tearing modes would be stabilized. The dominant linear evolution depends on the relative growth of these two modes and on their respective initial perturbations [Chen, 1997]. In the nonlinear stage, the interaction of these processes is an unresolved question that depends on the primary instability process. The results of this interaction can be expected to depend strongly on the initial conditions and boundary conditions, namely, conditions where (1) KH modes represent the initial or primary process or (2) magnetic reconnection is the primary process. Here we focus on the first condition, and the second condition is discussed in our companion paper (hereafter referred to as KH2).

It is well known that nonlinear KH waves can drive magnetic reconnection in two dimensions. However, in two dimensions a simple current sheet with shear flow can only be unstable to either KH modes or to the tearing mode. Therefore, the presented investigation, which considers the simultaneous evolution of reconnection and KH modes, requires three spatial dimensions. Section 2 introduces the numerical model used in this study and also describes how to measure the KH growth rate and reconnection rate in this configuration. In section 3, we first present the overall dynamics, then show the modulation of magnetic reconnection by the primary KH modes, and examine the influence of several critical parameters, that is, the initial shear flow value, the guide magnetic field, and the KH wave number. Finally, we also investigate the influence of Hall physics. Section 4 presents a summary and discussion.

2. Numerical Methods

2.1. Numerical Models

In this study, the full set of resistive Hall MHD equations are numerically solved by using leapfrog method [Otto, 1990, 2001]. In the computations all quantities are normalized to the typical values, that is, the length scales L to a typical length L_o, the density ρ to ρ_o = n_o m_o, with the number density n_o and the ion mass m_o, the magnetic field B to B_o, the velocity V to the typical Alfvén velocity V_A = B_o (μ_o ρ_o)^{-1/2}, the thermal pressure p to P_o = B_o^2/(2μ_o), and the time t to a typical Alfvén transit time T_A = L_o/V_A. The Hall parameter l = d/L_o is the ratio of ion inertial length d to the typical length L_o. The values for the normalization of the simulation units are summarized in Table 1.

We present the results from a series of simulation cases to study the interaction between magnetic reconnection and KH modes. The simulation domain is a volume with |x| ≤ L_x = 30, |y| ≤ L_y = 20, and |z| ≤ L_z = 40 and is resolved by using 103 × 203 × 103 grid points with a nonuniform grid along the x and z directions. To sufficiently resolve the diffusion region, the best resolution is set to 0.1 and 0.2 in the x and z direction in the diffusion region.

The initial equilibrium is a one-dimensional modified Harris sheet, where magnetic field B = B_0̂ e_x − B_0 tanh(x)̂ e_y, bulk velocity V = −V_A tanh(x)̂ e_x, thermal pressure p = ρ_o + 1 − B^2, and plasma density

<table>
<thead>
<tr>
<th>Table 1. Simulation Normalization</th>
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<tbody>
<tr>
<td>Normalization of the Simulation Units</td>
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<tr>
<td>Magnetic field (B_o)</td>
</tr>
<tr>
<td>Number density (n_o)</td>
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<tr>
<td>Length scale (L_o)</td>
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<tr>
<td>Alfvén velocity (V_A)</td>
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<tr>
<td>Time (T_A)</td>
</tr>
</tbody>
</table>

### References

\( p = \rho_0 + \delta \rho \tanh(x) \). Here \( B_{0x} = 1, B_{0z} = 0.25, \rho_0 = 1, \delta \rho = 0.1, B_{0z} \) is the guide field for magnetic reconnection, and \( V_{0x} \) is the initial shear flow magnitude. For convenience, we use the fast mode Mach number \( M_f = V_{0x}/V_f \) to represent this speed, where \( V_f = \sqrt{(B_{0x}^2 + \gamma \rho_0^2/2)/\rho_0} \) is the average fast mode speed. Because the growth and evolution of KH waves strongly depend on the initial shear flow magnitude, which drives the instability, and on the magnetic field magnitude along the wave vector, which can stabilize the mode, we choose these in the range \( M_f \in [0, 1] \) and \( B_{0x} \in [0, 0.5] \). The reference case uses \( B_{0x} = 1, B_{0z} = 0, \) and \( V_{0x} = 0.5 \). Figure 1 shows a sketch of the 3-D system, where magnetic shear, shear flow, and the structure of the KH wave are indicated. The spine region indicated by the green line in Figure 1 is the region between KH vortices \( \text{[Otto and Fairfield, 2000]} \) and is not related to the topological term “spine” of three-dimensional magnetic field topology \( \text{[e.g., Birn and Priest, 2007]} \).

In order to select the KH mode as the primary process, the system is triggered by a KH type perturbation, which is chosen as \( \delta \psi = \left[ \nabla \psi(x, y) \times \hat{e}_z \right] f(z) \). Here function \( f(z) \) is given by

\[
f(z) = \frac{1}{2} \left[ \tanh \left( \frac{z + z_0}{d} \right) - \tanh \left( \frac{z - z_0}{d} \right) \right],
\]

with \( z_0 = 15 \) and \( d = 3 \) and is used to localize the perturbation in the region where \( z < 15 \). For single mode the stream function \( \psi \) is given by

\[
\psi(x, y) = -\delta \nu \cos(ky) \tanh(x/2),
\]

where the amplitude of the perturbation \( \delta \nu \) is 0.2 and the wave number \( k = \pi L_z^{-1} \) is determined by the size of the simulation box in the \( y \) direction \( (L_z) \). Note that this perturbation is not a normal mode, that is, a solution of the linearized equations, but the spectrum of the perturbation has a dominant contribution to the normal mode with the chosen wave number. At the magnetopause, a realistic perturbation usually contains multiple wave modes. For select results of this study, the system is seeded by multiple modes, i.e.,

\[
\psi(x, y) = \sum_i \left( -\delta \nu / n_i \right) \cos(ky) \tanh(x/2),
\]

where \( n_i \) is the number of wave for each specific wave mode in the simulation box, \( k_i = n_i \pi L_z^{-1} \) is its wave number, and \( \delta \nu_i \) is its amplitude. Most results in this study use single-mode perturbations unless stated otherwise.

It is natural to use periodic boundary conditions along the \( y \) direction. Closed boundary \( (B_y = V_y = 0) \) conditions are applied in the \( x \) direction. However, the simulation box is chosen sufficiently wide that boundary effects can be ignored. At the real magnetopause, the magnetosheath magnetic field lines are moving with the solar wind, and magnetospheric field footprints stick to the Earth’s ionosphere, that is, at large distances from the equatorial plane. For our local simulation, the top and bottom boundaries represent these unperturbed regions, where magnetic field lines are moving with the plasma, due to the frozen-in condition. Thus, it is required to maintain the initial flow to mimic the real magnetopause at the top and bottom boundary. A straightforward method is fixing the \( z \) boundary values during the simulation. However, the fixed boundary conditions can easily reflect the wave causing unnecessary perturbation. A more sophisticated solution is adding an artificial friction term on the right-hand side of the momentum equation to maintain the initial shear flow and damp the Alfvén wave. In our simulation, this term is given by \( -\gamma(z) [\rho \mathbf{V} - \rho(0) \mathbf{V}(0)] \), where

![Figure 1. Sketch of the system geometry, where magnetic shear, shear flow, and the structure of the KH wave are indicated.](image-url)
\(\rho(0)\) and \(V(0)\) are the initial plasma density and velocity, respectively. The friction term should have no effect inside of the simulation box; therefore, the friction coefficient is localized near the z boundary and given by

\[\nu(z) = \frac{\nu_0}{2} \left[ 2 - \tanh \left( \frac{z + z_c}{dz_c} \right) + \tanh \left( \frac{z - z_c}{dz_c} \right) \right],\]

where \(\nu_0 = 1, z_c = 0.75L_p\), and \(dz_c = 3\).

At the real magnetopause, the collisionless plasma implies a zero resistivity. However, it is well established that reconnection occurs in the collisionless plasma of the magnetopause and the magnetotail provided that the corresponding current sheets are sufficiently thin. Thus, in the simulation, three current and plasma parameter-dependent resistivity models are applied, which switch on a resistivity if a critical current density is surpassed:

\begin{align*}
\text{Model 1:} & & \eta_1 = \eta_0 \sqrt{I - \frac{1}{2} j^2 H(j - j_c)} + \eta_b, \\
\text{Model 2:} & & \eta_2 = \eta_0 \sqrt{\frac{1}{2} j^2 H(j - j_c)} + \eta_b, \\
\text{Model 3:} & & \eta_3 = \eta_0 \left( j^2 - j_c^2 \right) H(j - j_c) + \eta_b.
\end{align*}

Here \(\eta_0 = 0.001\) is the background resistivity, \(\eta_b = 0.05\), and \(j\) and \(j_c = \rho V_e = \sqrt{2\rho p}\) are the current density and the critical current density, respectively. \(H(X)\) is the Heaviside step function. Thus, the resistivity is switched on only when (where) the current density exceeds the critical value, \(j_c\), which is determined by the plasma temperature and density. The chosen background resistivity in the magnetosphere is small such that the current density can assume rather high values. However, a large current density is equivalent to a large current carrier drift velocity \(V_d = V_i - V_e\), where \(V_i\) and \(V_e\) are ion velocity and electron velocity, respectively. This drift velocity is evaluated by \(j/j\). When the drift velocity is faster than some typical value (e.g., ion-acoustic speed), microinstabilities lead to turbulence limiting a further increase of the drift velocity. This implies an exchange of electron and ion momentum of the current carriers, which is equivalent to a resistivity. Thus, a resistivity is switched on only when the drift speed \(V_d\) is faster than a critical speed \(V_c = ac_s\), where \(c_s = \sqrt{(\gamma p)/2\rho}\) is the ion-acoustic speed, \(\gamma\) is the heat capacity ratio, and \(a\) is of order unity when the ion inertial scale is of the order of the normalization length \(L_p\). In reality, \(a\) is determined by the actual onset conditions for current-driven turbulence in a strong current. As will be demonstrated, the exact choice has minor influence on the macroscopic dynamics, and the presented results use \(a = \sqrt{4/\gamma}\). The purpose of using different resistivity models is not to justify any specific resistivity model but to test the robustness of our simulation results. A meaningful MHD simulation result should be fairly insensitive to the specific resistivity model or the exact choice of the parameters in the model. Our results demonstrate that the overall dynamical evolution is determined by the basic configuration rather than the particular resistivity or detailed physics of the diffusion region. A case with a uniform resistivity \(\eta = 0.005\) is included to compare with other resistivity models. Therefore, unless stated otherwise, we always use Model 1.

### 2.2. Growth Rate and Reconnection Rate

The KH mode growth rate and the reconnection rate are two of the most important physical quantities to characterize the dynamics of the system. Assuming an incompressible plasma, a discontinuous change of the tangential velocity, and the presence of different densities and tangential magnetic fields across a plasma boundary, the linear theory [Chandrasekhar, 1961] shows that the KH mode growth rate \(q\) is

\[q = \sqrt{a_1 a_2 \left[ (V_1 - V_2) \cdot k \right]^2 - a_1 (V_{1z} \cdot k)^2 - a_2 (V_{2z} \cdot k)^2},\]

where the indices refer to the two sides of the shear flow layer, \(a_i = \rho_i/\rho_{i+1}\), \(k\) is the wave vector of the perturbation, and \(V_{ui} = B_i/\sqrt{\rho_i}\) is the Alfvén velocity.

In the simulation, the growth of KH mode is indicated by \(\Delta V_e = \max(V_e) - \min(V_e)\), and the growth rate \(q\) is measured by exponential fitting \(\Delta V_e\) in the linear stage. In principle, one can use other quantity (e.g., \(V_i\) or \(B_i\)), however, most of them require to a separation of equilibrium and perturbation. Although \(B_i = 0\) in the equilibrium state, it gets modified more strongly once reconnection sets in. Therefore, \(V_e\) appears to be the best quantity to identify KH growth.

To identify the reconnected (open) magnetic flux, field lines are traced from the top boundary \((z = L_p)\). Figure 2 presents the selected magnetic field lines obtained from the reference case at \(t = 160\). Note that the
Figure 2. A perspective view of the selected magnetic field lines obtained at $t = 160$ for reference case. The red and black lines are the closed field lines, and the green, blue, magenta, and orange lines are the open field lines. The color index in the reference planes represents the magnetic $B_z$ component, and black arrows indicate the plasma bulk velocity.

Simulation box is periodically extrapolated in the $\pm y$ direction in Figure 2. The magnetic $B_z$ component ranging from $-1$ to $1$ is indicated by the color index (from purple to dark yellow) in the equatorial and bottom planes. Black arrows represent the plasma bulk velocity. The closed field lines (e.g., the red and black lines in Figure 2) extend from the top boundary to the bottom boundary have the endpoints on the same side of the magnetopause boundary ($x = 0$). The open field lines (e.g., the green, blue, magenta, and orange lines in Figure 2) from the top boundary on the magnetosheath side ($x > 0$) extend toward the equatorial plane and connect to the field lines on the magnetosphere side, which extend back to the top boundary because the magnetospheric field has the opposite direction along $z$. Thus, all open field lines started from the top boundary on the magnetosheath side ($x > 0$) have endpoints also at the top boundary on the magnetosphere side ($x < 0$). By integrating all of the positive (or negative) open flux (along $z$) at the top boundary, one can obtain the total open flux

$$\Phi = \frac{1}{2} \int_{\text{open}} |B_z(x, y, L_y)| \, dx \, dy,$$

where the integral is taken over the open flux at the top boundary plane. In order to compare with two-dimensional reconnection theory, this reconnection rate is normalized to the system size $2L_y$. Therefore, the normalized reconnection rate $r$ is defined by

$$r = \frac{1}{2L_y} \frac{d\Phi}{dt}.$$

This definition can underestimate the local reconnection electric field significantly but provides an appropriate measure for the large-scale reconnection process. In this study, 160,000 magnetic field lines are traced from top boundary in the rectangle of $[-10, 10] \times [-20, 20]$, which covers the whole open flux. Furthermore, the convergence of the results is confirmed by using more dense field lines tracing (640,000 lines).

The frozen-in condition implies that the magnetic field is equipotential if the resistivity is zero. Any local diffusion and specifically magnetic reconnection, which breaks the frozen-in condition, generates a parallel electric field component $E_\parallel$. Therefore, the field-aligned electric potential difference

$$\Delta \phi = \int E_\parallel \, ds,$$

where $ds$ is an infinitesimal length along the magnetic field line, indicates whether the magnetic field line goes through a diffusion region (newly reconnected) or not [Hesse and Schindler, 1988]. Theoretically, the maximum average parallel electric field along the magnetic field also represents the reconnection rate in three dimension. However, for real applications, this method is highly sensitive to the identification of the
singular field line with the highest parallel potential, and very small deviations from this field line generate large errors. Therefore, the field-aligned electric potential difference is used only to qualitatively and semiquantitatively represent the local reconnection rate.

Figure 3. The growth of the KHI, open magnetic flux, and normalized reconnection rate for the uniform resistivity case (yellow), resistivity Models 1 to 3 (red, green, and blue, respectively), and the two-dimensional KH instability case (black).

Note that in the current configuration, reconnection operates mainly in the xz planes (with antiparallel field components in the z direction). A location with $B_z = 0$ along a field line generally implies a change of direction or deflection along z of the respective field line. Therefore, the number of $B_z = 0$ points (deflection points) indicates how often this field line has been reconnected. In this study, a magnetic field line with single/no deflection point is called as simple open/closed field line; a magnetic field line with multiple deflection points is indicative of a flux rope structure.

3. Simulation Results

3.1. Overall Dynamics

Based on the simulation results, three-dimensional KH modes with a perpendicular antiparallel magnetic field have the following features: (1) a growth rate lower than the linear theoretical value, (2) a high normalized reconnection rate close to the Petschek rate, and (3) a limited open flux. Figure 3 shows the growth of the Kelvin-Helmholtz instability (KHI), open magnetic flux, and normalized reconnection rate for the different resistivity models under the conditions of single wave mode (i.e., wavelength is 40) and a shear flow of $0.5V_f$ without an initial guide field. A two-dimensional case under the same conditions is included for comparison. This two-dimensional case has a marginally faster growth but a much earlier saturation. The three-dimensional KH growth rate 0.043 is lower than the theoretical value 0.078 from equation (1).

Although the uniform resistivity case is a little slower than other resistivity models, the overall KH mode dynamics is insensitive to the specific resistivity model. The enhancement of $\Delta V_z$ in the nonlinear stage of the three-dimensional case is likely caused by the onset of reconnection.

Two explanations for this lower growth rate are the finite width of shear layer and compressibility [Miura and Pritchett, 1982]. A third important cause for stabilization is the localization of the wave in the z direction, which leads to a loss of wave energy by radiation of Alfvén waves. In the three-dimensional configuration, the plasma velocity caused by the wave ($v_x, v_y$) is confined to a vicinity of the equatorial plane and drags the magnetic field lines across the boundary. However, the building magnetic tension in three dimensions tends to pull the magnetic field line back to its original position (unless this field line is “broken” by magnetic reconnection).
Figure 4. The evolution of the KH mode. The color index represents the current density in equatorial plane at $t = 80, 100, \text{and } 160$ for the reference case, and green arrows indicate the bulk velocity in the $xy$ plane.

An alternative but equivalent explanation is the generation of Alfvén waves through the growing KH waves, which travel in the positive and negative $z$ directions and extract energy from the KH mode. Assuming a KH wavelength of $\lambda$ and a half-width $D_z$ of the KH unstable region, the perturbation energy per unit time associated with the wave, $E_{KH}$, is $\frac{1}{2} \rho D_\lambda^2 \delta V^2 D_z q$, where $\delta V$ is the perturbed velocity amplitude, $q \sim \kappa (2\pi \lambda^{-1}) V_0$ is the KH growth rate, and $\kappa < 1$ measures the reduction of the growth rate from the ideal limit. The energy transport by Alfvén waves through the Poynting flux $P_s$ is $\mu_0^{-1} \delta V \delta B \lambda^2$, where $B$ is the unperturbed magnetic field in the $z$ direction, $\delta B$ is the perturbed magnetic field amplitude, and $\delta V = \delta B / \sqrt{\mu_0 \rho}$. Marginal growth of the KH wave requires $E_{KH} \geq P_s$, which yields $D_z \geq (\kappa \pi)^{-1} (V_a / V_0) \lambda$ for marginal stability. Here $V_a$ is the background Alfvén speed based on $B$. The estimate also shows that KH growth is uninhibited for $D_z \gg \lambda$, in which case energy transport through Poynting flux is negligible and the system approaches the two-dimensional KH evolution.

The strong antiparallel magnetic field represents a configuration that is susceptible to magnetic reconnection. The onset of large amplitude KH waves deform the current layer, which widens in the center of the vortex and thins in the spine of the KH wave. As a result, the width of the current layer becomes comparable to the diffusion length, satisfying the condition for the onset of resistivity and causing magnetic reconnection.

Figure 4 illustrates the evolution of current density (color index) and the in-plane bulk velocity (green arrows) in the equatorial plane at $t = 80, 100, \text{and } 160$ for the reference case. Figure 4 (left) shows that a narrow vortex is formed in the region of $y \in (-10, 5)$ at $t = 80$, and the thin current layer connected to this vortex region is the spine region. Figure 4 (middle) shows that the growing KH mode winds the spine region current layer around the nonlinear vortex and generates multiple current layers in the vortex region at $t = 100$. Figure 4 (right) shows the saturation of KH mode at $t = 160$. The multiple current layers in the $y > 12$ region often observed in our simulations are secondary KH modes, which eventually are convected into the main vortex. Note, the spine region usually has the highest current densities, such that reconnection is triggered first and maximizes in this region.

Figure 3 (middle and bottom) shows that magnetic reconnection strongly increases when the KH modes reach their nonlinear stage (after $t = 80$). The uniform resistivity case (yellow line) is shifted by about 20 Alfvén times due to slightly slower KH growth, likely because of the widening of the shear layer by the uniform diffusion. However, all cases (except for the 2-D case) show the same behavior in terms of the saturation of open magnetic flux within 10% of the maximum value. All simulation cases assume a highest reconnection rate of 0.1, which is similar to the Petschek reconnection rate, and is largely independent of the resistivity model. Note that our uniform resistivity case uses the same resistivity as Geospace Environment Modeling (GEM) magnetic reconnection challenge by Birn and Hesse [2001], which has a much
lower reconnection rate. This demonstrates that the enhanced reconnection rate is not due to the specific resistivity model. The physical reason for the high reconnection rate is likely the converging normal velocity (i.e., $V_x$) by the KH mode, which causes a compression of the current sheet in the spine region and carries more flux into the diffusion region once reconnection operates. This convergent flow has been noted by Otto and Fairfield [2000] as the cause for concentrating magnetic flux in the spine region. Note that the normalized reconnection rate drops when a certain amount of magnetic flux has been reconnected for all cases, indicating a physical process that limits the total amount of open flux. The saturation of the open magnetic flux occurs at the same time when the longest wavelength KH wave starts to saturate nonlinearly. The likely physical reason is the dissipation and widening of the boundary by the late stage nonlinear KH mode, which also widens the current layer and switches off reconnection. The final amount reconnected flux is almost identical for all the cases, which implies that the limited amount of open flux is determined by the size of KH mode. We will further discuss the relation between the size of the KH mode and the limitation of open flux in section 3.2.

To better illustrate the reconnection onset process, we plot the maximum current density, resistivity, parallel electric field, and normalized reconnection rate from the reference case in Figure 5. It shows the current density rapidly increasing to 1.6 until $t \approx 40$, when resistivity, and consequentially the parallel electric field, are switched on. This indicates that the current density has reached the critical value. As we expect, the maximum parallel electric field, as an indication of the maximum local reconnection rate, has a behavior similar to the normalized reconnection rate. Note, $|E|_\parallel$ indicates the local character while $r$ is a global measure as defined above. Nevertheless, the two quantities compare fairly well. The large fluctuations in maximum current density and $E_\parallel$ after about $t = 80$ indicate that the nonlinear KH waves have broken up the continuous and smooth current layer into multiple patches where reconnection occurs.

The complexity of the reconnection process driven by the KH mode is illustrated in Figure 6, showing the magnitude of the field-aligned electric potential difference in the equatorial plane and top boundary at $t = 160$ for the reference case. To improve contrast, the color bar only covers $|\Delta \phi| < 0.2$. The footprints of magnetic field lines from Figure 2 in the equatorial plane are plotted in the Figure 6 (left). The bright patchy spots indicate ongoing reconnection, which is consistent with the high current density, see Figure 4 (right). The re-reconnection process allows the open field line to become closed field line again, which is likely the case for the red line in Figure 2. The red line crosses the equatorial plane several times in the vortex, which indicates reconnection between the flux rope and other open flux. The blue line with multiple twists is a typical open field line through the vortex. This field line crosses the whole simulation domain and has likely been reconnected multiple times. In contrast, the green line, as a typical field line through the spine region, is an open field line that has been reconnected once. This implies that the spine region is the source region for simple open magnetic field topology. The magenta and orange lines are the examples of earlier reconnected simple open magnetic field lines, which are convected away from the equatorial plane. Their footprints are plotted in Figure 6 (right). One should keep in mind, the large-scale and ongoing patchy reconnection can change the amount of open flux with positive and negative values in the
The normalized reconnection rate, allowing the possibility of negative global reconnection particularly at late times in the evolution.

The topological structure of magnetic field lines is illustrated in Figure 7, which is the map of six different types of magnetic field lines in $z = -5, 0, \text{ and } 20$ at $t = 160$ for reference case. The color dots represent the footprints of magnetic field lines in Figure 2. As expected, the newly reconnected field lines in the spine region are mostly simple open field lines. Patchy reconnection in the vortex region generates complex flux ropes. However, in the plane away from the equatorial plane, open flux is mostly simple reconnected fields originating from the spine region and accumulating in the outflow region of reconnection. Complex flux ropes cover only the edges of open flux areas, indicating that patchy reconnection plays a minor role for the net flux transport for most of the evolution. Note that the complex flux ropes dominate in the equatorial plane because of the weaker magnetic field and multiple crossings of the equatorial plane. This is also consistent with Figure 6, in which the bright area is large in the equatorial plane and much smaller in the top boundary. Note that this result is obviously sensitive to the position of the location of the cross section relative to the equatorial plane (highest KH amplitudes and patchy vortex reconnection).

Once a field line is reconnected, the footprint at top/bottom boundary on the magnetosheath side is moving in the negative $y$ direction, while the footprint on the magnetosphere side is moving in the positive $y$ direction. This represents the fact that at the real magnetopause one end of the open field is carried tailward by the solar wind, while the other end is stuck in the ionosphere. This mechanism generates a magnetic field $B_y$ component, and thereby elongating open magnetic field lines as illustrated by the orange and magenta field lines in Figure 2. The basic mechanism is already present in two-dimensional reconnection [La Belle-Hamer et al., 1995]. It is important to realize that the accumulation of $B_y$ flux generates a significant field-aligned current [Ma and Otto, 2013], which in turn can cause reconnection. For instance, the magenta and orange lines in Figure 2 show a moderate field-aligned potential difference (see the Figure 6 (right)). However, reconnection in these regions, which bound the KH unstable interaction in the Northern and Southern Hemispheres, occurs between open field lines. For instance, the magenta and orange lines could merge and generate a new pair of field lines with interchanged foot point locations. However, this magnetic
Figure 7. The map of magnetic field line topology in \( z = -5, 0, \) and 20 at \( t = 160 \) for the reference case. Here the six different types of field lines are (1) simple closed field line (white), (2) closed field line with flux ropes (black), (3) simple open field line connected to the top boundary (pink), (4) open field line with flux ropes connected to the top boundary (yellow), (5) simple open field line connected to the bottom boundary (blue), and (6) open field line with flux ropes connected to the bottom boundary (orange). The color dots represent the footprints of magnetic field lines in Figure 2.

reconnection only involves the open magnetic field lines of the same topology and does not contribute to the normalized reconnection rate.

3.2. Influence of the Shear Flow, Guide Field Component, and KH Mode Wave Number

As indicated in equation (1), two-dimensional incompressible KH growth largely depends on the magnitude of the shear flow, the magnetic field component along the \( \mathbf{k} \) vector, and the wavelength \( \lambda \). In the current configuration, the magnetic field component along the \( \mathbf{k} \) vector is the magnetic \( B_y \) component. We summarize the dependence on these three quantities in Figure 8, showing the KHI growth rate, maximum normalized reconnection rate, and maximum open magnetic flux for different magnitudes of shear flow (Figure 8 (top), \( B_y = 0, \lambda = 40 \)), initial guide field component (Figure 8 (middle), \( M_f = 0.5, \lambda = 40 \)), and wavelengths (Figure 8 (bottom), \( M_f = 0.5, B_y = 0 \)). Here the scale of growth rate and reconnection rate is on the left side in black, and the scale of open flux is on the right side in blue. It is expected that our three-dimensional KH mode linear growth has similar properties as for the two-dimensional configurations [Miura and Pritchett, 1982], because reconnection operates mainly in the nonlinear stage. However, as shown in Figure 3, the KH mode is indeed significantly modified by reconnection in the nonlinear stage.

It appears that certain aspects of the dynamics are insensitive to the shear flow magnitude. Except for the \( M_f = 1 \) case, the KH growth rate increases with increasing of shear flow magnitude in a small range from 0.02 to 0.04 as expected. While there is some increase of the maximum reconnection rate, this rate is about 0.1 for all cases with \( M_s \geq 0.4 \), and the total open flux saturates at about 250, except for \( M_s = 1 \). The \( M_s = 1 \) case indicates that the KH mode is unstable up to a shear flow magnitude of fast mode speed \( V_f \). The physical interpretation is that information cannot propagate from the upstream obstacle (vortex) to the downstream (opposite side). There is no nonlinear KH wave observed for \( M_s \leq 0.2 \) case, which implies a low-critical shear flow magnitude is required to overcome compressibility and magnetic field tension. Note that the initial configuration used an under-critical width of the current sheet, meaning that no resistivity other than the very small background is present initially. Also, the spectrum of the chosen initial perturbation does not contain the normal mode of the tearing instability, a situation that is investigated in detail in a separate paper (KH2). Otherwise reconnection would have started from the beginning of the simulation.

The initial guide field magnitude is highly important for the dynamical evolution. According to equation (1), the increasing guide field decreases the KH growth rate, and the cutoff value for \( M_s = 0.5 \) is 0.55. The
Figure 8. The KH growth rate (black dot), maximum normalized reconnection rate (black square), and maximum reconnected magnetic flux (blue diamond) for different magnitude of (top) shear flow and (bottom) initial guide field component. Here the scale of growth and reconnection rate is on the left side; the scale of open flux is on the right side.

Simulation results confirm that a sufficient guide field component stabilizes the KH mode. For an increasing guide field the wave takes longer to reach its nonlinear stage and saturation occurs at a smaller amplitude and size of the KH vortex. Therefore, both the total open magnetic flux and the normalized reconnection rate decrease. For the $B_{\phi_0} \geq 0.4$ cases, there is no KH vortex observed in the simulations, and thus no magnetic reconnection.

The magnetic $B_y$ component generating mechanism discussed in section 3.1 is unlikely to affect the linear KH growth rate, because reconnection operates mostly in the nonlinear stage. Note that this mechanism generates positive $B_y$ in the $z > 0$ region, and negative $B_y$ in the $z < 0$, according to the geometry. The presence of an initial guide field breaks this antisymmetry. In our simulation, the KH-produced magnetic $B_y$ component is positive in the $z > 0$ region, and negative in the $z < 0$ region. Therefore, the initial presence of a guide field component causes a hemispheric asymmetry and different stability conditions as illustrated in Figure 9. A well-developed KH vortex and a secondary vortex are observed in the $z = -5.6$ plane, while a vortex in the $z = 5.6$ plane has a smaller amplitude. This asymmetry can affect possible conjugate auroral observations, which is also found and discussed in more detail in our KH2 paper. For in situ observation, the $B_y$ asymmetry implies that a well-developed KH wave may be present only in the Northern or in the Southern Hemisphere depending on the sign of $B_y$.

The dispersion relation of our three-dimensional KH mode is consistent with the two-dimensional configuration [Miura and Pritchett, 1982] and shows the fastest growth for $2ka \approx 0.63$ (see Figure 8 (bottom)), where $a$ is the half width of the shear flow. Different wavelengths are achieved by changing the box size ($L_y \in [5, 20]$); therefore, the maximum open flux is normalized by $20/L_y$ for comparison. Figure 8
demonstrated that the reconnection rate is proportional to the wavelength for short wavelengths ($2ka > 0.63$). However, for long wavelength ($2ka < 0.63$) cases, the reconnection rate and the normalized open flux become largely insensitive to the wavelength. Specifically, this illustrates again that the total amount of open flux is determined by the wavelength of dominant nonlinear KH mode unless the magnetic field $B_{0z}$ component is large. The increase of total flux by double the simulation domain in the $x$ direction ($L_x = 60$, $L_y = 20$, and $L_z = 40$) has no influence on saturation of open flux. This demonstrates that the saturation of total open flux is not due to the limitation of available open flux. A larger simulation domain in the $z$ direction ($L_x = 30$, $L_y = 20$, and $L_z = 80$) increases the saturated magnetic flux slightly by 20%, because increase $D_z$ destabilizes the $\lambda = 40$ mode, as discussed in section 3.1.

At the Earth’s magnetopause, the typical KH wavelength is about 2 to 6 $R_E$. Thus, the long wavelength limit appears more appropriate for the magnetospheric boundary where the typical magnetopause width is about 800 km [Berchem and Russell, 1982; Dunlop et al., 2001]. However, as discussed in section 3.1, the wavelength is limited by the localization of KH wave activity along the $z$ direction. A case with a large simulation box ($L_x = 40$, $L_y = 60$, $M_r = 0.5$, and $B_{0z} = 0$) is used to illustrate this limitation. The evolution of KH modes in this large simulation box is represented in Figure 10, which shows the magnetic $B_z$ component (color index) and the in-plane bulk velocity (black arrows) in the equatorial plane at $t = 48$, 64, and 188. Note that the initial main spectrum of KH perturbation is the $\lambda = 80$ mode. However, this mode only appears as a surface wave with a very tiny amplitude at the very beginning of the simulation (not shown in Figure 10). Subsequently four shorter wavelength modes associated with the most unstable KH mode (Figure 8) develop in the simulation (Figure 10 left)). Late in the simulation, the longest wavelength in the three KH modes is about 40, which agrees with the limitation discussed in section 3.1. The total open flux normalized by the box size is about 233, consistent with the $\lambda = 40$ case in Figure 8.

Coalescence of KH vortices is often considered as a mechanism of the evolution from shorter wavelength KH modes to the longer wavelength KH modes by different authors (see summary in Miura [1995b]). However, the simultaneous but slower growth of longer wavelength modes and the decay/diffusion of shorter nonlinear waves is another plausible scenario for this evolution, because coalescence and linear growth both occur...
on the ideal fluid timescales. For example, in Figure 10, the long wavelength mode \((y \in (0, 25))\) containing the two small vortices at \(t = 120\) evolves to a large vortex at \(t = 180\), while the two small vortices decay during this process. Any perturbation at the real magnetopause usually contains a spectrum of modes. Figure 11 shows the evolution of KH modes seeded by multiple spectrum \(n_1 = 1, n_2 = 4, \delta v_y = \delta v_z = 0.1, M_i = 0.5,\) and \(B_{z0} = 0\). The small vortices associated \(n_2 = 4\) decay at the boundary of the \(n_1 = 1\) mode at \(t = 56\), and the \(n_1 = 1\) mode dominates the system at \(t = 124\), which is consistent with the prior discussion. The total amount of open magnetic flux of 233 in this multiple wave spectrum case agrees with the single mode with \(\lambda = 40\) case, which again confirms that the total reconnected flux is limited by the longest nonlinear wavelength mode in the system.

### 3.3. Influence of Hall Physics

Including the Hall term in Ohm’s law implies the separation of the ion and electron velocity, and the frozen-in condition only applies to the electrons, which often carry most of the current in thin current sheets and in field-aligned currents. Hall physics generates faster magnetic reconnection [Birn et al., 2001; Otto, 2001] typically with the Petschek rate. The typical width of the magnetospheric boundary is about 800 km or 5 to 10 ion inertial scales [Berchem and Russell, 1982; Dunlop et al., 2001]. Fast magnetic reconnection requires a width of the diffusion region (current layer) comparable to or smaller than the ion inertial scale. However, typical KH observations indicate wavelengths between 2 and 8\(R_e\), around the terminator or 120 to 480\(d_o\), although shorter wavelength are likely to occur as well. In this section, three selected cases are chosen to investigate effects caused by Hall physics. For computational reasons, we consider a KH wavelength of 33\(d_o\) with a box size of \(L_y = 10\) (implying \(l = 0.6\)). Note that Huba [1994] demonstrates that the Hall term breaks the dawn-dusk symmetry for KH waves present in MHD for KH wave lengths of 3.8\(d_o\), i.e., shorter than...
Figure 11. The magnetic $B_z$ component in $z = 0$ plane at $t = 46, 56$, and $124$ for multiple spectrum case. The black arrows present bulk velocity in the equatorial plane.

typically observed and even shorter than the typical magnetopause width. The first case with a Hall parameter of $l = 0.6$ represents the dawnside ($B_{z0} = 1.0$), and the second case with the same parameter represents the duskside ($B_{z0} = -1.0$). For comparison, the third case is the MHD case ($l = 0$). All other parameters are the same as in the reference case. Figure 12 presents the growth of the KHI (Figure 12 (top)), open magnetic flux (Figure 12 (middle)), and normalized reconnection rate (Figure 12 (bottom)) for these three cases. It appears

Figure 12. (top) The growth of the KHI, (middle) reconnected magnetic flux, and (bottom) normalized reconnection rate for different Hall parameter and $B_z$. 
that the overall dynamics is almost identical for the three cases. Although reconnection is initially faster and have more fluctuations in the Hall cases, the overall difference in the reconnection rates and in the amount of open flux is relatively small. This suggests that Hall physics does not play a critical role in reconnection during the nonlinear interaction with the KH waves different, for instance, from the results of the simple reconnection geometry in the GEM reconnection challenge [Ma and Bhattacharjee, 2001; Otto, 2001]. In all cases including the MHD case, the average reconnection rate is approximately the Petschek rate illustrating that MHD reconnection in the presence of nonlinear KH waves is fast. Finally, we found the overall evolution does not show a significant dawn-dusk asymmetry. Note that the KH wavelength in the study by Huba [1994] is only a few ion inertial scales, and therefore, inertial effects leading to the dawn-dusk asymmetry are important. Whether or not KH waves actually develop on such small kinetic scales at the real magnetopause is unresolved in the absence of any observational evidence.

4. Summary and Discussion

For southward IMF conditions, the interaction between KH modes and magnetic reconnection is important because it can potentially determine the plasma transport across the Earth’s magnetospheric boundary. Here we have examined this interaction systematically for conditions in which KH waves are the primary instability process in this interaction. Our main results can be summarized as follows.

1. Fast magnetic reconnection is driven and strongly modified by the nonlinear KH modes. Reconnection rates are comparable to Petschek reconnection even without the inclusion of Hall physics.
2. The total amount of open flux is limited by the longest wavelength mode in the system.
3. The spine region is the source of simple open magnetic flux, which provides the majority of open flux.
4. Patchy reconnection in the vortex region generates complex flux ropes. However, it does not provide the major contribution of net flux transport.
5. Marginal stability of KH wave requires that the width of the localization in the $z$ direction is comparable to the KH wavelength.

The nonlinear interaction between magnetic reconnection and KH waves onset conditions are investigated by examining the influence of the shear flow, the initial guide field component, and the KH mode wave number. However, most theoretical and numerical approaches [e.g., Chandrasekhar, 1961; Miura and Pritchett, 1982] including the present study treat the KH instability as an initial value problem. There is no well-defined “initial state” at the real magnetopause, and the width of transition layer is always influenced by a combination of viscous and diffusion processes such as gyroviscosity and including KH waves when the mode operates. The approach to treat this as an initial value problem is justified, if all other viscous processes (excluding KH waves) generate a much thinner boundary layer than expected for KH waves (otherwise the stability cannot operate). It is also reasonable to assume that this boundary width is of the order of typically observed dayside magnetopause boundary width, but caution should be used to literally apply the concept of a maximum growth rate for KH modes at the magnetopause. Here we have focused mostly on the long wavelength limit to understand the basic properties of the nonlinear interaction between magnetic reconnection and KH waves.

Our results suggest that onset conditions for nonlinear KH wave are frequently satisfied at the Earth’s magnetopause for southward IMF conditions. The magnetosheath velocity exceeds the low-critical shear flow ($0.3V_\text{sw}$) a small distance from subsolar point. Although, a guide field component is present almost everywhere at the dayside magnetopause, a three-dimensional configuration allows the $k$ vector to adjust to the most unstable mode, that is, the direction mostly perpendicular to the magnetic field. The simulation suggests that the total open flux for a well-developed KH vortex is about $250 B_0 L_0^2$ for a nonlinear wave with wavelength of $40 L_0$, which yields about $4 \times 10^6$ Wb open flux produced by a wave of $\lambda = 4 R_E$ based on our normalization. At the real magnetopause, typical KH waves velocities are about $V_{\text{sw}} = 200$ km s$^{-1}$, which takes $\tau = \lambda/V_{\text{sw}} = 128$ s to travel its own wave length (after that the next wave comes). This yields a global reconnection rate of $2 \times (4 \times 10^6$ Wb)/128 s $\approx 60$ kV, where the factor 2 assumes KH waves on the dawnside and duskside.

Our simulation results make several predictions for observational signatures. They suggest that the vicinity of the equatorial plane is dominated by patchy reconnection generated in the KH vortices, with a mixture of cold magnetosheath and hot magnetospheric plasma in combination with multiple current layers is expected. In contrast, above or below the equatorial plane simple reconnected flux is dominant.
Reconnection at the spin region shows a rapid increase of the reconnection rate likely consistent with signatures of magnetic flux transfer events and fast jets away from the equatorial plane. A significant variation of the magnetic \( B_y \) component (or \( x \) component in the GSM coordinate near the flank region) is expected in this reconnection outflow region. At some distance from the equatorial plane this outflow region is expected to have typical reconnection layer properties where shear flow and \( B_y \) satisfy the Walén relation and can be the source for significant field-aligned currents [Ma and Otto, 2013].

The influence of density and magnetic asymmetry can be expected to change the detailed quantitative results and will be part of future work. However, we believe that basic results such as the fast-driven reconnection, the flux limitation, and the characteristics of the magnetic topology of the presented results also have validity in more general configurations. The results of this study apply not only to the Earth’s magnetospheric flank for southward IMF conditions but are also important in the high-latitude magnetopause for magnetic \( B_y \) component (GSE coordinate) dominant IMF conditions. Furthermore, this magnetic and flow sheath configuration can also be found at Saturn’s and Jupiter’s magnetopauses. Therefore, our results also provide guidance and reference for studies on the solar wind interaction with Saturn’s and Jupiter’s magnetopauses.

References


