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A 3+1 Decomposition of the Minimal Standard-Model Extension Gravitational Sector

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The 3+1 (ADM) formulation of General Relativity is used in, for example, canonical quantum gravity and numerical relativity. Here we present a 3+1 decomposition of the minimal Standard-Model Extension gravity Lagrangian. By choosing the leaves of foliation to lie along a timelike vector field we write the theory in a form which will allow for comparison and matching to other gravity models.

1. Introduction

Local Lorentz invariance is one of the cornerstones of General Relativity (GR) and modern physics. As such it is an excellent probe of new physics, and Lorentz violation is a large and active area of research.¹ The Standard-Model Extension (SME) is an often-used effective field theory framework which includes all Lorentz and CPT violating terms.^{2–4}

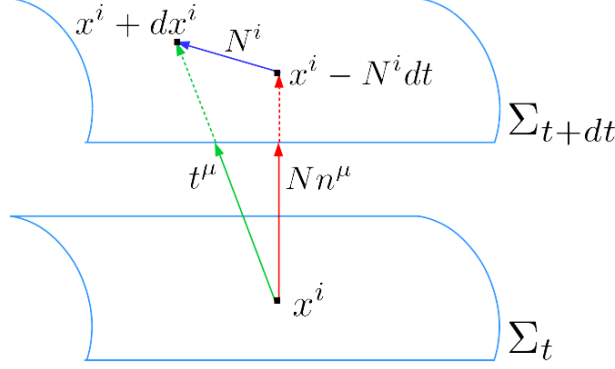
The 3+1 (ADM) version of GR is used in for example canonical quantum gravity and numerical relativity.^{5,6} Here we present a 3+1 decomposition of the minimal SME gravity Lagrangian in the case of explicit Lorentz-symmetry breaking. By choosing the hypersurfaces to be spatial, we write the framework in a form which will allow for comparison and matching to other gravity models.

2. The Decomposition

Using the ADM variables, the metric reads:

$$ds^2 = -N^2 dt^2 + \gamma_{ij} (dx^i + N^i dt) (dx^j + N^j dt), \quad (1)$$

where N is the lapse function and N^i is the shift vector. These ADM variables relate points on different constant-time hypersurfaces (see Fig-


 Fig. 1. Constant-time hypersurfaces Σ along with the ADM variables.

ure 1), Decomposition of the manifold $\mathcal{M} \rightarrow \Sigma \times \mathbb{R}$ induces the metric $\gamma^{\mu\nu} = g^{\mu\nu} + n^\mu n^\nu$, where $n^\mu = (1/N, -N^i/N)$ is a vector normal to the foliation. The minimal gravitational sector of the SME reads as:^{4,7}

$$\mathcal{L}_{\text{mSME}} = \frac{\sqrt{-g}}{2\kappa} [-uR + s^{\mu\nu} R_{\mu\nu}^T + t^{\mu\nu\alpha\beta} W_{\mu\nu\alpha\beta}], \quad (2)$$

where $\kappa = 8\pi G$, $R_{\mu\nu}^T$ is the trace-free Ricci tensor, and $W_{\mu\nu\alpha\beta}$ is the Weyl tensor. In the isotropic limit we can write the above Lagrangian as:

$$\mathcal{L}_{\text{mSME,iso}} = \frac{\sqrt{-g}}{2\kappa} [{}^{(4)}s_{\mu\nu} {}^{(4)}R^{\mu\nu}], \quad (3)$$

where a superscript (4) denotes quantities defined on \mathcal{M} . Here, we focus on explicit symmetry breaking so that dynamical terms in the action vanish.⁸ The above Lagrangian can be rewritten as:

$$\mathcal{L}_{\text{mSME,iso}} = \frac{\sqrt{-g}}{2\kappa} {}^{(4)}s^{\mu\nu} \left[\gamma^\alpha_\mu \gamma^\beta_\nu {}^{(4)}R_{\alpha\beta} + n_\mu n_\nu n^\alpha n^\beta {}^{(4)}R_{\alpha\beta} - 2\gamma^\alpha_\mu n_\nu n^\beta {}^{(4)}R_{\alpha\beta} \right] \quad (4)$$

and by using the Gauss, Gauss-Codazzi, and Ricci equations we can write down the fully decomposed formulation of the gravitational sector (GR + minimal SME):

$$\begin{aligned} \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{mSME,iso}} = & \frac{\sqrt{-g}}{2\kappa} \left[\left(1 + n^\alpha n^\beta {}^{(4)}s_{\alpha\beta} \right) (K^{\alpha\beta} K_{\alpha\beta} - K^2) + \right. \\ & \left. + \left(\frac{1}{3} {}^{(4)}s^i_i \right) \mathcal{R} + \nabla_\mu \left(1 + \frac{1}{3} {}^{(4)}s^i_i - n^\alpha n^\beta {}^{(4)}s_{\alpha\beta} \right) (a^\mu + n^\mu K) \right] \quad (5) \end{aligned}$$

where \mathcal{L}_n is the Lie derivative along the vector field n^μ , D_μ is the covariant derivative associated with the induced metric $\gamma_{\mu\nu}$, ${}^{(4)}s_i^i$ is the trace of the spatial part of $s^{\mu\nu}$, and $K_{\mu\nu}$ denotes the extrinsic curvature of the foliation. Moreover, we define the *acceleration vector* $a_\mu = D_\mu \ln N$ and the three-dimensional Ricci scalar \mathcal{R} . GR is recovered when $s^{\mu\nu} \rightarrow 0$.

3. Discussion & Conclusions

Using standard tools in numerical relativity theory we have derived a 3+1 decomposition of the minimal SME gravity Lagrangian in the isotropic limit. We make no linearised gravity approximations, and thus this is an *exact* result. This complements other exact studies of the SME.⁹ Our results can be used in ongoing work on identifying the dynamical degrees of freedom in the explicit symmetry breaking case and matching to proposed models of quantum gravity.

Acknowledgments

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