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A Measure of the Hurst Exponent Variability on Ground Based Magnetometer Data for Quiet and Active Magnetospheric Periods

Dario O. Cersosimo

Embry-Riddle Aeronautical University - Daytona Beach

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A MEASURE OF THE HURST EXPONENT VARIABILITY ON GROUND BASED MAGNETOMETER DATA FOR QUIET AND ACTIVE MAGNETOSPHERIC PERIODS

By

Darío O. Cersosimo

A Thesis Submitted to the
Physical Sciences Department
In Partial Fulfillment of the Requirements for the Degree of
Master of Science in Space Science

Embry-Riddle Aeronautical University
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Darío O. Cersosimo

This thesis was prepared under the direction of the candidate's thesis committee chair, Dr. James Wanliss, Department of Physical Sciences, and has been approved by the members of his thesis committee. It was submitted to the Department of Physical Sciences and was accepted in partial fulfillment of the requirements for the Degree of

Master of Science in Space Sciences

THESIS COMMITTEE:

Dr. James Wanliss, Chair

Dr. John Olivero, Member

Dr. Mahmut Reyhanoglu, Member

Dr. Hong Liu, Member

MSSpS Graduate Program Coordinator

Department Chair, Physical Sciences

Dr. John Watret, Associate Chancellor, Daytona Beach Campus
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ABSTRACT

Author: Dario O. Cersosimo

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Ground-based magnetometer data were analyzed for the period of 1991-2001. The data were classified into periods of quiet and active magnetospheric activity. Those periods classified as quiet required that Kp ≤ 1 for not less than 48 consecutive hours and active periods required a Kp ≥ 4 for not less than 24 consecutive hours. Detrended fluctuation analysis was employed to analyze 40 events. A monofractal approach was used to identify differences in the Hurst exponent of quiet and active events. No statistical differences were found using this approach since both types of events displayed quasi-random walk behavior. A second approach determined the temporal variations in the Hurst exponent for each event. The Hurst exponent is temporally dynamic -- active events are more correlated than quiet events -- suggesting a multifractional rather than monofractal behavior. The results are useful to suggest an appropriate model of magnetic field fluctuations.
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CHAPTER I

INTRODUCTION

1.1 The magnetosphere

The Earth's magnetosphere is a semi permeable cavity caused by the Earth's dipolar magnetic field confined in the interplanetary plasma environment. Most of the plasma from the solar wind flows around this cavity at supersonic speeds and the pressure balance between the solar wind and the Earth's magnetic field determines the size of the magnetosphere (see Figure 1). On the day side the magnetosphere extends up to 10 Earth radii, while in the night side the magnetotail extends several hundred Earth radii. The boundary that separates the magnetosphere from the interplanetary plasma environment is the magnetopause. Due to the supersonic speeds of the solar wind, a shock is produced in front of the dayside magnetopause (bow shock) generating a layer of compressed, heated and turbulent plasma called the magnetosheath. The magnetosphere is divided into several regions depending on composition, energies and density of the plasmas that populate them. Magnetospheric plasmas consist mainly of protons and electrons with other minor species such as He$^+$, O$^+$ from ionospheric origin, and He$^{++}$ carried by the solar wind [Baumjoham and Treumann, 1996]. The ionosphere is the conducting region of the atmosphere and is the main source of magnetospheric plasma, where ionized species are produced due to the UV and X-ray radiation incident from the Sun. Next in
proximity to Earth are the Van Allen Radiation Belts, which are regions where energetic particles are trapped by the Earth’s magnetic field drift. Such particles are protons, electrons, O⁺, C⁺, H⁺ and He++ ions [Baumjoham and Treumann, 1996]. These particles have energies ranging from 1 keV to over 100 MeV. Most of the energy and density is concentrated at the equatorial plane. At higher latitudes particle loss is caused by interactions with the neutral atmosphere. Approximately at the same region in space as the radiation belts there is a dense cold-plasma population called the plasmasphere. Typical energies in the plasmasphere are ~1 eV and densities of 10³ cm⁻³. At about 3-5 RE in the night side a sharp drop in the proton density marks the plasmapause, the outer edge of the plasmasphere. The size of the plasmasphere increases with geomagnetic activity and the plasmapause become less defined. The magnetotail is a vast region composed of four major plasma regions: the plasma sheet, the plasma sheet-boundary layer, the plasma mantle and the tail lobes.

The plasma sheet consists of particles with energies on the order of 1 keV for electrons and 10 keV for protons. Particle densities are between 0.1 - 1 cm⁻³. During geomagnetic activity, some of the particles are precipitated into the high latitude ionosphere causing auroras. At the same time ionospheric ions are injected into the magnetosphere by mechanisms not well understood [Parks, 2004]. The plasma sheet boundary layer (PSBL) is characterized by confined high-speed ion flow [Eastman et al., 1985]. High-energy ion beams travel along this boundary directed sunward and antisunward. The PSBL and the plasma mantle act as transport regions of the magnetotail [Eastman et al., 1985]. The tail
lobe is a region with rarefied plasma located at the outer part of the magnetotail. Cold ions from ionospheric origin are often observed [Kivelson and Russell, 1995; Baumjoham and Treumann, 1996; Eastman et al., 1985]. The plasma mantle is a region that partially covers the high-latitude magnetosphere. Particle densities range from 0.01 – 1 cm$^{-3}$ with energies on the order of 100 eV.

Figure 1.1 The Earth’s magnetosphere (from Williams et al., 1992).

Stresses applied from the solar wind to the magnetopause produce a system of currents in different regions. The forces caused by these currents distort the magnetic field stretching the field lines downstream into the long magnetotail [Baker, 1997]. The magnetopause current (Chapman-Ferraro current) is directed from dawn to dusk over the dayside.
equatorial plane caused by the ions and electrons that partially penetrate into the magnetosphere. Another current system that separates the antiparallel magnetic field lines at the night side along the equatorial plane is called the plasma sheet current. A third current system, called the ring current, encircles the Earth at the magnetic equator. Particles drift around the Earth across the magnetic field lines. Ions drift westward and electrons eastward. Typical energies range from 10 to 250 keV. Birkeland currents, also known as magnetic field-aligned currents, flow on high latitudes from space to the Earth on the dawn region and then return to space through the dusk region.

1.2 Space weather

The solar wind continuously transfers mass, energy and momentum into the magnetospheric cavity. This transfer is far from steady state even during solar minimum and high changes in energy cause the magnetosphere to move from its relative ground state equilibrium into more exited states where the energy is suddenly dissipated in a series of dynamical processes known as space storms.

Space storms are global geomagnetic disturbances caused by the interaction between solar wind and the charged particles in the near-earth space plasma environment. They are the most dramatic space weather phenomenon that significantly impact modern technology such as satellites, communication and power transmission systems. The development and morphology of space storms is well understood, but discussion is still open regarding the exact causes of magnetic storms [Gonzalez et al., 1994]. It is agreed
that coronal mass ejections (CME), co rotating streams and strong and prolonged values of the southward interplanetary magnetic field (IMF) are among the most important factors in the development of space storms [Tsurutani et al., 1992; Richardson et al., 2001; Huttunen et al., 2003], but these factors alone are not sufficient nor necessary for the storm occurrence or development [Gonzalez et al., 1994; Kamide et al., 1998; Daglis et al., 2003]. Internal magnetospheric conditions, along with coupling and feedback between the ionosphere and magnetosphere, also play important roles in the initiation and development of magnetic storms, and the interaction between these two spheres have been shown to be highly nonlinear [Daglis et al., 2003].

Over the years, several geomagnetic indices have been developed to monitor geomagnetic activity. Within the most used are the disturbance storm time index (Dst), the planetary index (Kp) and the aurora electrojet index with its variations (AE, AU and AL). These indices provide global information about current magnetospheric activity based on different inputs at different locations around the globe. For example, the Dst index measures the strength of the equatorial ring current that causes a decrease in the horizontal component of the magnetic field. It is derived hourly from data collected at four magnetic observatories: Hermanus, Kakioka, Honolulu, and San Juan. These observatories are located at middle and low latitudes around the globe away from the influences caused by the auroral electrojets. For each observatory a baseline horizontal magnetic field is defined by eliminating secular and diurnal variations. For each observatory the disturbance variation $D(t)$ is defined by:
\[ D(t) = \Delta H(t) - Sq(t) \]

Where \( \Delta H(t) \) is the change in horizontal component of the magnetic field and \( Sq(t) \) is the solar quiet daily variation. Finally the Dst is obtained by:

\[ Dst(t) = \frac{D(t)}{\cos \delta_i}, \]

where the denominator is the average of the cosines of the dipole latitudes \( \delta_i \) \((i=1,4)\), of the observatories that contribute to the average. The Kp index, introduced by Bartels in 1949 is used to measure the magnitude of local disturbances in the Earth’s magnetic field component caused by solar particle radiation by “filtering” all other regular and irregular disturbances. The index scale ranges from 0 to 9 in 28 intervals and the indices are standardized for all of the stations (Ks). Conversion tables specific for each station were created in order to obtain the same Ks distributions in all the observatories. Finally Kp is defined as the average of Ks values obtained at 13 observatories distributed at latitudes ranging from 44° to 60° in the northern and southern hemispheres [Kivelson and Russell, 1995]. Figure 1.2 shows the measured Dst and Kp values during the space storm of March 13, 1989. During the early hours of March 13, the Dst index started to decrease rapidly reaching a minimum value of -589 nT on the first hours of March 14. The Kp
reached a maximum of 9 at about the same time, indicative of the intensity and latitudinal extent of this storm.

Figure 1.2 a) Dst and b) Kp values during the space storm of March 13, 1989. During the peak of the storm the Dst reached a minimum value of -589 nT, resulting in the most intense storm of that year causing billions of dollars in damages.

1.3 Space weather prediction

The study and forecasting of space weather has become of major importance in recent years. Space storms caused problems in electrical systems since the mid 19th century (e.g. Boteler et al., [1998]). Since the beginning of the space era, intensive space weather
had caused substantial damage in space and ground-based systems. In consequence navigation, communication, reconnaissance, and energy systems had been damaged causing adverse effects in the economy of the affected regions [Daglis et al., 2003]. The March 13, 1989 storm (Figure 1.2) caused the collapse of the Hydro-Québec power system affecting the power service of 6 million Québec residents and billions of dollars in damage [Allen et al., 1989; Boteler et al., 1998; Odenwald, 2002]. Thus a better understanding of the Sun-Earth connections is imperative in order to develop reliable forecasting and warning systems. Major problems associated with the occurrence of space storms are to identify time, location and intensity of the storm. Once these objectives are met, space weather forecasting will help to prevent damages on sensitive ground-based and space-based systems.

1.3.1 Current attempts in space weather prediction

Attempts in quantitative prediction of geomagnetic activity from solar wind data were published by Russell et al., [1974] and Burton et al., [1975]. They found an empirical relationship between Dst and the product of the solar wind velocity and the southward IMF ($V B_s$), [Kamide et al., 1998]. Linear prediction filters were first developed by Iyemori et al., [1979]. This technique uses solar wind and IMF to predict geomagnetic indices. In this case the product of the solar wind velocity $V$ and the southward magnetic field $B_s$ is the system input and a geomagnetic index such as Dst is the output [Kugblenu et al., 1999; Kamide et al., 1998, and references therein]. Studies using linear prediction
filters showed that the processes governing the solar wind-magnetosphere coupling are mostly nonlinear.

Neural network techniques are computational models with learning ability, i.e. they can be trained by using various examples with common relationships. They were used to predict geomagnetic indices and showed to be successful in the reproduction of the storm phases [Kugblenu et al. 1999; Kamide et al., 1998, and references there]. Other methods focused on energy conservation properties of the magnetosphere. These models are based on statistical energy input functions and global loss time scales, and have been very successful in predicting Dst during storms. An example of this type of method is the Ring Current-Atmosphere Interaction Model (RAM) [Daglis et al., 2003].

The main problem with many of these methods is that they typically use solar wind parameters as input and a geomagnetic index is the predicted output. The solar wind input is typically taken from a single isolated in situ satellite and thus provide an inadequate monitor of solar wind variability over the scale size of the magnetosphere. Geomagnetic indices are calculated from data measured in observatories around the world. As mentioned earlier, the Dst index is calculated from four observatories at mid-low latitudes and on the other hand the Kp index is calculated from observatories at high latitudes. Then the output of these indices will give us an idea of the geomagnetic activity from a global perspective. But if we are interested in the local aspects of geomagnetic activity, i.e. to forecast the geomagnetic conditions for Hydro-Quebec or other power
utilities, we need to develop ways to understand the geomagnetic activity in a more localized way. It has been shown that during geomagnetic activity, intensity depends on geographic location and time [Weigel et al., 2002].

Ground-based indices [Takalo et al., 1999; Wanliss, 2004] and individual magnetometer stations [Vörös, 2000; Weigel et al., 2002; Wanliss and Reynolds, 2003] are clearly an excellent indicator of space weather conditions. Part of the reason is undoubtedly the property of the earth's magnetic field lines to focus and converge as they approach the earth. These field lines extend far into space and since they are connected to the earth, nonlinear plasma processes that occur far away are mapped all the way down to the earth. Observation of ground-based magnetometer stations can thus serve as a remote sensing tool of distant magnetospheric processes.

**1.3.2 What is next?**

Complex geosystems such as the magnetosphere might be modeled through the use of new mathematical and statistical techniques. These techniques involve the development of stochastic models capable of describing Earth's magnetic field time series and other processes via non-Gaussian probability densities. A proper characterization of the behavior of the time series can thus serve to guide the development of future models used in space physics modeling.
1.4 Objectives

Previous analyses have used global statistics to study the differences between quiet and active magnetospheric times [Wanliss 2004, 2005]. The problem with this is that space weather is not homogeneous around the Earth and so global studies only give an average behavior rather than local information.

In this thesis my objective is to characterize changes in the nonlinear statistics of the Earth’s magnetic field, by means of the Hurst exponent, measured from a single ground-based magnetometer station. Analysis is performed as a function of the geomagnetic activity. The goal is to gain an understanding of the local behavior of the magnetic field, for differing geomagnetic activity. I expect that the highly nonlinear processes involved in the magnetospheric dynamics can be approached using ideas from statistical physics. Segments of the local magnetic field time series can be analyzed and characterized based on its correlation properties. The level of correlation in time may indicate if the series has dependence from previous events or it follows a random walk behavior. The changes in the statistics can be used as a local indicator of the magnetospheric conditions, which may be useful to develop reliable warning and forecasting systems using information not available in geomagnetic indices.

A second objective is to determine the long-range statistical behavior of the geomagnetic field at a local observation site. If the time series can be described as a particular statistical process – Brownian motion for example – then this knowledge can be used for
future space weather modeling purposes. The statistical structure of the magnetometer time series will provide key clues for the development of mathematical models of these time series.

The work performed and the results found are presented in the following pages. Chapter II gives an introduction to the concept of Brownian motion with its variations and the method to be employed to analyze the series. Tests using synthetic fractional Brownian motion (fBm) and multifractional Brownian motion (mfBm) data are performed to determine the abilities of the method and to draw a picture of the results that we might expect from real data. In the last section a brief summary comments on how the results obtained in this chapter can be employed for the analysis of the magnetic field data in the next chapter. Chapter III describes the criteria used to select the data and the different approaches used to analyze the data. Results are presented and briefly discussed. Finally Chapter IV discusses the results and compares them with those obtained by different authors. A brief discussion about future research is presented in the last section of this chapter.
CHAPTER II

METHOD OF ANALYSIS

2.1 Brownian motion

The motion of small particles suspended in a fluid due to the collisions with molecules that have a Maxwellian velocity distribution follows a random walk process. Jan Ingenhousz first observed this process in 1785, but in 1827 Robert Brown rediscovered it when he was looking at pollen grains suspended in water through a microscope lens. Since then such a random walk process is known as Brownian motion. Albert Einstein developed the first mathematical explanation of Brownian motion in 1905 using kinetic theory. Brownian trajectories are continuous and of infinite length between any two points. Brownian motion can also be found in time series where a particular process varies randomly with time. The center panel of Figure 2.1 shows an example of synthetic Brownian motion data. Here the signal is generalized by its amplitude variation as a function of time. The probability that the amplitude of this signal increases in the next interval of time \((t+\Delta t)\) is 50%. In other words, it is an uncorrelated process where a future event is not influenced by the present event. Brownian motion has self-affine scale invariance, which implies that its statistical properties are not affected by changing the time scale by a factor \(T\) and rescaling the spatial coordinates by a factor \(T^H\), where \(H=1/2\)
[Hergarten, 2002]. Here H is called the Hurst exponent, also known as the self-similarity index.

2.2 Fractional Brownian motion

Fractional Brownian motion (fBm) was first introduced by Kolmogorov, [1940] and Mandelbrot and Van Ness, [1968]. It is the generalization of Brownian motion with an arbitrary H value different than 1/2. This means that the probability of an event next to a previous one is not 50%. The exponent H determines the probability for a particular event to occur as well as the roughness in the signal. Notice that in figure 2.1 the smoothness of the plot increases as H does. The more persistent the time series, the more defined the trend becomes.

Figure 2.1 Synthetic fBm data for different Hurst exponents: H=0.3 (top), H=0.5 (center) and H=0.8 (bottom). The method used to generate these data has been taken from code written by Coeurjolly [2000].
Specifically, fBm is the process defined as the fractional integration of Gaussian white noise. When a particular time series has $H > 1/2$, it exhibits long-term persistence and memory, which implies that there exist correlation between events along different time scales.

Generally, if a process is increasing it will probably continue to increase at the next time interval $t+\Delta t$ (Figure 2.1). On the other hand if $H < 1/2$ there is antipersistence in the time series. In other words, if the process is increasing it will probably decrease in the next time interval $t+\Delta t$. Figure 2.1 shows various examples of fBm for different H values. Notice that as H increases the plots are smoother and the trends become stronger due to the strong correlation for H values closer to 1.

2.3 Detrended fluctuation analysis

Coupling between the solar wind and the magnetosphere is inherently nonlinear but also nonstationary. Traditional techniques such as the power spectrum are unable to distinguish between stationary and non-stationary data and require infinite length time-series. Wanliss [2004, 2005] used detrended fluctuation analysis (DFA) to determine the scaling properties of the symmetric-horizontal (SYM-H) index during active and quiet magnetospheric activity over two solar cycles. He found strong evidence that different statistical processes govern the SYM-H index behavior during quiet and active intervals in the magnetosphere. SYM-H is essentially a high-resolution version of Dst described in Chapter I (section 1.2).
DFA is a novel technique developed by Peng et al., [1995] to accurately quantify long-range power-law correlations embedded in nonstationary time series [e.g. Chen et al., 2002]. The technique consists of a modified root mean square analysis of the random walk, designed specifically to detect long-range correlations in non-stationary nonlinear data.

In DFA the time average of the time series is subtracted from the original series and then it is integrated:

\[ y(k) = \sum_{i=1}^{k} [B(i) - \bar{B}] \]

\( B(i) \) is the value of the magnetic field at the \( i^{th} \) time interval and \( \bar{B} \) is the average value of the magnetic field in the time series. Once the series is integrated, it is divided into boxes of equal size \( n \), as shown in Figure 2.2.
Figure 2.2 The time series is divided into boxes of equal size $n$ and then a least squares line is fit to the data in each box (dashed lines). These lines represent the local trend (from Wanliss, 2004).

In each box a linear least squares line is fit to the data, representing the trend of the series in that particular box. The next step is to remove the local trend $y_n(k)$ of the new time series $y(k)$ in each box. In Figure 2.2 linear trends are shown (dashed lines) but in principle higher order polynomial trends can also be used. The characteristic size of the fluctuations $F(n)$, is then calculated as the root mean squared deviation between $y(k)$ and its trend in each box,

$$F(n) = \sqrt{\frac{1}{N} \sum_{k=1}^{N} [y(k) - y_n(k)]^2}$$
The process is repeated over all time scales (box sizes). The presence of scaling is indicated by a power-law relationship between $F(n)$ and $n$

$$F(n) \propto n^a$$

where $a=H$, within the set $[0,1]$, is the scaling exponent. The slope of a log-log plot of $F(n)$ vs. $n$ indicates the value of the scaling exponent. If $a=0.5$ then the signal is white noise. A value of $a<0.5$ indicates that the data is uncorrelated (random walk) and if $0.5<a<1$ then there is correlation in the time series (i.e. long term memory). When $a=1$ the series is $1/f$ noise and $a>1$ indicates that the series is correlated but is no longer of the power-law form. If $a=1.5$ it indicates Brown noise which is the integration of white noise [Peng et al., 1995].

2.4 Testing the method

In order to develop experience with the numerical analysis, several data sets with known scaling exponent were first analyzed. Synthetic fBm data of size $N$ was generated with different scaling values using Wood-Chan's circulant method [Wood and Chan, 1994] and then analyzed using the DFA algorithm to test different properties. Coeurjolly, [2000] describes and analyzes the method of Wood-Chan used to generate synthetic fBm. It is showed to be fast and accurate in the generation of fBm for large values of $N$ [e.g. Coeurjolly, 2000].
The data was tested by comparing the fractional error between the expected scaling value of the synthetic data and the results returned by DFA. The trends were removed by applying linear and quadratic polynomial fits to the series. The percent of difference between these values decreases significantly after $10^4$ points and no marked differences exist when comparing linear and quadratic fits.

The analysis was done for 50 different fBm series for values of $H$ ranging from $H=0.4$ to $H=0.9$ in 0.1 increments. Each of the 50 series were analyzed by DFA and then determined the fractional error,

$$
diff = \frac{|H - H_{DFA}|}{H},$$

and the relative error,

$$\rho = \frac{dH_{DFA}}{H},$$

where $dH$ is the uncertainty in the determination of $H$. The average in the error calculations was determined as a function of the number of points ($n$) in the series for different $H$. Figure 2.3 top panel shows the averaged fractional error between the expected $H$ value generated by Wood-Chan's fBm generator and the value returned by DFA. The panel at the bottom shows the relative error in the DFA calculation. Both plots
show that the error is about 0.2% or less after \( n \geq 10^4 \). In general, the error in the calculated exponent increase as the length of the time-series is reduced. A similar procedure was applied using a second order polynomial to remove the trends, but it did not show a significant difference at the expense of the higher computation time it required.

Figure 2.3 Top panel displays the averaged fractional error between the expected H values generated by Wood-Chan's fBm generator and the value returned by DFA (H ranges from H=0.4 to H=0.9 in increments of 0.1). Bottom panel shows the relative error in the DFA calculation. Both plots show that the error is about 0.2% or less after \( n \geq 10^4 \).
2.4.1 Synthetic fractional Brownian time series

The sample in Figure 2.4 represents fBm data with $H=0.7$ for 40000 data points (top left). The Hurst exponent for the entire data set was determined using DFA (bottom left). Here the fluctuation versus the box size is clearly linear on a log-log plot, indicating fBm. Recall that the slope of this line gives the Hurst exponent $H$. The results returned by DFA are in good agreement with those expected from the fBm generator.

One can also analyze the scaling exponent as a function of time. This is done by a) selecting the window size for which $H$ will be calculated. b) Calculating $H$ that corresponds to a certain time $t$: $H(t)$. c) Stepping the window a certain distance $\Delta t$ along the original time series. d) Taking a new subsection of the series and calculating $H(t+\Delta t)$. e) Repeat steps (b) – (e). The top right panel in Figure 2.4 shows the variation in the Hurst exponent as a function of time for different lengths of the data window. Windows of different sizes represent the number of data points used to determine the Hurst exponent at a particular interval of time in the series (i.e. 5000, 10000, 15000 and 20000). Here is clearly shown that fBm has constant scaling properties along the time axis as expected. In this plot the value of the Hurst exponent oscillates around the expected value ($H=0.7$). These small oscillations may be caused by the size of the data window, or the accuracy of the fBm generator. The error is shown in the bottom right panel of figure 2.4. Here we observe that on average the error decreases as the windows size increases. A window size of 5000 results in significant error.
Figure 2.4 Synthetic fBm data generated using Wood-Chan’s circulant fBm generator. The top left panel shows the synthetic time series with a Hurst exponent of $H=0.7$. The bottom left panel shows the results obtained from DFA, $H=0.70\pm0.02$. On the top right panel are the results for the time dependent DFA for different window sizes (5000, 10000, 15000 and 20000). The bottom right panel shows the uncertainty $dH$ in the determination of $H(t)$. Notice that as the window size increases $dH$ is smaller.

Figure 2.5 (left) shows how the mean value of $H(t)$ changes with window size. We observe that the mean approaches the expected value as the number of data windows increase. Also the mean error decreases as the window size increases. These results may suggest that mean $H(t)$ approaches to the actual $H$ as the window sizes approaches to the size of the entire time series. The results suggest that to have errors $dH < 0.03$, a window size larger than 5000 points must be used. In this work, for all future calculations of $H(t)$, a window size of 10000 will be used.
Figure 2.5 The left panel shows the averaged $H(t)$ value for the different window sizes. On the right panel the average $dH$ is displayed as a function of the window size. Notice how the uncertainty $dH$ approaches to the value determined for the entire time series (Figure 2.3 bottom left panel), while the mean $H$ value seems to approach the Hurst exponent shown in the same figure. Note: the actual value should be $H=0.7$.

2.4.2 Synthetic multifractional time series

Multifractional Brownian motion (mfBm) is an extension of fBm where its properties vanish with time i.e., mfBm is non-stationary and the process is not self similar and it may have an infinite number of scaling exponents. Synthetic mfBm data was generated and then analyzed using DFA. The simplest kind of mfBm is obtained by stitching together series of fBm that have different Hurst exponents. The synthetic series was divided in three equal segments with different scaling values chosen arbitrarily for each segment.

Figure 2.6a shows a synthetic time series generated with three different scaling values, $H=0.3$, $H=0.8$ and $H=0.6$. The landscape roughness changes as the scaling value changes. Smoother patterns are caused by higher values of $H$ while rough patterns are
caused by lower H values (i.e., H=0.3). In panel (c) the Hurst exponent of the entire series is determined to be H=0.35 ± 0.05. Several tests were performed with this method and the results showed that the Hurst exponent is strongly influenced by the segment in the series that is least correlated (i.e. with the smallest Hurst exponent). A more detailed analysis is shown on panel (b) of Figure 2.6 where the value of H was determined as a function of time. Here the changes in scaling values are well defined. Clear jumps can be observed when H changes from H=0.3 to H =0.8 and again from H=0.8 to H=0.6. In this subplot the H(t) values were calculated for different window sizes. These jumps occur at different regions on the series depending on the size of window. For a window size of 5000 points the delay occurs around 2000 points after the Hurst exponent changes from H=0.3 to H=0.8. For larger window sizes the delay is larger and accuracy in the temporal DFA seems to be reduced. Notice that the segment with H=0.8 is not reflected in the results shown in panel (b) when the window size is 20000. For the transition from H=0.8 to H=0.6 it can be observed that no delay exists in the determination of H(t) among the different window sizes. This might suggest that DFA is highly influenced by relative lower H values in the series. On panel (d) the error in the Hurst exponent is plotted as a function of time. Notice that the shape of the four plots is similar, except for a delay factor. The uncertainty is relatively low from t=1 to t=20000; after this point the global uncertainty rises at different rates inversely proportional to the window size. The maximum of the peaks also seems to be inversely related to the window size and a sharp decrease in the uncertainty occurs when H(t) in panel (b) becomes stable. Finally when H(t) changes from H=0.8 to H=0.6 the uncertainty dH(t) follows the same process but
with a negative slope. Notice that the uncertainty determined for a window size of $n = 5000$ stabilizes faster than the uncertainty determined using larger window sizes but also it has higher peaks, i.e. larger uncertainty.

![Multifractional synthetic data](image)

**Figure 2.6** (a) Synthetic multifractional Brownian motion time series for three Hurst exponent values: $H=0.3$, $H=0.8$ and $H=0.6$. (b) Time dependent DFA of the synthetic time series using window sizes of 5000, 10000, 15000 and 20000. Panel (c) shows the results obtained for DFA of the entire time series. The Hurst exponent showed a value of $H=0.35\pm0.05$ inconsistent with those values obtained for the time dependent DFA. In panel (d) we plot the uncertainty in $H(t)$ for different window sizes.
2.5 Test results

The previous tests show the results when DFA is applied to fractional and multifractional Brownian data. Looking at the roughness of the time series, we can observe clear differences between both sets. First of all, we can easily distinguish different H values by looking at the roughness of the data. Constant roughness occurs in fBm series while in the mfBm the roughness can change with time (discrete changes at 2 points in time are shown in Figure 2.6a). The H(t) plots in figures 2.4 and 2.6 top right panels are good indicators of the value of the Hurst exponent as a function of time but we can observe that the error dH(t) increases when changes in the Hurst exponent occurs. On the other hand, when H is calculated for the entire event we obtain good accuracy with fBm data but for mfBm the analysis is strongly influenced by the lowest H value in the time series. These results are consistent with figures 2.5 and 2.7. In Figure 2.5 the error dH decreases as the window size increases in fBm data, but the opposite occurs with mfBm data (Figure 2.7). As the window size increases, the error dH also increases but at the same time the average H(t) linearly decreases. These results strongly suggest that DFA results from mfBm data must be treated with caution.

In the first place we find DFA of the entire data set does not give useful information about the temporal scaling fluctuations in the mfBm data set (e. g. Figure 2.6c). Moreover, the results of the temporal DFA for mfBm are influenced by the window size to a higher degree than those results obtained for fBm (Figure 2.7). Thus there is the
question about how to determine the appropriate window size in order to accurately calculate the value of H in mfBm data.

Figure 2.7 The left panel shows the averaged H(t) value for the different window sizes (i.e. 5000, 7500, 10000, 12500, 15000 and 20000). On the right panel the average dH(t) is displayed as a function of the window size. Notice how the mean uncertainty dH increases beyond the value determined for the entire time series (Figure 2.4 bottom left), while the mean H value seems to approach the Hurst exponent shown in the same figure (H=0.35, dH=0.05).

2.6 Operational tools for space weather analysis

In this chapter we have studied the properties of fBm and mfBm in order to develop operational tools for space weather analysis and prediction based on a nonlinear statistical approach. In summary, we performed similar analysis as in section 2.5 for mfBm with different permutations of H for the segments of the whole series. We find that:

- If the series is made of 3 different fBm segments, the calculated H for a mfBm series trends towards the least correlated Hurst exponent found in a subset of the entire series.
• H(t) experiences impulsive jumps at times when the Hurst exponent changes in the original series.

• A large error dH(t) occurs at the transition between different Hurst exponents in the original series.

• The slope of the error at the transition between different Hurst exponents is positive or negative if the Hurst exponent in the original time series increases or decreases, respectively.
CHAPTER III

ANALYSIS AND RESULTS

3.1 Introduction

This section describes the criteria used for the selection of space physics data and data analysis. We analyzed the ground magnetometer data obtained from the Canadian Space Agency (CANOPUS) magnetometer array during periods of quiet magnetospheric activity and periods with moderate to high magnetospheric activity. The goal is to understand the statistical differences between ground magnetometer data during quiet and active intervals. Since magnetic fields from the earth map out to space, the variability of the signal acts as a remote sensor of behavior along the field line and gives information about space weather.

3.2 Data selection

We chose the Kp index, described previously in section 1.2, as our indicator of the level of magnetospheric activity. This index has the property that it is derived from measurements taken at the same latitudes were the data to be analyzed in this work is gathered. Several methods for the classification of geomagnetic activity using geomagnetic indices have been proposed and used by different authors. Bartels, [1963] used the criteria for selecting quiet and active events based on Kp values. In general, he
proposed that a \( Kp \leq 1 \) is indicator of quiet periods and a \( Kp \geq 4 \) indicates disturbed periods [Bartels, 1963; Rangarajan and Iyemori, 1997]. Gosling et al., [1991] uses \( Kp \) to classify several levels of geomagnetic activity in more detailed fashion ranging from small storms (typically substorms) to major storms. In this work our interest focuses on two geomagnetic states: active and quiet. Thus we adopt Bartels [1963] method to classify levels of geomagnetic activity.

\[\text{Figure 3.1 Mean Kp values for all the events analyzed from 1991-2001.}\]

Data selected for quiet times (QT) were based on those periods between 1991 and 2001 where the \( Kp \leq 1 \) for not less than two days. The average length of the quiet events selected was 2.6 days. On the other hand, active events were selected from those periods
of time having a Kp ≥ 4 for no less than a day. Twenty active events matching these criteria were selected with an average length of 2.2 days. The length of each event is determined only by continuous intervals where the Kp matches the criterion. Once the Kp value moves outside the criterion, it sets the boundaries to that particular event. Figure 3.1 shows the mean Kp values of all the selected events, active and quiet, chronologically from 1991 to 2001. Most of the active events are close to a solar maximum (1991) while the majority of the quiet events occur within solar minimum (1997).

We thus selected a total 40 events with the given criteria using the CANOPUS Fort Churchill Station (FCHU) as the primary data source. The reason behind the selection of this source is its geographic location (58.76N and 265.91W), which is frequently in the auroral oval. This location has the particular advantage that the data for the selected events is consistent with the Kp index. Moreover, in future these events can also be studied using photometer and riometer data broadening the range of the analysis. Data from this station are consistent and also it is part a longitudinal array of magnetometers, which may be useful to provide a future comparative aspect in the data analysis in conjunction with data from other stations located at the same meridian.

3.2 Data conditioning

Prior to applying DFA the magnetic field components were convolved to obtain the total magnetic field. Data gaps were found and removed from the time series and then the series was stitched together. Previous studies reported by Chen et al., [2002] showed that
performing DFA on highly uncorrelated signals (i.e. $H \approx 0.1$) are more affected by removed segments than highly correlated signals (i.e. $H \approx 0.9$). As more segments are removed in non-correlated signals they suffer from crossover becoming white noise ($H=0.5$) [Chen et al., 2002].

In order to limit spurious results we only considered events with small data gaps. The mean amount of gaps is about 699 for QT representing an average 1.36% of information lost. The active events have a larger mean of data gaps than quiet events (i.e. 940) and the average event length is 2.2 days. Only two events have significant amount of data gaps (i.e. ~17% of data is lost). The remaining 18 events have 0.4% of gaps, which represents a small amount of data loss. These provide confidence that statistical analysis will not be largely deteriorated by the incomplete data set. Once the data was conditioned, DFA was performed in two different ways. First DFA was implemented for the entire event in order to determine the Hurst exponent for the entire data set. The second analysis examines time fluctuations on the scaling exponent.

### 3.2.1 Power spectrum density

The presence of possible scaling in natural signals can be partially determined by implementation of a power spectral density analysis. The slope $\beta$ obtained from the plot of PSD vs. frequency determines the level of correlation of the signal $P(f) \propto f^{-\beta}$. If $-1 < \beta < 1$ then the signal is fractional Gaussian noise (fGn) and it is stationary, which means that the signal is statistically invariant by translation in time. If the signal is fBm, it
exhibits power-law scaling with slope $1 < \beta < 3$. In this case the signal is nonstationary but has stationary increments over a range of scales. The relationship between $H$ in fBm and $\beta$ is given by

$$H = \frac{1}{2}(\beta - 1)$$

The results in Figure 3.2, which show power spectral density on a log-log scale, represent the PSD analysis of the bulk magnetic field measured at Fort Churchill station for two selected active and quiet magnetospheric events. The left panel displays the quiet event of 1999 starting the day of year 31, and the right panel displays the active event of 1995 starting on the day of year 122. Both results suggest that the signals are fBm with $H=0.40$ and $H=0.45$ respectively but, further analysis is needed to determine more accurately the scaling exponent of these signals.

Figure 3.2 Power spectral density plot for the quiet event of 1999, day of year 31 with $\beta=1.8$ (left) and active event of 1995 day of year 122 with $\beta=1.9$ (right).
Estimates of $H$ via PSD are poorly accurate in determining the scaling properties of nonstationary signals. This is due mainly because PSD is unable to distinguish between stationary and nonstationary signals. An example of this case is in Figure 3.3, which shows PSD plots for two different signals: a) is a stationary signal where two different frequencies are present at all times and c) is a nonstationary signal with two different frequencies. The power spectrum analysis (panels (b) and (d)), is similar for both signals and no information is present regarding to the different natural behavior of these two time series. Based on this reason it is suggested that power spectrum should not be used as the only tool in the analysis of nonstationary time series [Stanley et al., 1999]. We use DFA to determine the values of $H$, since it is not affected by nonstationarity in the time series.

![Figure 3.3](image)

**Figure 3.3** Panel (a) represents a stationary time series of two intrinsic different frequencies. Panel (b) represents the power spectral analysis for the signal on panel (a). In panel (c) a nonstationary signal is created and its power spectral analysis is displayed on panel (d). Observe that these two signals differ physically but their power spectrum is almost identical.
3.3 Whole event analysis

Detrended fluctuation analysis was applied to determine the scaling exponent of quiet and active events. Once the data was conditioned, the DFA algorithm was implemented on each event. For each of the 20 quiet and active events selected from the FCHU ground magnetometer station, an analysis of its integral scaling properties was performed. The purpose of this type of analysis is to see if there is correlation within the time scale of a whole particular event. This section summarizes the results obtained for the analysis of quiet and active times, where DFA was performed over the entire event.

3.3.1 Dependence of the event length

The size of the analyzed data set extends to the entire period where the Kp conditions are met. Consequently this produces a wide range of event lengths. This subsection briefly presents an analysis to determine whether the different sizes of the data affect the results from DFA.

We looked at the variations of the scaling values as a function of the event length for quiet and active times. Figure 3.4 shows the results for quiet times and active times for the entire event analysis. The plot clearly shows that the distributions of H are not affected by the duration of a particular event and for both, quiet and active, a range of H from about 0.4 – 0.6 is found.
Figure 3.4 Distributions of the Hurst exponent for quiet and active events vs. the event length. No direct evidence was found suggesting that the Hurst exponent determined is affected by the event’s length. The average Hurst exponent for quiet times is $<H_Q> = 0.52 \pm 0.06$ and for active times $<H_A> = 0.51 \pm 0.05$.

3.3.2 Quiet events

For each of the 20 quiet events selected from FCHU ground magnetometer station, an analysis of its integral scaling properties was performed. The purpose of this type of analysis is to see if there is correlation within the time scale of a particular event.

As an example, solar wind and IMF conditions for the quiet event of 1999, day of year 31.2 are shown in Figure 3.5. Panels (a) to (c) show the three components x, y and z of the IMF respectively, the solar wind velocity shown in panel (d) decreases steadily as the quiet event develops in time. The relatively high Dst index (panel e) indicates that no significant ring current has developed, an indicator that magnetospheric activity is low at
middle latitudes. On panel (f) the Kp index indicates very low values of geomagnetic activity at auroral latitudes.

Figure 3.5 Solar wind conditions measured for the quiet event of 1999, day of year 31. Panels a) to c) show the three x, y and z components of the IMF respectively, d) show the solar wind bulk speed, e) shows the Dst index and f) displays the value of the Kp index during the event.
Once the magnetospheric conditions were placed into context the next step is to proceed with the analysis of statistics for this particular event. Figure 3.6 shows the values for the Hurst exponent obtained for all the quiet events. The results from Figure 3.6 display a wide range of values that includes antipersistent fBm to persistent fBm with a mean $H_{Q1}=0.52\pm0.06$. Figure 3.7 shows the plot of the power law relationship for a single event. The average slope in the plot indicates the value of the Hurst exponent $H$. It is evident that $H$ (slope) is not constant along the entire series, it starts with a relative high correlation and as $n$ increases the correlation is lost. This loss of correlation is called crossover [e. g. Chen et al., 2002]. Crossover is the loss of memory properties for a given time scale. For quiet events it seems that memory properties are lost after a certain $n$ and thus little knowledge can be obtained by looking at the event as a whole.
3.3.3 Active events

Before getting submerged in the results of DFA for the active events, let’s present briefly the space weather conditions dominating within a particular active event. The solar wind conditions and IMF condition for the active event of 1995, day of year 122.5 are presented in Figure 3.9. Panels (a) to (c) show the x, y, and z, components of the IMF. In this case the x and y components of the IMF appear to be moderately disturbed while the z component intermittently oscillates with peak to peak amplitudes of about 5 nT. The solar wind velocity (panel d) shows moderate values near 700 Km/s for nearly the entire event. On panel e) the Dst displays values near −50 nT indicative of occurrence of small to moderate storm level [Gonzalez et al., 1994] and in panel (f) the Kp index rises up to 6.
within the first five hours of this active event. Notice that at no time does Kp have values that go below 4.

Figure 3.8 Solar wind conditions measured for the active event of 1995, day of year 122.5. Panels a) to c) show the three x, y and z components of the IMF respectively, d) show the solar wind bulk speed, e) shows the Dst index and f) displays the value of the Kp index during the event.
Results obtained for active events when DFA is applied to the whole event showed the same characteristics than those obtained for quiet events. Figure 3.9 displays the Hurst exponent determined for all the active events analyzed. The mean Hurst exponent was $H_{AT} = 0.51 \pm 0.05$ for 20 events, consistent with a random walk. The average length for active events is 2.1 days. On Figure 3.10 the linear fit shows the fluctuation versus box-size for an active event measured at Fort Churchill on 1995 day of year 122. The slope of this curve in log-log space gives Hurst exponent H. As for quiet events, the results for the entire interval show the presence of crossover, which may cause the low correlation seen so far.

![Active Events Scaling Values](image)

**Figure 3.9** Hurst exponent for all the active events analyzed in chronological order.
3.3.4 Discussion of the crossovers

So far the analysis for the entire event does not show any significant differences between quiet and active times (i.e., $H_{QT}=0.52\pm0.06$ and $H_{AT}=0.51\pm0.05$). However we find that in all cases crossover occurs for quiet and active times in slightly different fashion. Visual inspection of figure 3.7 suggests that the logarithmic curve, which slope represents the estimated Hurst exponent for that particular quiet event, can be approximated by two straight lines of different slopes. The point where these two lines intersect might represent the breaking point; i.e. where the statistics changes. Active events are somewhat more complicated since, as displayed in figure 3.10, the curvature seems to be

Figure 3.10 Detrended fluctuation analysis for the active event of 1995, day of year 122 at Fort Churchill magnetometer station. The Hurst exponent for that event was determined to be $H=0.51\pm0.05$. 

![DFA active event during 1995 days 122.5 to 124.3 @ FCHU](image)

$F(n)$

$H = 0.5079$

$\Delta H = 0.0520$
more constant and a natural breaking point is harder to identify for most of the active events because it simply may not exist.

The intersection point was determined by two linear least square fits of the data at approximately $n=10$ to $n=100$ and $n=200$ to $n=1000$ in all the events. The intersection of these best fits gives the approximated crossover point. The results were averaged for each type of event — quiet and active — and the error was determined via the standard deviation. For quiet events it was found that the mean breaking point occurs at $<n_{QT}> = 107\pm 20$, and for active events happens at $<n_{AT}> = 118\pm 13$.

While physical meaning of this analysis may not be clear, the presence of crossover may be an indicative that the determination of the Hurst exponent of an entire particular event is not an accurate way to determine differences between the statistics of quiet and active events. In support of this reasoning we have that the averaged scaling values for each event is basically the same, $H=0.5$. To clarify this idea the students-t test was employed to determine whether the quiet and active values come from the same population. It was found $p=0.7830$, $t=0.2774$. These results imply that there is no difference in the statistics for quiet and active events and variations are mainly due to random processes. This result is surprising because when we look at the distribution functions, quiet and active events are significantly different. Figure 3.11 displays the distribution function for quiet and active times. The dot-dash curve represents the averaged distribution function for 20 active events while the solid curve represents the 20 quiet events analyzed. The
distribution functions of the events analyzed does not fit into a Gaussian because the presence of heavy tails on both data sets.

![Distribution functions for the active and quiet events analyzed](image)

**Figure 3.11** Distribution functions for the active and quiet events analyzed. The solid line represents the averaged distributions for the 20 quiet events and the dash-dot line represents the averaged distributions for the 20 active events.

3.4 Time dependent analysis

Since we found no difference between quiet and active behaviors for whole event analysis, in this section we perform time dependent analysis by looking at the evolution of the series to see if information is buried in the details. In the previous chapter we examined the behavior of data in which H changes in patches along the time series.
Typical fBm data has constant scaling values for different lengths of the series as shown in section 2.4.1. However mfBm data will show fluctuations in the value of H as a function of time. In order to get more information about a single event we propose a new novel method of implementing DFA. This method consists in observing the behavior of H as a function of time (H(t)) as was introduced in the previous chapter (section 2.4.2). Figure 3.12 shows the distributions of the average H(t) determined for each event as a function of the event length. As previously demonstrated for the whole event analysis, no evidence is found that the average time dependent value of the Hurst exponent depends on the overall length of the raw series.

![Time averaged Hurst exponent](image.png)

**Figure 3.12** Distributions of the time average Hurst coefficients for quiet and active events vs. the event length. No direct dependences were found that the average Hurst exponent is affected by the duration of the event. The average Hurst exponent for quiet times is $<H_{QT}>=0.73\pm0.05$ and for active times $<H_{AT}>=0.87\pm0.06$. 
In this section we implement a time dependent DFA with steps of 100 points to determine \( H(t) \) as explained in chapter 2.4.2. A window size of 10000 data points is used to determine the \( H(t) \) and the uncertainty \( dH(t) \) at the time corresponding to the end of the segment. The process is repeated along the entire data set.

**Figure 3.13** Magnetic field measured at Fort Churchill station for the active time of 1999, day of year 31 (top). Hurst exponent \( (H(t)) \) determined by the time dependent DFA (center) and uncertainty \( (dH(t)) \) in the \( H(t) \) calculation (bottom).
3.4.1 Quiet events

In Figure 3.13 we observe the temporal variations in H for the quiet event of 1999, day of year 31 previously shown in Figure 3.8. The top panel displays the mean of the bulk magnetic field, the center panel shows H(t) and the bottom panel is the uncertainty dH(t) in the determination of H(t) for each interval. According to the techniques developed in Chapter 2 we can discern several changes in the Hurst exponent via the jump in dH around 1.2, 1.8, 3.0 and perhaps 3.6x10^4 in the time variable. These changes are not as clear in H(t), but this is consistent with what we learned from chapter 2, namely, sensitivity is most clearly indicated by dH(t).

![Time averaged scaling values during quiet events](image)

**Figure 3.14** Time averaged Hurst exponent for all the quiet events analyzed.
The values for $H(t)$ fluctuates within a $H=0.6$ to $H=0.9$ but with a consistent $H > 0.5$. For all the quiet events analyzed the time averaged Hurst exponent, $H(t)_{QT}=0.73 \pm 0.05$ was higher than $H_{QT}$ obtained for the DFA from the entire time series. Figure 3.14 shows the mean values of $H(t)$ and dH(t) for each quiet event.

### 3.4.2 Active events

Active events were defined as those events with $Kp \geq 4$ for at least a 24-hour period. Using the same criterion explained at the beginning of this section, it was found that the time dependent analysis showed greater correlation than the whole series analysis for the active events. Figure 3.15 shows the magnetic field (top) for an event that lasted 1.8 days with mean $Kp=5.2$. The Hurst exponent for the first 10000 points showed high correlation $H(t)\sim0.9$ (center panel). This oscillates along the entire event while the uncertainty is $\sim20\%$ and varies in a similar manner than $H(t)$. A smooth transition takes place during the first 25000 points of $H(t)$ and dH(t). This type of behavior might be due to a gradual change in the Hurst exponent during that period. Notice how the roughness of the magnetic field changes (Figure 3.15 top). After interval number 25000 more dramatic changes in $H(t)$ occur as can be seen in the center and bottom panels of Figure 3.15.
Figure 3.15 Magnetic field measured at Fort Churchill station for the active time of 1995, day of year 122 (top). Hurst exponent determined by the time dependent DFA (center) and uncertainty in the H(t) value (bottom).

Figure 3.16 displays the time averaged Hurst exponent for the twenty active events analyzed. Here the events appear to be highly correlated with an average Hurst exponent of $H(t)_{AT} = 0.87 \pm 0.06$. 
3.5 Summary

Here we comment on the results obtained for quiet and active events. It was found that non-time dependent analysis of quiet and active events has poor correlation while the time dependent analysis shows, in average, higher correlation but also significant fluctuations. The time independent whole event results for QT and AT were all similar irrespective of length of the series. Both types of events resulted in $H \approx 0.5$ for the time independent analysis indicating a random walk.

Probably a major interrogative concern regards the results obtained in the DFA for the entire event. The evident crossover shown in Figures 3.7 and 3.10 suggest that memory properties are not conserved along the time scale of a particular event and they vanish. In
other words, it seems that short-term memory dominates over the long-term memory relative to the event scale length [Peng et al., 1995]. Cannon et al., [1997] found that when real time signals are contaminated with white noise, the estimates of H are biased towards $H=0.5$ (we found: $H_{AT}=0.51\pm0.05$ and $H_{QT}=0.52\pm0.06$).

For the time dependent analysis we found that active events are more correlated (larger H-values) than quiet events but overlap exists between both types of events at $H=0.8$ (Figures 3.14 and 3.16). When we zoomed in to focus on the details of the time dependent analysis of H we find quite a difference. The students-t test applied to the time dependent analysis for QT and AT found $p=2.79\times10^{-9}$ and $t=-7.7557$, which implies that the differences in the statistics of the averaged $H(t)$ for the computed quiet and active events are significant. Moreover, values of $H(t)$ higher than whole event H were found in all the events are direct evidence that there exist a short-term memory that dominates over the long-term memory.
CHAPTER IV

CONCLUSION

4.1 Current results

In this work I presented a fractional and multifractional approach in the analysis of the Earth’s bulk magnetic field time series obtained from a high-latitude magnetic observatory. Previous works reported the existence of multiscale statistics in a variety of geomagnetic indices [Vörös, 2000; Hnat et al., 2003; Wanliss, 2004; Wanliss, 2005] and in the interplanetary magnetic field [Burlaga and Mish, 1987; Burlaga, 1991; Burlaga, 1996]. In this thesis detrended fluctuation analysis was used to determine the presence of long-range statistical correlations in ground based magnetometer data. This technique was preferred over other classical methods since it is able to deal with nonstationarities in time series [Chen et al., 2002; Kantelhardt et al., 2002; Wanliss, 2004; Wanliss, 2005].

4.2 What we found

The magnetic field time series was classified into active and quiet events following the criteria described in chapter 3. The remaining time series that did not fit into the criteria were discarded. It resulted in twenty time series from quiet events and twenty from active events along a ten-year period from 1991 to 2001. The main motivation in separating the
data into active and quiet times was to study the different local fractal processes that might be involved in the development of geomagnetic activity. The research was focused to a singular geographic location (i.e. Fort Churchill station from the CANOPUS magnetometer array). Since these processes may vary not only in time but also spatially [Weigel et al., 2002], to gain a proper understanding of the global behavior makes it necessary to first understand local behavior. Othani et al., [1995] and Consolini and De Michelis et al., [2000] previously reported the existence of changes in statistics related to different levels of magnetospheric activity. They examined the scaling properties of the magnetic field fluctuations in the magnetotail and found evidence of multifractionality in these fluctuations with a Hurst coefficient of $H \sim 0.5$ before current disruption and $H \sim 0.7$ after current disruption. Wanliss, [2005] reported weak differences in the Hurst exponent of the SYM-H time series for quiet and active events: $H_{QT} = 0.53 \pm 0.04$ and $H_{AT} = 0.59 \pm 0.04$ respectively.

Two different approaches were applied in the data analysis. The first identified the Hurst exponent characteristic for the entire event time scale (fBm approach) expecting that the time series might be classified with distinct values of the Hurst exponent for quiet and active events. The second measured temporal fluctuations of the Hurst exponent in patches through the event (mfBm approach). These two particular approaches were intended to identify the characteristic time scales for the magnetic field fluctuations during active and quiet magnetospheric conditions.
From the fBm whole event approach no clear evidence was found for changes on the 
statistics during quiet or active events. The results were in general the same: 
$H_{QT}=0.52\pm0.06$ and $H_{AT}=0.51\pm0.05$ for quiet events and active events respectively. 
However these results should be considered with caution since the crossover found in the 
log-log space may be indicative of possible white noise contamination. These sources of 
noise may be related to the way that gaps in the row data were handled: i.e. the gaps 
where removed and the series was stitched together or they may be an intrinsic part of the 
data. However the citation of the students-t test confirms that statistical differences 
between events are not significant and thus the hypothesis of a single scaling value 
characterizing the statistics for each of the two events analyzed is discarded.

When we assess the problem considering that the time series is multifractional, the results 
are different. The time dependent analysis performed for $H(t)$ revealed clear differences 
in the data statistics for quiet and active times. These temporal variations of $H$ take place 
during time intervals shorter than the duration of a particular event. Notice that in Figures 
3.13 and 3.15, significant changes in the value $H(t)$ occurs in periods shorter than 1 day 
(1 day = 17280 data samples). Both sets of data (i.e. quiet and active events) showed 
strong correlations far from a random walk. The mean Hurst exponent $H(t)$ for quiet 
events was $H(t)_{QT}=0.73\pm0.05$ and for active events $H(t)_{AT}=0.87\pm0.06$. As these results 
show, the correlation is high while the error in the determination of $H(t)$ is about 6%, and 
overlap exists between the values of the temporal $H$ for both types of event. Again the 
students-t test was performed, and the returned results were consistent with our
expectations (i.e., quiet and active event’s data set comes from different types of populations). The multifractional approach clearly showed that the dynamics of the statistics of the local magnetic field is not steady and dynamically changes through different levels of correlation, suggesting that this correlation increases as the level of geomagnetic activity increases. It appears that the magnetic field at a single high latitude location is best described as a mfBm rather than as a fBm. This finding is a helpful guide for predictive models.

4.3 Future work

Further research may focus on the time where a transition from quiet to active event occurs. These types of observations may be useful to determine the dynamics of the statistics in a continuum time series that are known to have two different statistical processes. The explicit analysis of the x, y and z components of the magnetic field will give a better insight to the statistical behavior of storm development since each of the components are affected in a different fashion given certain magnetospheric conditions.

The spatial distribution of the statistics may serve in the future as a tool for tracking the possible magnetic disturbances by using data from other instruments. The CANOPUS longitudinal magnetometer array is ideal for this type of observation. This array contains seven stations along the meridian 275° and extends from 50 to 70 degrees of latitude. Other magnetometer arrays distributed along North America will enrich these observations.
REFERENCES


Wood and Chan, Simulation of stationary Gaussian processes in [0,1]