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Recent Developments in Spacetime-Symmetry Tests in Gravity

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We summarize theoretical and experimental work on tests of CPT and local Lorentz symmetry in gravity. Recent developments include extending the effective field theory framework into the nonlinear regime of gravity.

1. Introduction

Motivated by potentially detectable but minuscule signatures from Planck-scale or other new physics, there has been a substantial increase in tests of spacetime symmetry in gravity in recent years.[1,2] Some novel hypothetical effects that break local Lorentz symmetry and CPT symmetry in gravitational experiments as well as solar system and astrophysical observations have been studied in recent works.[3] Much of this work uses the effective field theory framework, the Standard-Model Extension (SME), that includes gravitational couplings.[4,5] In other cases, the parameters in specific hypothetical models of Lorentz violation in gravity have been tested.[6]

2. Framework

The general framework of the SME in the pure-gravity sector can be realized as the Einstein-Hilbert action plus a series of terms formed from indexed coefficients, explicit or dynamical, contracted with increasing powers of curvature and torsion. Each term in this series maintains observer invariance of physics, while breaking "particle" invariance, with respect to local Lorentz symmetry and diffeomorphism symmetry.[5]

One interesting and practical subset of the SME is a general description of CPT and Lorentz violation that is provided by an expansion valid for linearized gravity ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$). For instance, in this approximation the Lagrange density for General Relativity (GR) plus the mass dimension 4 and 5 operators controlling local Lorentz and CPT violation are given by[7–9]

$$\mathcal{L} = -\tfrac{1}{4\kappa}(h^{\mu\nu}G_{\mu\nu} - \overline{s}^{\mu\kappa}h^{\nu\lambda}\mathcal{G}_{\mu\nu\kappa\lambda} + \tfrac{1}{4}h_{\mu\nu}(q^{(5)})^{\mu\rho\alpha\nu\beta\sigma\gamma}\partial_\beta R_{\rho\alpha\sigma\gamma} + ...), \quad (1)$$

where $\kappa = 8\pi G_N$, and the double dual curvature $\mathcal{G}$ and the Riemann curvature $R_{\rho\alpha\sigma\gamma}$ are linearized in $h_{\mu\nu}$. This lagrange density maintains linearized diffeomorphism invariance, though generalizations exist[10], and $\overline{s}_{\mu\nu}$ and $(q^{(5)})^{\mu\rho\alpha\nu\beta\sigma\gamma}$ are the coefficients controlling the degree of symmetry breaking (they are zero in GR).

3. Experiment and Observation

The mass dimension 4 Lagrange density, the minimal gravity SME, has now been studied in a plethora of tests. The best controlled and simultaneous parameter-fitting limits come from lunar laser ranging[11], and other laboratory experiments such as gravimetry.[12] These place limits on the $\overline{s}_{\mu\nu}$ coefficients at the level of approximately $10^{-7} - 10^{-8}$ on the 3 $\overline{s}_{TJ}$ and $10^{-10} - 10^{-11}$ on 5 of the $\overline{s}_{JK}$ coefficients. Stronger limits can be countenanced from distant cosmic rays[14], and one combination of coefficients is bounded at 10^{-15} by the multimessenger neutron star inspiral event in 2017.[13] Other searches for these coefficients include ones with pulsars.[15]

For the mass dimension 5 coefficients in (1) that break CPT symmetry, the post-Newtonian phenomenology includes a velocity-dependent inverse cubic force. This leads to an extra term in the relative acceleration of two bodies given by[16]

$$\begin{aligned}\delta a^j = \frac{G_N M v^k}{r^3}\big(&15 n^l n^m n^n n_{[j}K_{k]lmn} \\ &+9 n^l n^m K_{[jk]lm} - 9 n_{[j}K_{k]llm}n^m - 3K_{[jk]ll}\big),\end{aligned} \quad (2)$$

where K_{jklm} are linear combinations of the coefficients q in the lagrange density (1), $\vec{r}$ is the separation between the bodies and $\hat{n} = \vec{r}/r$.

Measurements of the mass dimension 5 coefficients in (2) are currently scarce. There is one constraint on a combination of dimension 5 and 6 coefficients from Ref. 9 in searches for dispersion of gravitational waves from distant sources and analysis with multiple gravitational wave events is underway.[19] Disentangled constraints on the K_{jklm} coefficients from analysis of pulsar observations exists at the level of 10^6 meters.[20] This leaves room for potentially large, "countershaded" symmetry breaking to exist in nature.[21] Higher-order terms in the series, at mass dimension 6 and beyond, have been constrained in short-range gravity tests.[22]

4. Extension to the nonlinear regime

While the general form for linearized gravity has been explored, only several works have explored the general SME framework beyond linearized gravity.[23] One approach is to extend the general lagrange density for linearized gravity (which is quadratic order in the metric fluctuations) to include terms of cubic and higher order terms. If we adopt the point of view of spontaneous symmetry breaking (SSB), one must consider the dynamics of the coefficients for Lorentz violation. Considering the case of a symmetric two tensor $s_{\mu\nu}$ being the Lorentz-breaking field, it is expanded in the SSB scenario as $s_{\mu\nu} = \overline{s}_{\mu\nu} + \tilde{s}_{\mu\nu}$, where $\overline{s}_{\mu\nu}$ are the vacuum expectation values and $\tilde{s}_{\mu\nu}$ are the fluctuations. The Lagrange density is a series $\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + ...$ where (2) and (3) indicate the order in fluctuations $h_{\mu\nu}$ or $\tilde{s}_{\mu\nu}$. A general conservation law[18], contained in equation (9) of Ref. 17, can be used to constrain the terms in the series. In the example of $s_{\mu\nu}$ it takes the form

$$\partial_\beta \left(\frac{\delta \mathcal{L}}{\delta h_{\gamma\beta}} \right) + \Gamma^\gamma{}_{\alpha\beta} \left(\frac{\delta \mathcal{L}}{\delta h_{\alpha\beta}} \right) + g^{\delta\gamma} s_{\delta\alpha} \partial_\beta \left(\frac{\delta \mathcal{L}}{\delta \tilde{s}_{\alpha\beta}} \right) + g^{\delta\gamma} \tilde{\Gamma}_{\delta\alpha\beta} \frac{\delta \mathcal{L}}{\delta h_{\alpha\gamma}} = 0, \quad (3)$$

where $\tilde{\Gamma}_{\delta\alpha\beta} = (\partial_\alpha \tilde{s}_{\beta\delta} + \partial_\beta \tilde{s}_{\alpha\delta} - \partial_\delta \tilde{s}_{\alpha\beta})/2$. This equation holds "off-shell", assuming the action obtained from $\mathcal{L}$ is diffeomorphism invariant.

In the case of the minimal SME with just $s_{\mu\nu}$, the Lagrange density is constructed from all possible contractions of generic terms of the quadratic form $\overline{s}_{\alpha\beta} h_{\gamma\delta} \partial_\epsilon \partial_\zeta h_{\eta\theta}, \tilde{s}_{\alpha\beta} \partial_\gamma \partial_\delta \tilde{s}_{\epsilon\zeta}, \tilde{s}_{\alpha\beta} \partial_\gamma \partial_\delta h_{\epsilon\zeta}, ...,$ the cubic form $\overline{s}_{\alpha\beta} h_{\gamma\delta} h_{\epsilon\zeta} \partial_\eta \partial_\theta h_{\kappa\lambda}, \overline{s}_{\alpha\beta} h_{\gamma\delta} \partial_\epsilon h_{\zeta\eta} \partial_\theta h_{\kappa\lambda}, h_{\alpha\beta} \tilde{s}_{\gamma\delta} \partial_\epsilon \partial_\zeta \tilde{s}_{\theta\kappa}, ...,$ and potential terms. The sum of all such terms, each with an arbitrary parameter, is inserted into (3) and the resulting linear equations for the parameters are solved. What remains, up to total derivative terms in the action, are a set of independently diffeomorphism invariant terms. As an example of such a term produced by this expansion, we find to cubic order

$$\begin{aligned} \mathcal{L} \supset\ & \overline{s}_{\alpha\beta} \tilde{s}^{\alpha\beta} R^{(1)} + \tfrac{1}{2} \tilde{s}_{\alpha\beta} \tilde{s}^{\alpha\beta} R^{(1)} - 2 h^{\alpha\beta} \overline{s}_\alpha{}^\gamma \tilde{s}_{\beta\gamma} R^{(1)} \\ & + \overline{s}_{\alpha\beta} \tilde{s}^{\alpha\beta} (\Gamma_{\alpha\beta\gamma} \Gamma^{\alpha\beta\gamma} - \Gamma^{\alpha\beta}{}_\beta \Gamma_{\alpha\gamma}{}^\gamma + \tfrac{1}{2} h^\alpha{}_\alpha R^{(1)} - 2 h^{\alpha\beta} R^{(1)}_{\alpha\beta}), \quad (4) \end{aligned}$$

where the (1) superscript indicates linear order in $h_{\mu\nu}$ and the connection coefficients are at linear order. Note that this construction generally includes dynamical terms for the fluctuations and so does not assume "decoupling".[24]

The construction including all such terms allows exploration of the regime in gravity where nonlinearities need to be considered.[25,26] This includes higher order post-Newtonian gravity in weak-field systems and developing a multipole expansion for gravitational waves affected by Lorentz violation.

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