Dissertations and Theses

Fall 2012

Neural Network Fatigue Life Prediction in Steel I-Beams Using Mathematically Modeled Acoustic Emission Data

Prathikshen Nambiar Selvadorai

Embry-Riddle Aeronautical University - Daytona Beach

Follow this and additional works at: https://commons.erau.edu/edt

Part of the Aerospace Engineering Commons, and the Materials Science and Engineering Commons

Scholarly Commons Citation


https://commons.erau.edu/edt/129

This Thesis - Open Access is brought to you for free and open access by Scholarly Commons. It has been accepted for inclusion in Dissertations and Theses by an authorized administrator of Scholarly Commons. For more information, please contact commons@erau.edu.
NEURAL NETWORK FATIGUE LIFE PREDICTION IN STEEL I-BEAMS USING MATHEMATICALLY MODELED ACOUSTIC EMISSION DATA

by

Prathikshen N. Selvadorai

A Thesis Submitted to the Graduate Studies Office in Partial Fulfillment of the Requirements for the Degree of Master of Science in Aerospace Engineering

Embry-Riddle Aeronautical University

Daytona Beach, Florida

Fall 2012
NEURAL NETWORK FATIGUE LIFE PREDICTION IN STEEL I-BEAMS USING
MATHEMATICALLY MODELED ACOUSTIC EMISSION DATA

by

Prathikshen N. Selvadorai

This thesis was prepared under the direction of the candidate’s thesis committee chairman, Dr. Fady F. Barsoum, Department of Mechanical Engineering, and has been approved by the members of his thesis committee. It was submitted to the School of Graduate Studies and Research and was accepted in partial fulfillment of the requirements for the degree of Master of Science in Aerospace Engineering.

THESIS COMMITTEE:

Dr. Fady F. Barsoum
Chairman

Dr. Dae Won Kim
Member

Dr. Yi Zhao
Member

Dr. Tasos Lyrintzis
Graduate Program Coordinator

Dr. Robert Oxley
Department Chair, Aerospace Engineering

12/12/12
Date

12/12/12
Date
ACKNOWLEDGMENTS

First and foremost, a very special thanks to my mother (Prema Nambiar), for without her love, support and care this thesis would never have been possible. To paraphrase Isaac Newton, if I have seen any further, it is by standing on the shoulder of giants, all of whom mentioned in this section. But the biggest giant of them all is my father (Selvadorai Krishnasamy). I am extremely grateful for all his love, wisdom and motivation. I also thank my brother (Kaarthikeshen Selvadorai) for his love and enlightening resoluteness about the power of hard work and determination.

This work is the culmination of a year’s effort in conducting abundant mathematical analysis coupled with essential physical principles. It could not have been written without the capable assistance of highly respected Dr. Eric Hill and Dr. Fady Barsoum. They have provided inspiration and guidance for many applied ideas and support at critical phases of this work. They have also provided me with plentiful of advice and knowledge of how to be a better engineer as well as other aspects of life. I also like to thank Dr. Dae Won Kim and Dr. Yi Zhao for being in my thesis committee. Their supervision of my work has been invaluable.

I am also indebted to my Japanese brother, Jun Shishino. He will always be remembered for his patience, support and useful insights provided throughout this work. Without his thoughts and encouragement, I would not have been able to shepherd this enormous endeavor to completion. My sincere gratitude also extends to Ning Leung for his guidance in the early days of this work.

Finally, I would like to thank Anad Babu and Kam Keong Foo for simply being my friends. I greatly value their friendship and deeply appreciate their belief in me.

I dedicate this thesis to my late grandmother, Letchumy Nair.

“\textit{I know numbers are beautiful. If they aren't beautiful, nothing is.}”

- Paul Erdős
ABSTRACT

Author: Prathikshen Nambiar Selvadorai

Title: Neural Network Fatigue Life Prediction in Steel I-Beams Using Mathematically Modeled Acoustic Emission Data

Institution: Embry-Riddle Aeronautical University

Degree: Master of Science in Aerospace Engineering

Year: 2012

The purpose of this research is to predict fatigue cracking in metal beams using mathematically modeled acoustic emission (AE) data. The AE data was collected from nine samples of steel I-beam that were subjected to three-point bending caused by cyclic loading. The data gathered during these tests were filtered in order to remove long duration hits, multiple hit data, and obvious outliers. Based on the duration, energy, amplitude, and average frequency of the AE hits, the filtered data were classified into the various failure mechanisms of metals using NeuralWorks® Professional II/Plus software based self-organizing map (SOM) neural network. The parameters from mathematically modeled AE failure mechanism data were used to predict plastic deformation data. Amplitude data from classified plastic deformation data is mathematically modeled herein using bounded Johnson distributions and Weibull distribution.

A backpropagation neural network (BPNN) is generated using MATLAB®. This BPNN is able to predict the number of cycles that ultimately cause the steel I-beams to fail via five different models of plastic deformation data. These five models are data without any mathematical modeling and four which are mathematically modeled using three methods of bounded Johnson distribution (Slifker and Shapiro, Mage and Linearization) and Weibull distribution. Currently, the best method is the Linearization method that has prediction error not more than 17%. Multiple linear regression (MLR) analysis is also performed on the four sets of mathematically modeled plastic deformation data as named above using the bounded Johnson and Weibull shape parameters. The MLR gives the best prediction for the Linearized method which has a prediction error not more than 2%. The final conclusion made is that both BPNN and MLR are excellent tools for accurate fatigue life cycle prediction.
TABLE OF CONTENTS

ACKNOWLEDGMENTS ............................................................................................................. 1

ABSTRACT ................................................................................................................................. 2

TABLE OF CONTENTS .................................................................................................................. 3

LIST OF FIGURES ....................................................................................................................... 8

LIST OF TABLES .......................................................................................................................... 14

NOMENCLATURE ......................................................................................................................... 17

CHAPTER 1: INTRODUCTION ..................................................................................................... 18

1.1 Overview .................................................................................................................................. 18

1.1.1 Problem Statement ............................................................................................................... 18

1.1.2 Literature Review ................................................................................................................. 18

1.2 Acoustic Emission Nondestructive Testing ................................................................................ 20

1.3 Acoustic Emission .................................................................................................................... 21

1.4 AE Signal Parameters .............................................................................................................. 23

1.5 Failure Mechanisms in Metal Structures ................................................................................ 24

1.6 Kohonen Self-Organizing Map (KSOM) .................................................................................. 26

1.7 Histogram Bin Size Selection .................................................................................................. 29

1.8 Johnson Distribution ............................................................................................................... 29

1.8.1 Slifker and Shapiro’s Method ............................................................................................. 30

1.8.2 Mage’s Method ................................................................................................................... 34

1.8.3 Linearization Method ......................................................................................................... 36

1.9 Weibull Distribution ................................................................................................................ 37

1.10 Expected Frequency of Occurrence ...................................................................................... 40

1.11 Chi-square Hypothesis Testing .............................................................................................. 41

1.12 Back Propagating Neural Network (BPNN) ............................................................................. 41
1.12.1 Background.......................................................................................................................... 41
1.12.2 BPNN with MATLAB........................................................................................................ 43
1.13 Multiple Linear Regression (MLR) ....................................................................................... 46
1.14 Analysis Of Variance (ANOVA) ............................................................................................ 48

CHAPTER 2: EXPERIMENT........................................................................................................... 50
2.1 Experimental Setup................................................................................................................. 50
2.2 Noise Test .............................................................................................................................. 53
2.3 Data Acquisition System........................................................................................................ 55
2.4 Experimental Results ............................................................................................................ 57

CHAPTER 3: KOHONEN SELF-ORGANIZING MAP (KSOM) ANALYSIS.......................... 59
3.1 Overview................................................................................................................................. 59
3.2 Data Storing .......................................................................................................................... 59
3.3 Data Filtering ......................................................................................................................... 60
3.4 Data Pre-Analysis in KSOM.................................................................................................. 61
3.5 Verification Criterion Analysis.............................................................................................. 62
3.6 Data Classification ................................................................................................................ 64
3.7 Data Validation ..................................................................................................................... 67

CHAPTER 4: MATHEMATICAL MODELING ANALYSIS............................................................ 69
4.1 Overview................................................................................................................................. 69
4.2 Bin Size Selection ................................................................................................................ 69
4.3 Plastic Deformation Amplitude Data..................................................................................... 71
4.4 Bounded Johnson Distribution Mathematical Modelling ................................................... 72
   4.4.1 Bounded Johnson (SB) Distribution by Slifker and Shapiro’s Method ....................... 72
   4.4.2 Bounded Johnson (SB) Distribution by Mage’s Method ............................................. 75
   4.4.3 Bounded Johnson (SB) Distribution by Linearization Method .............................. 78
F.6 JohnsonMage.m ........................................................................................................... 196
F.7 JohnsonLinear.m ........................................................................................................... 201
F.8 Weibull.m ..................................................................................................................... 204
F.9 BPNN.m ....................................................................................................................... 207
F.10 SlifkerAndShapiroMLR.m .......................................................................................... 211
F.11 MageMLR.m ............................................................................................................... 212
F.12 LinearMLR.m ............................................................................................................. 213
F.13 WeibullMLR.m .......................................................................................................... 214

APPENDIX G: ANOVA RESULTS ...................................................................................... 215

G.1 Bounded Johnson Distribution (Slifker and Shapiro) ANOVA Results ............... 215
G.2 Bounded Johnson Distribution (Mage) ANOVA Results ............................................ 220
G.3 Bounded Johnson Distribution (Linearization) ANOVA Results ............................ 227
G.4 Weibull Distribution ANOVA Results ...................................................................... 234
LIST OF FIGURES

Figure 1.1: AE System [17] ........................................................................................................... 21
Figure 1.2: Typical AE Data Acquisition System [18] ................................................................. 22
Figure 1.3: AE Piezoelectric Transducer [19] .............................................................................. 22
Figure 1.4: AE Signal ................................................................................................................... 24
Figure 1.5: Fatigue Cracking Modes ............................................................................................ 25
Figure 1.6: Kohonen Network [20] ............................................................................................. 27
Figure 1.7: Data Classification Using Kohonen Self-Organizing Map [16] ............................... 28
Figure 1.8: Sample Linear Line Plot of Bounded Johnson Distribution .................................. 37
Figure 1.9: Sample Logarithmic Plot of Weibull Distribution .................................................... 39
Figure 1.10: Sample Linear Line Plot of Weibull Distribution .................................................... 39
Figure 1.11: Back Propagation Neural Network [27] ................................................................ 42
Figure 1.12: Transfer Functions [28] ......................................................................................... 43
Figure 1.13: MATLAB BPNN GUI ............................................................................................ 46
Figure 2.1: Engineering Sketch of the Standard S4 x 7.7 I-beam [14] .................................... 50
Figure 2.2: 150 kHz Acoustic Emission Transducers ................................................................. 51
Figure 2.3: I-Beam Subjected To Cyclic Loading ........................................................................ 52
Figure 2.4: I-Beam Test Structure .............................................................................................. 52
Figure 2.5: Average Frequency Histogram for a Bar Noise Test [14] ...................................... 53
Figure 2.6: Duration versus Counts with Noise Overlap [14] ..................................................... 54
Figure 2.7: Amplitude Histogram for a Bar Noise Test [14] ....................................................... 54
Figure 2.8: Pocket AE System [34] ........................................................................................... 55
Figure 2.9: Pocket AE Waveform Parameters [35] ................................................................. 57
Figure 2.10: Fatigue Cracked I-Beam ....................................................................................... 57
Figure 3.1: Partial Data Sample of Beam 1 in Microsoft Excel ............................................... 59
Figure 3.2: Filtered Data of Beam 1 .............................................................................................. 61
Figure 3.3: Partial Data Sample of Beam 1 in Notepad ............................................................. 62
Figure 3.4: Verification Criterion Analysis for Beam 1 ............................................................... 63
Figure 3.5: NeuralWorks Professional II/Plus start-up page ................................................. 65
Figure 3.6: KSOM EDAF with 5 Classifications for Beam 1 .................................................... 66
Figure 4.1: Plastic Deformation Histogram with Bin Size of 2 for Beam 1 ............................ 69
Figure 4.2: Plastic Deformation Histogram with Bin Size of 0.5 for Beam 1 ........................................ 70
Figure 4.3: Plastic Deformation Histogram with Bin Size of 1 for Beam 1 ........................................ 70
Figure 4.4: SB Distribution by Slifker and Shapiro’s Method for Beam 1 ........................................ 73
Figure 4.5: SB Distribution by Mage’s Method for Beam 1 ................................................................. 76
Figure 4.6: Linear Line Plot for Beam 1 ................................................................................................. 78
Figure 4.7: SB Distribution by Linearization Method for Beam 1 ......................................................... 80
Figure 4.8: Weibull Linear Plot for Beam 1 ............................................................................................ 83
Figure 4.9: Weibull Distribution (Fixed $\gamma$) for Beam 1 ................................................................. 84
Figure 6.1: Flow Chart of Major Processes Undertaken ................................................................. 101
Figure A.1: Filtered Data of Beam 1 ........................................................................................................ 107
Figure A.2: Filtered Data of Beam 2 ....................................................................................................... 107
Figure A.3: Filtered Data of Beam 3 ....................................................................................................... 108
Figure A.4: Filtered Data of Beam 4 ....................................................................................................... 108
Figure A.5: Filtered Data of Beam 5 ....................................................................................................... 109
Figure A.6: Filtered Data of Beam 6 ....................................................................................................... 109
Figure A.7: Filtered Data of Beam 7 ....................................................................................................... 110
Figure A.8: Filtered Data of Beam 8 ....................................................................................................... 110
Figure A.9: Filtered Data of Beam 9 ....................................................................................................... 111
Figure B.1: Verification Criterion Analysis for Beam 1 ................................................................. 112
Figure B.2: Verification Criterion Analysis for Beam 2 ................................................................. 112
Figure B.3: Verification Criterion Analysis for Beam 3 ................................................................. 113
Figure B.4: Verification Criterion Analysis for Beam 4 ................................................................. 113
Figure B.5: Verification Criterion Analysis for Beam 5 ................................................................. 114
Figure B.6: Verification Criterion Analysis for Beam 6 ................................................................. 114
Figure B.7: Verification Criterion Analysis for Beam 7 ................................................................. 115
Figure B.8: Verification Criterion Analysis for Beam 8 ................................................................. 115
Figure B.9: Verification Criterion Analysis for Beam 9 ................................................................. 116
Figure B.10: KSOM EDAF with 3 Classifications for Beam 1 ...................................................... 116
Figure B.11: KSOM EDAF with 3 Classifications for Beam 2 ...................................................... 117
Figure B.12: KSOM EDAF with 3 Classifications for Beam 3 ...................................................... 117
Figure B.13: KSOM EDAF with 3 Classifications for Beam 4 ...................................................... 118
Figure D.18: Plastic Deformation Histogram with Bin Size of 0.5 for Beam 9 .................................. 144
Figure D.19: Plastic Deformation Histogram with Bin Size of 1 for Beam 1 ................................. 145
Figure D.20: Plastic Deformation Histogram with Bin Size of 1 for Beam 2 ................................. 145
Figure D.21: Plastic Deformation Histogram with Bin Size of 1 for Beam 3 ................................. 146
Figure D.22: Plastic Deformation Histogram with Bin Size of 1 for Beam 4 ................................. 146
Figure D.23: Plastic Deformation Histogram with Bin Size of 1 for Beam 5 ................................. 147
Figure D.24: Plastic Deformation Histogram with Bin Size of 1 for Beam 6 ................................. 147
Figure D.25: Plastic Deformation Histogram with Bin Size of 1 for Beam 7 ................................. 148
Figure D.26: Plastic Deformation Histogram with Bin Size of 1 for Beam 8 ................................. 148
Figure D.27: Plastic Deformation Histogram with Bin Size of 1 for Beam 9 ................................. 149
Figure E.1: SB Distribution by Slifker and Shapiro’s Method for Beam 1 .................................. 150
Figure E.2: SB Distribution by Slifker and Shapiro’s Method for Beam 2 .................................. 150
Figure E.3: SB Distribution by Slifker and Shapiro’s Method for Beam 3 .................................. 151
Figure E.4: SB Distribution by Slifker and Shapiro’s Method for Beam 4 .................................. 151
Figure E.5: SB Distribution by Slifker and Shapiro’s Method for Beam 5 .................................. 152
Figure E.6: SB Distribution by Slifker and Shapiro’s Method for Beam 6 .................................. 152
Figure E.7: SB Distribution by Slifker and Shapiro’s Method for Beam 7 .................................. 153
Figure E.8: SB Distribution by Slifker and Shapiro’s Method for Beam 8 .................................. 153
Figure E.9: SB Distribution by Slifker and Shapiro’s Method for Beam 9 .................................. 154
Figure E.10: SB Distribution by Mage’s Method for Beam 1 ..................................................... 154
Figure E.11: SB Distribution by Mage’s Method for Beam 2 ..................................................... 155
Figure E.12: SB Distribution by Mage’s Method for Beam 3 ..................................................... 155
Figure E.13: SB Distribution by Mage’s Method for Beam 4 ..................................................... 156
Figure E.14: SB Distribution by Mage’s Method for Beam 5 ..................................................... 156
Figure E.15: SB Distribution by Mage’s Method for Beam 6 ..................................................... 157
Figure E.16: SB Distribution by Mage’s Method for Beam 7 ..................................................... 157
Figure E.17: SB Distribution by Mage’s Method for Beam 8 ..................................................... 158
Figure E.18: SB Distribution by Mage’s Method for Beam 9 ..................................................... 158
Figure E.19: Linear Line Plot for Beam 1 ................................................................................. 159
Figure E.20: Linear Line Plot for Beam 2 ................................................................................. 159
Figure E.21: Linear Line Plot for Beam 3 ................................................................................. 160
Figure E.22: Linear Line Plot for Beam 4 ................................................................. 160
Figure E.23: Linear Line Plot for Beam 5 ................................................................. 161
Figure E.24: Linear Line Plot for Beam 6 ................................................................. 161
Figure E.25: Linear Line Plot for Beam 7 ................................................................. 162
Figure E.26: Linear Line Plot for Beam 8 ................................................................. 162
Figure E.27: Linear Line Plot for Beam 9 ................................................................. 163
Figure E.28: SB Distribution by Linearization Method for Beam 1 ....................... 163
Figure E.29: SB Distribution by Linearization Method for Beam 2 ....................... 164
Figure E.30: SB Distribution by Linearization Method for Beam 3 ....................... 164
Figure E.31: SB Distribution by Linearization Method for Beam 4 ....................... 165
Figure E.32: SB Distribution by Linearization Method for Beam 5 ....................... 165
Figure E.33: SB Distribution by Linearization Method for Beam 6 ....................... 166
Figure E.34: SB Distribution by Linearization Method for Beam 7 ....................... 166
Figure E.35: SB Distribution by Linearization Method for Beam 8 ....................... 167
Figure E.36: SB Distribution by Linearization Method for Beam 9 ....................... 167
Figure E.37: Weibull Linear Plot for Beam 1 .......................................................... 168
Figure E.38: Weibull Linear Plot for Beam 2 .......................................................... 168
Figure E.39: Weibull Linear Plot for Beam 3 .......................................................... 169
Figure E.40: Weibull Linear Plot for Beam 4 .......................................................... 169
Figure E.41: Weibull Linear Plot for Beam 5 .......................................................... 170
Figure E.42: Weibull Linear Plot for Beam 6 .......................................................... 170
Figure E.43: Weibull Linear Plot for Beam 7 .......................................................... 171
Figure E.44: Weibull Linear Plot for Beam 8 .......................................................... 171
Figure E.45: Weibull Linear Plot for Beam 9 .......................................................... 172
Figure E.46: Weibull Distribution for Beam 1 .......................................................... 172
Figure E.47: Weibull Distribution for Beam 2 .......................................................... 173
Figure E.48: Weibull Distribution for Beam 3 .......................................................... 173
Figure E.49: Weibull Distribution for Beam 4 .......................................................... 174
Figure E.50: Weibull Distribution for Beam 5 .......................................................... 174
Figure E.51: Weibull Distribution for Beam 6 .......................................................... 175
Figure E.52: Weibull Distribution for Beam 7 .......................................................... 175
Figure E.53: Weibull Distribution for Beam 8 ................................................................. 176
Figure E.54: Weibull Distribution for Beam 9 ................................................................. 176
LIST OF TABLES

Table 1.1: AE Parameters ................................................................. 24
Table 1.2: Characteristics of Failure Mechanisms of Metals ......................... 26
Table 1.3: ‘trainlm’ Function Parameters .............................................. 44
Table 1.4: ANOVA Results .................................................................. 49
Table 2.1: Pocket AE Setup Parameters ................................................. 56
Table 2.2: Experimental Number of Cycles to Catastrophic Failure ................ 58
Table 3.1: AE Data Filtration Methods .................................................. 60
Table 3.2: Voting Value Results ............................................................ 64
Table 3.3: Energy Analysis .................................................................. 68
Table 3.4: Duration Analysis .................................................................. 68
Table 3.5: Amplitude Analysis ............................................................... 68
Table 4.1: Plastic Deformation Data ....................................................... 72
Table 4.2: Best Slifker and Shapiro’s Method Configuration ......................... 73
Table 4.3: Mathematical Modeled Plastic Deformation Data (Slifker and Shapiro) 74
Table 4.4: Best Mage’s Method Configuration ........................................... 75
Table 4.5: Mathematical Modeled Plastic Deformation Data (Mage Method) .... 77
Table 4.6: Best Linearization Method Configuration .................................... 79
Table 4.7: Mathematical Modeled Plastic Deformation Data (Linearization Method) 81
Table 4.8: Statistical Analysis of Chi-square ($\chi^2$) Values ......................... 82
Table 4.9: Weibull Distribution Parameters (Fixed $\gamma$) ............................. 84
Table 4.10: Mathematical Modeled Plastic Deformation Data (Weibull) .......... 85
Table 5.1: Neural Network Function Parameters ....................................... 88
Table 5.2: BPNN Prediction Results (Non-mathematically Modeled) .............. 89
Table 5.3: BPNN Prediction Results (Slifker and Shapiro’s Method) ................ 89
Table 5.4: BPNN Prediction Results (Mage’s Method) ................................ 90
Table 5.5: BPNN Prediction Results (Linearization Method) ......................... 90
Table 5.6: BPNN Prediction Results (Weibull Distribution – Fixed $\gamma$) ......... 91
Table 5.7: Statistical Analysis of the Absolute Errors .................................. 91
Table 5.8: Independent MLR Variables .................................................. 93
Table 5.9: ANOVA Results for Stepwise Regression (Slifker and Shapiro’s Method) 94
Table C.21: Amplitude Analysis................................................................. 133
Table C.22: Energy Analysis ..................................................................... 134
Table C.23: Duration Analysis ................................................................. 134
Table C.24: Amplitude Analysis................................................................. 134
Table C.25: Energy Analysis ..................................................................... 135
Table C.26: Duration Analysis ................................................................. 135
Table C.27: Amplitude Analysis................................................................. 135
Table G.1: ANOVA Results – First Analysis (Slifker and Shapiro).......... 217
Table G.2: ANOVA Results – Final Analysis (Slifker and Shapiro) .......... 219
Table G.3: ANOVA Results – First Analysis (Mage) .................................. 222
Table G.4: ANOVA Results – Second Analysis (Mage) ............................ 224
Table G.5: ANOVA Results – Final Analysis (Mage) ................................. 226
Table G.6: ANOVA Results – First Analysis (Linearization) ..................... 229
Table G.7: ANOVA Results – Second Analysis (Linearization) ................. 231
Table G.8: ANOVA Results – Final Analysis (Linearization) ..................... 233
Table G.9: ANOVA Results – First Analysis (Weibull) ............................. 234
Table G.10: ANOVA Results – Second Analysis (Weibull) ....................... 234
Table G.11: ANOVA Results – Third Analysis (Weibull) ......................... 235
Table G.12: ANOVA Results – Final Analysis (Weibull) ......................... 235
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AE</td>
<td>Acoustic Emission</td>
</tr>
<tr>
<td>ANOVA</td>
<td>Analysis of Variance</td>
</tr>
<tr>
<td>BPNN</td>
<td>Backpropagation Neural Network</td>
</tr>
<tr>
<td>D</td>
<td>Dimension</td>
</tr>
<tr>
<td>DB</td>
<td>Davies-Bouldin</td>
</tr>
<tr>
<td>DF</td>
<td>Degrees of Freedom</td>
</tr>
<tr>
<td>GDR</td>
<td>Generalization Delta Rule</td>
</tr>
<tr>
<td>GUI</td>
<td>Graphical User Interface</td>
</tr>
<tr>
<td>KSOM</td>
<td>Kohonen Self-Organizing Map</td>
</tr>
<tr>
<td>MHD</td>
<td>Multiple Hit Data</td>
</tr>
<tr>
<td>MLR</td>
<td>Multiple Linear Regression</td>
</tr>
<tr>
<td>MR</td>
<td>Multiple Regression</td>
</tr>
<tr>
<td>NDT</td>
<td>Nondestructive Testing</td>
</tr>
<tr>
<td>PE</td>
<td>Processing Element</td>
</tr>
<tr>
<td>SB</td>
<td>Bounded Johnson Distribution</td>
</tr>
<tr>
<td>SHD</td>
<td>Single Hit Data</td>
</tr>
<tr>
<td>SL</td>
<td>Lognormal Johnson Distribution</td>
</tr>
<tr>
<td>SN</td>
<td>Normal Distribution</td>
</tr>
<tr>
<td>SOM</td>
<td>Self-Organizing Map</td>
</tr>
<tr>
<td>SR</td>
<td>Stepwise Regression</td>
</tr>
<tr>
<td>SS</td>
<td>Sum of Squares</td>
</tr>
<tr>
<td>SU</td>
<td>Unbounded Johnson Distribution</td>
</tr>
<tr>
<td>SW</td>
<td>Silhouette Width</td>
</tr>
<tr>
<td>T</td>
<td>Tou</td>
</tr>
<tr>
<td>VC</td>
<td>Verification Criterion</td>
</tr>
<tr>
<td>Z</td>
<td>Standard Normal Variate</td>
</tr>
</tbody>
</table>
CHAPTER 1: INTRODUCTION

1.1 Overview

1.1.1 Problem Statement

The purpose of this research is to mathematically model plastic deformation data extracted from acoustic emission (AE) data acquired from fatigue test on steel I beams caused by cyclic loading. The mathematically modeled data is then analyzed using backpropagating neural network (BPNN) and multiple linear regression (MLR) to predict the number of cycles to catastrophic failure. Catastrophic failure is a phenomenon where a structure undergoes total collapse and is unable to withstand anymore load.

If the percentage error between the predicted results and the theoretical results is low enough, it is envisioned that this research will be further developed into an operational electronic device that can predict the time to catastrophic failure of any metals. This device can reduce the cost of laborious and time consuming maintenance procedures and increase safety in aerospace and mechanical applications.

1.1.2 Literature Review

In 1990, Walker and Hill [1, 2] mathematically modeled matrix cracking data from AE nondestructive testing on unidirectional graphite/epoxy tensile via Weibull distribution. The mathematically modeled data was analyzed through a neural network. This work demonstrated the feasibility of predicting ultimate strength of simple composite structures by mathematically modeling the data.

In the same year, Khamitov, Gorkunov, and Bartenev [3] investigated crystallographic anisotropy on magnetoelastic acoustic emission signals in interrelation with linear magnetostriction in a nickel monocrystal. The mathematical model proposed for the measuring channel sets up a direct proportional dependence between the rms voltage of the mean absolute error signals and the linear magnetostriction of ferromagnets.

In 1998, Chunguang, Xinyi, and Jishou [4] set up mathematical models for time domain parameters of acoustic emission sources and signals of turning tool crack in turning cutting
process. The methods are based on the generalized Hooke's law and acoustic emission energy model of tool crack extension. Based on Hooke’s energy conservation principle, by means of Parseval’s theorem, the time domain mathematical model of acoustic emission signal in the process of turning tool crack was successfully established.

Also in 1998, Vaughn and Hill [5] used AE to monitor in-flight fatigue crack growth. With a neural network they were able to distinguish plastic deformation, fatigue cracking and rubbing noises in a Piper PA-28 aircraft engine cowling during flight. Using a SOM neural network and AE quantification parameter data, Rovik and Hill [6, 7], monitored fatigue crack growth in the vertical tail of a Cessna T-303 as the aircraft performed various in-flight maneuvers.

In the same year, initially Ballard and Hill [8, 9] undertook fatigue life prediction in inconel and stainless steel bellows from early fatigue cycle AE data by using backpropagating neural network (BPNN). Later, multivariate statistical analysis was also employed.

In 2001, Cumberbatch and Fitt used acoustic emission nondestructive testing on metal cylinders overwrapped by continuous-filament fibre-reinforced plastic (FRP). The acoustic emission data is mathematically modeled data using the linear elasticity theory for the metal liners and a simple “tension band” model for the fibre wrapping. [10]

In 2010, Wotzka, Boczar, and Fracz used mathematical modeling techniques to describe acoustic emission signals generated by partial discharges occurring in oil immersed electric power transformers. [11] The mathematical models used are the sigmoid function and the exponential function.

By employing AE and KSOM technique; plastic deformation, fatigue cracking and rubbing noises were successfully classified as failure mechanisms in 7075-T6 aluminum using basic plots such as amplitude histogram, amplitude versus average frequency and duration versus counts by Okur in his thesis in Spring 2010. [12]

In transversely loaded steel testing, the experimental procedures and fatigue cycle prediction by using fatigue cracking data was explored by Korcak et al. [13] In his thesis, Korcak utilized KSOM and BPNN via NeuralWorks Professional II/Plus software in Spring 2010. [14]
AE data can also be mathematically modeled using bounded Johnson distribution. This was done by Izuka in his thesis in Fall 2010. [15] Izuka mathematically modeled matrix cracking data from composite overwrapped pressure vessels which was subjected to internal pressures until catastrophic failure. The mathematical model used was Slifker and Shapiro, and Mage method of the bounded Johnson distribution.

In 2011, Hill and the author [16] analyzed AE data from the tail rotor gearbox, primarily in the bevel gear which were undergoing fatigue cracking caused by high frequency rotary motion. The AE data were successfully classified into plastic deformation, plane stress fatigue cracking, plain strain fatigue cracking and rubbing noises.

This research uses the same data that have been utilized by Korcak [14] in his thesis. In his thesis, the acoustic emission data are split into 4 categories which are first quarter fatigue life (0-25 %), second quarter fatigue life (25-50 %), third quarter fatigue life (50-75 %) and semi-random fatigue life. The first quarter fatigue life primarily consists of plastic deformation data and small amounts of fatigue cracking data. The worst case BPNN absolute error of the failure prediction result using only the first quarter fatigue life was 18.4 %.

The present research utilizes the KSOM feature in the NeuralWorks® Professional II/Plus software coupled with verification criterion methods to classify the data and extract only plastic deformation data. The Johnson and Weibull distributions are used to mathematically model the plastic deformation data. Linearization method of the bounded Johnson distribution is introduced to mathematically model the data. Slifker and Shapiro, and Mage method is also performed to compare the effectiveness of all three methods of the bounded Johnson distribution. Finally, MATLAB® based BPNN and MLR are used to predict fatigue life cycle of steel I beams using the mathematically modeled plastic deformation data.

1.2 Acoustic Emission Nondestructive Testing

AE is a popular choice of nondestructive testing (NDT) method in many industries. AE NDT is a volumetric testing method which can detect changes within a material and not only on its surface. This method can measure and detect crack initiation and crack growth rate, internal cracking, boiling or cavitations, friction or wear, plastic deformation, and phase transformation of any material from composites to metals which are being subjected to a structural load or
stress. This method of testing is also a passive technique which can be performed on a test specimen or structure without impeding on its operations.

The advantage of AE testing is its capability of locating structural discontinuities and flaws in any part of the structure from a single surface point of the structure. Other advantages of AE testing are remote and continuous (real time) surveillance of a specimen, ability to keep permanent record of failure, equipment portability, and triangulation technique to locate flaws with multiple transducers.

The disadvantage of AE testing is that transducers must be placed on the specimens’ surface and are also subjected to wave attenuation issues. Highly ductile material also yield low amplitude AE emissions which are hard to detect. The test specimen must also be stressed or operating since AE testing can only detect structural flaws that are growing. This is also called the Kaiser effect. Another disadvantage is that AE testing is very sensitive which causes it to pick up unwanted noises.

1.3 Acoustic Emission

AE is the transient elastic wave generated by the rapid release of energy from sources within a material which are subjected to loading. These waves will propagate through the material and can be detected at the surface of the material by using piezoelectric transducers. These transducers convert wave motion into low level and high impedance electrical signals which can be recorded via a data acquisition system.

A simple representation of an acoustic emission instrumentation is illustrated Figure 1.1.

![Figure 1.1: AE System](image)

Figure 1.1: AE System [17]
Figure 1.2 is a block diagram of all the components associated with AE instrumentation. The crack is an example of an AE event or signal which is a local material change giving rise to acoustic emission. This signal is picked up by an AE transducer. A typical transducer used for AE research is the R15-150 kHz or the R15i-150 kHz with integral pre-amplifier, also known as the piezoelectric transducer. Some transducers come with integral pre-amplifiers to shorten the wire from the transducers to the pre-amplifier thus reducing the probability of picking up radio transmissions. Often, the AE transducer and the pre-amplifier can be visualized as a single block. Figure 1.3 shows a piezoelectric transducer.

Figure 1.2: Typical AE Data Acquisition System [18]

Figure 1.3: AE Piezoelectric Transducer [19]
The signal then passes through a filter after being acquired by the transducer. Filters eliminate mechanical (low frequency) and electromagnetic (high frequency) noises. There are highpass or bandpass filters that allow the user to set an operating frequency range which is typically between 100 kHz and 1 MHz. The typical frequency range for AE testing is 100 to 300 kHz. The amplifier amplifies the amplitude of the signal which can be up to 120 dB. This amplification is related in a logarithmic function to the voltage of the signal. The AE signal input voltage of 1 µV is set as the reference level of 0 dB at the transducer before any amplification.

AE hit is defined as the detection and measurement of an AE signal on a channel. AE hit description is a digital (numerical) description of an AE hit. Signal conditioner and event detector takes the parametric inputs which are environmental variables (load, pressure and temperature) that are measured and stored as part of the AE hit description. Raw AE signal is now converted to a digital numerical description of an event, comprising one or more signal.

The final block is the computer data storage post processor which visualizes the signals in plots such as history plots, channel plots, point plots, diagnostic plots or distribution functions (differential amplitude distribution or cumulative amplitude distribution).

1.4 AE Signal Parameters

An AE signal can be quantified by five main acoustic emission parameters namely amplitude, duration, counts, rise time and energy as illustrated in Figure 1.4. Table 1.1 summarizes all these parameters and their definitions.

These parameters enable AE signals to be statistically analyzed. Different types of failure mechanisms can be established or interpreted by their respective statistical characteristics of the signals.
Table 1.1: AE Parameters

<table>
<thead>
<tr>
<th>AE Parameter</th>
<th>Notation</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude</td>
<td>A</td>
<td>Largest voltage peak</td>
<td>dB</td>
</tr>
<tr>
<td>Count</td>
<td>C</td>
<td>Number of times the signal crosses the detection threshold</td>
<td>#hits</td>
</tr>
<tr>
<td>Duration</td>
<td>D</td>
<td>Time elapsed between initial and final signal that crosses the detection threshold</td>
<td>µs</td>
</tr>
<tr>
<td>Energy</td>
<td>E</td>
<td>Measured Area Under the Rectified Signal Envelope (MARSE)</td>
<td>energy counts</td>
</tr>
<tr>
<td>Rise Time</td>
<td>R</td>
<td>Time elapsed between the initial signal to the highest amplitude signal</td>
<td>µs</td>
</tr>
</tbody>
</table>

1.5 Failure Mechanisms in Metal Structures

Failure mechanisms are agents that cause structures to undergo deterioration of its usefulness or complete breakdown. The most common failure mechanisms in metals are plastic deformation and fatigue cracking (Plane Strain/Mode 1 and Plane Stress/Mode 3).
Plastic deformation is the most common occurring failure mechanism in metallic materials that are being subjected to loads. When at a certain area of a material undergoes strain hardening after multiple localized plastic deformations, a crack may occur at that location. If loads (cyclic loading or uniaxial loading) are constantly applied to the material, the crack will pass the initiation stage and will keep growing until the material witness catastrophic failure.

The two modes of fatigue cracking are shown in Figure 1.5. Initially plane strain or Mode 1 fatigue cracking occurs which rips an opening in a material and continues to propagate directly into the material. After that the crack may flip direction and start to tear the material. This change of direction and tearing is called plane stress or Mode 3 fatigue cracking.

![Fatigue Cracking Modes](image)

**Figure 1.5: Fatigue Cracking Modes**

Apart from the failure mechanisms there is a third type of mechanism that has to be included, which is noise. Noise is signals produced by causes other than AE and are irrelevant to the purpose of AE testing. Even though this mechanism does not involve in the integrity of the material, it has to be included since the characteristics of this mechanism have to be established in order to successfully eliminate it from the data. However, it is impossible to get rid of all noise data.

The two main types of noise that may disrupt AE testing are mechanical noise (friction or fretting) and electrical noise (electromagnetic or radio frequency interference). The frequencies
of these noises can be used to distinguish between the types of noises since mechanical noises have much lower frequencies compared to electrical noises.

Table 1.2 shows the characteristics of all the mechanisms.

**Table 1.2: Characteristics of Failure Mechanisms of Metals**

<table>
<thead>
<tr>
<th>Mechanisms</th>
<th>Amplitude</th>
<th>Duration</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatigue Cracking</td>
<td>High</td>
<td>Medium</td>
<td>High</td>
</tr>
<tr>
<td>Plastic Deformation</td>
<td>Medium</td>
<td>Short</td>
<td>Low</td>
</tr>
<tr>
<td>Noise</td>
<td>Low</td>
<td>Long</td>
<td>Medium</td>
</tr>
</tbody>
</table>

### 1.6 Kohonen Self-Organizing Map (KSOM)

KSOM is an artificial neural network which is competitive; single layered and is a type of unsupervised learning which is able to identify underlying structure of multi-dimensional input data which is specified by the user. The KSOM then generates low dimensional (usually one or two), discretized representation of the input data which preserves its neighbourhood relations which is also called a topological map. In other words, KSOM analyzes all the input data and groups similar data together. This process is very similar to neurons in a mammalian brain which has the tendencies to cluster into specific groups.

A SOM consist of an input layer and a Kohonen layer. Figure 1.6 illustrates a Kohonen Network where the top layer is the Kohonen layer and the bottom is the input layer.
Each processing element, PE or node in the Kohonen layer represents a separate classification. The input nodes are independent of each other. But all input nodes are connected to the PEs via a weight matrix.

During iteration, the weight matrix is randomly assigned to the input nodes. The SOM then learns by minimizing the Euclidean distance between the weights and the input nodes. As the learning progresses, more PEs with the closest weight vectors to the input vectors are clustered together. This competitive learning progresses till it incorporates all input data, and the Kohonen layer will begin to take shape which classifies all the different characteristics of the input data.

The input parameters and the Kohonen processing layer can be controlled by the user. The input parameters are the AE parameters. An additional input parameter can be created which is the average frequency (kHz) where it is nothing more than the ratio of counts to duration. The Kohonen layer or the output data consist of the failure mechanisms.

Figure 1.7 below is an example of a well classified test data which consists of four input parameters and five Kohonen nodes or failure mechanisms indicated by different colors. The four input parameters are duration, energy, amplitude and average frequency. Average frequency is counts divided by duration. Plane strain and plane stress are fatigue cracking. Multiple hit data (MHD) is considered noise where 2 or more AE hits are clustered into one signal. There are two
types of MHD classified in this data namely MHD Type 1 and MHD Type 2. The number of failure mechanisms can be increased if the desired number of output classification is unknown but the KSOM will not necessarily use all the possible classification.

![Figure 1.7: Data Classification Using Kohonen Self-Organizing Map [16]](image)

There are 3 embedded plots in Figure 1.7. The most important analyses process is to associate which color represents a specific failure mechanisms since KSOM will not directly establish this. From known characteristics of failure mechanisms as summarized in Table 1.2, the Duration vs. Amplitude plot is to be the most informative. For example, the color red has the lowest amplitude and duration; hence it is obvious that this group of data is plastic deformation. The other plots can help back up such conclusions.

Verification criterion methods are statistical algorithms that can be used to identify the optimum number of clusters or mechanisms that are present in a specific data set. No one verification methods are the best. Typically, a few verification methods are utilized and the results are combined to establish the number of classifications that exist in a data set. Three verification criterion methods that are very commonly used are Davies-Bouldin (DB) Criterion [21], Silhouette Width (SW) Criterion [21] and Tou (T) Criterion [22].
1.7 Histogram Bin Size Selection

Histogram is one of the best visual impression or graphical representation of the probability distribution of data. Histograms have distinct intervals between the maximum and minimum values. These intervals are called bins. The relationship between the number of bins in a histogram and its width are given by Equation 1. Equation 1 is also known as the ceiling function.

\[ k = \frac{Maximum \ x - minimum \ x}{h} \]  

(1)

where \( k \) is the number of bins, \( h \) is the width of bin and \( x \) is the data value.

There are many methods to determine the value of ‘\( k \)’ or ‘\( h \)’. The two main methods are square-root choice (Equation 2) and Scott’s normal reference rule (Equation 3). Equation 2 is applied as the default choice in Microsoft Excel histograms.

\[ k = \sqrt{n} \]  

(2)

where \( k \) is the number of bins and \( n \) is the number of data points.

\[ h = \frac{3.5\sigma}{n^{1/3}} \]  

(3)

where \( \sigma \) is the standard deviation of data and \( n \) is the number of data points.

There is another method called the engineering choice where the bin size, \( h \) is simply set to be 1. The square root method causes the bin size, \( h \) to be larger than 1 and the Scott’s method causes \( h \) to be lesser than 1.

1.8 Johnson Distribution

The Johnson distribution is a statistical curve fitting model which transforms any continuous distribution into an altered form of the standard normal distribution (SN). This distribution basically transforms the standard normal variate, ‘\( z \)’. The SN distribution has a ‘\( z \)’ value based on the mean, \( \mu \) and standard deviation, \( \sigma \) of the data set but this is not the case for the Johnson
distributions where its ‘z’ is dependent upon four parameters where \( \gamma \) and \( \eta \) are shape parameters, \( \lambda \) is the scale parameter and \( \varepsilon \) is the location parameter.

There are three forms of Johnson distribution which are bounded (SB, or logistic transformation), unbounded (SU, or hyperbolic sine transformation) and lognormal (SL, or exponential transformation). Hence, the distribution is very versatile and is able to mathematically model various types of data. Slifker and Shapiro described the basic mathematical procedure governing form selection procedure and parameter estimation in 1978. [23] In 1979, Mage introduced explicit solution for SB parameter estimation based on four percentiles points. [24] The third method is called Linearization where the four parameters for SB distribution are directly estimated without determining the standard normal variate, ‘z’.

1.8.1 Slifker and Shapiro’s Method

Slifker and Shapiro began by first determining the type of distribution that has to be chosen which is dependent on the data sample. The first step is to select the standard normal variate, ‘z’. The selection of ‘z’ value is crucial since it will dictate the parameter estimation and shape of the distribution. Once the ‘z’ value is chosen, the data is broken down into four quantiles, \( \zeta; -3z, -z, z \) and \( 3z \) which are symmetrical about zero and equally spaced. The corresponding percentages, \( P_\zeta \) are calculated using the table of areas of the normal distribution. For each, \( \zeta \), the \( i^{th} \) order of observation, \( x_\zeta \) in the data is obtained using Equation 4.

\[
i = nP_\zeta + \frac{1}{2}
\]  

(4)

where \( i \) is the observation order of the data set and \( n \) is the number of data points.

After obtaining the data population based \( x_\zeta \) values, Slifker and Shapiro’s algorithm in form selection begin with calculating the \( m, n, p \) values as shown in Equations 5 to 7.

\[
m = x_{3z} - x_z
\]  

(5)

\[
n = x_{-z} - x_{-3z}
\]  

(6)

\[
p = x_z - x_{-z}
\]  

(7)

Hence, the form selection is based on the following cases:
\( mn/p^2 < 0.999 \), correlates to SB distribution

\( mn/p^2 > 1.001 \), correlates to SU distribution

\( 0.999 < mn/p^2 < 1.001 \), correlates to SL distribution

1.8.1.1 Bounded Johnson (SB) Distribution Parameter Estimation

This distribution is heavily dependent upon the ratios \( \frac{p}{m} \) and \( \frac{p}{n} \). The four parameters are determined as listed in the following equations.

\[
\eta = \cosh^{-1} \left[ \frac{1}{2} \left( \left( 1 + \frac{p}{m} \right) \left( 1 + \frac{p}{n} \right) \right)^{1/2} \right] ; (\eta > 0)
\]

\[
\gamma = \eta \sinh^{-1} \left[ \frac{\left( \frac{p}{n} - \frac{p}{m} \right) \left( \left( 1 + \frac{p}{m} \right) \left( 1 + \frac{p}{n} \right) - 4 \right)^{1/2}}{2 \left( \frac{p}{m} \cdot \frac{p}{n} - 1 \right)} \right] ; (\gamma > 0)
\]

\[
\lambda = \frac{p \left( \left( 1 + \frac{p}{m} \right) \left( 1 + \frac{p}{n} \right) - 2 \right)^{1/2}}{\frac{p}{m} \cdot \frac{p}{n} - 1} ; (\lambda > 0)
\]

\[
\xi = \frac{x_z + x_{-z}}{2} - \frac{\lambda}{2} + \frac{p \left( \frac{p}{n} - \frac{p}{m} \right)}{2 \left( \frac{p}{m} \cdot \frac{p}{n} - 1 \right)}
\]

where, the values \( m, n \) and \( p \) are calculated from Equation 5 to 7.

The standard normal variate, ‘\( z \)’ is determined by the following equation.

\[
z = \gamma + \eta \ln \left( \frac{x - \xi}{\lambda + \xi - x} \right)
\]

where \( x \) is the sample data and its domain is described below;

\[\xi < x < \xi + \lambda\]

The probability density function is given by Equation 13.
\[ f(x) = \frac{\eta}{\sqrt{2\pi}} \frac{\lambda}{(x - \varepsilon)(\lambda - x + \varepsilon)} e^{\exp\left\{ -\frac{1}{2} \left[ \gamma + \eta \ln\left( \frac{x - \varepsilon}{\lambda + \varepsilon - x} \right) \right]^2 \right\} } \]  

where \( x \) is the sample data.

### 1.8.1.2 Unbounded Johnson (SU) Distribution Parameter Estimation

This distribution is heavily dependent upon the ratios \( \frac{m}{p} \) and \( \frac{n}{p} \). In other words, SU distribution is dependent on the reciprocals of the SB distribution ratios. The four parameters are determined as listed in the following equations.

\[ \eta = \frac{2z}{\cosh^{-1}\left( \frac{1}{2} \left( \frac{m}{p} - \frac{n}{p} \right) \right)} ; (\eta > 0) \]  

\[ \gamma = \eta \sinh^{-1}\left( \frac{\frac{n}{p} - \frac{m}{p}}{2 \left( \frac{m}{p} \frac{n}{p} - 1 \right)^{1/2}} \right) \]  

\[ \lambda = \frac{2p \left( \frac{mn}{p^2} - 1 \right)^{1/2}}{\left( \frac{m}{p} + \frac{n}{p} - 2 \right) \left( \frac{m}{p} + \frac{n}{p} + 2 \right)^{1/2}} ; (\lambda > 0) \]  

\[ \varepsilon = \frac{x_z + x_{-z}}{2} + \frac{p \left( \frac{n}{p} - \frac{m}{p} \right)}{2 \left( \frac{m}{p} + \frac{n}{p} - 2 \right)} \]  

where, the values \( m, n \) and \( p \) are calculated from Equations 5 to 7.

The standard normal variate, ‘\( z \)’ is determined by the following equation.

\[ z = \gamma + \eta \ln\left( \frac{x - \varepsilon}{\lambda} \right) \]  

where \( x \) is the sample data and its domain is described below;

\[ -\infty < x < \infty \]

The probability density function is given by Equation 19.
where \( x \) is the sample data.

### 1.8.1.3 Lognormal Johnson (SL) Distribution Parameter Estimation

This distribution has the following characteristic described by Equation 20.

\[
\frac{n}{p} = \left( \frac{m}{p} \right)^{-1}
\]

The consequence of this relationship is that only three parameters are needed to determine the standard normal variate, ‘\( z \)’ and not four. This distribution is also known as three-parameter lognormal distribution. The three parameters are determined as listed in the following equations.

\[
\eta = \frac{2z}{ln \left( \frac{m}{p} \right)}
\]

\[
\gamma = \eta ln \left[ \frac{m}{p} \frac{p - 1}{p \left( \frac{m}{p} \right)^{1/2}} \right]
\]

\[
\varepsilon = \frac{x_2 + x_{-2}}{2} + \frac{p \left( \frac{m}{p} + 1 \right)}{2 \left( \frac{m}{p} - 1 \right)}
\]

where, the values \( m, n \) and \( p \) are calculated from Equations 5 to 7.

The standard normal variate, ‘\( z \)’ is determined by the following equation.

\[
z = \gamma + \eta ln(x - \varepsilon)
\]

where \( x \) is the sample data and its domain is described below;

\[
\varepsilon < x < \infty
\]

The probability density function is given by Equation 25.
\[ f(x) = \frac{\eta}{\sqrt{2\pi \varepsilon}} \cdot \frac{1}{\sqrt{x}} \cdot \exp \left\{ -\frac{1}{2} \eta^2 \left( \frac{\varepsilon + \eta \ln(x - \varepsilon)}{\eta} \right)^2 \right\} \]  \hspace{1cm} (25)

where \( x \) is the sample data.

If the parameter \( \varepsilon \) is 0, then this distribution transforms into a two-parameter lognormal distribution. The standard normal variate, ‘\( z \)’ is determined by the following equation.

\[ z = \gamma + \eta \ln(x) \]  \hspace{1cm} (26)

where \( x \) is the sample data and its domain is described below;

\[ x > 0 \]

The probability density function is determined by the following equation.

\[ f(x) = \frac{\eta}{\sqrt{2\pi \varepsilon}} \cdot \frac{1}{\sqrt{x}} \cdot \exp \left\{ -\frac{1}{2} \eta^2 \left( \frac{\varepsilon + \eta \ln(x - \varepsilon)}{\eta} \right)^2 \right\} \]  \hspace{1cm} (27)

where \( x \) is the sample data.

1.8.2 Mage’s Method

Mage introduced an explicit method in determining the four quantiles, \( \zeta \). In this case, they are not required to be symmetrical but they are still required to be equidistant. The quantiles are denoted as \( z_1, z_2, z_3 \) and \( z_4 \). This method still leads to the evaluation of the four shape parameters; \( \eta, \gamma, \lambda \) and \( \varepsilon \) which are vital in determining the shape of the probability density function. Equation 28 illustrates the relationship between the four quantiles.

\[ z_4 - z_3 = z_3 - z_2 = z_2 - z_1 \]  \hspace{1cm} (28)

This allows two quantiles to be set as independent variables and the other two as dependent variables which offer more flexibility in dictating the ultimate shape of the probability density function. Rearranging Equation 28 leads to two new equations, Equations 29 and 30 which illustrate the independent nature of \( z_1 \) and \( z_2 \).

\[ z_3 = 2z_2 - z_1 \]  \hspace{1cm} (29)

\[ z_4 = 3z_2 - 2z_1 \]  \hspace{1cm} (30)
After setting the four quantiles, \( \zeta; z_1, z_2, z_3 \) and \( z_4 \); the procedure to find the corresponding \( P_\zeta \) are identical with the Slifker and Shapiro’s method. The respective \( x_\zeta (x_1, x_2, x_3 \) and \( x_4 \)) are obtained using Equation 4.

Mage introduces 9 variables (\( a, b, c, d, e, f, \theta, \varphi \) and \( \tau \)) that is required to explicitly evaluate the four parameters (\( \eta, \gamma, \lambda \) and \( \varepsilon \)). The eight equations for the Mage variables are listed below.

\[
\begin{align*}
    a &= x_2 + x_4 - 2x_3 \\
    b &= x_3^2 - x_2x_4 \\
    c &= 2x_2x_3x_4 - (x_2 + x_4)x_3^2 \\
    d &= x_1 + x_3 - 2x_2 \\
    e &= x_2^2 - x_1x_3 \\
    f &= 2x_1x_2x_3 - (x_1 + x_3)x_2^2 \\
    \varphi &= \frac{cd - af}{bd - ae} \\
    \theta &= \frac{ce - bf}{bd - ae}
\end{align*}
\]

\[
\tau = -\frac{\varphi}{2} + \left(\frac{\varphi^2}{4} - \theta\right)^{\frac{1}{2}}
\]

From Equations 31 to 39, the four shape parameters can be evaluated from the equations listed below.

\[
\begin{align*}
    \lambda &= 2\left(\frac{\varphi^2}{4} - \theta\right)^{1/2} \\
    \varepsilon &= -\frac{\varphi}{2} - \left(\frac{\varphi^2}{4} - \theta\right)^{\frac{1}{2}}
\end{align*}
\]
The probability density function can be determined by using Equation 13.

**1.8.3 Linearization Method**

The Linearization method does not begin with the task of determining the standard normal variate, ‘z’ which moves along to determine the shape parameters. The Linearization method directly utilizes Equation 12. A plot of $y^*$ versus $x^*$ is established. Equations 44 and 45 describe $y^*$ and $x^*$.

$$y^* = z = \frac{x - \mu}{\sigma}$$  \hspace{1cm} (44)

where $z$ is the standard normal variate, $x$ is the data sample, $\mu$ is the mean of the data sample and $\sigma$ is the standard deviation of the data sample.

$$x^* = \ln\left(\frac{x - \varepsilon}{\lambda + \varepsilon - x}\right)$$ \hspace{1cm} (45)

where $x$ is the data sample, and $\lambda$ and $\varepsilon$ are the Johnson shape parameters.

The shape parameters $\lambda$ and $\varepsilon$ are iterated when the plot is generated until a linear line with the best coefficient of determination, $R^2$ is attained. The equation of this linear line will have the form as shown in Equation 46.

$$y^* = mx^* + c$$ \hspace{1cm} (46)

where $m$ is the gradient of the line and $c$ is the $y^*$-axis interception value.

Equation 46 is compared with Equation 12 where $\eta$ is the gradient of the line and $\gamma$ is the interception value as shown in Equations 47 and 48.

$$\eta = m$$ \hspace{1cm} (47)
At this point, all four shape parameters have been determined and the probability density function for the bonded Johnson distribution can be obtained from Equation 13. Figure 1.8 shows a sample of a linear line plot for the bounded Johnson distribution.

1.9 Weibull Distribution

Weibull distribution is a very flexible continuous probability distribution. This distribution utilizes three parameters (α, β and γ) to describe its probability density function. This is also called the three parameter Weibull distribution where its probability density function is described in Equation 49.

\[
f(x) = \frac{\alpha}{\beta} \left( \frac{x - \gamma}{\beta} \right)^{\alpha-1} \exp \left\{ - \left( \frac{x - \gamma}{\beta} \right)^{\alpha} \right\} \quad (49)
\]

where \( \alpha > 0 \) is the shape (or slope) parameter, \( \beta > 0 \) is the scale parameter, \( \gamma \) is location (or threshold) parameter and \( x \) is the sample data.

The domain of \( x \) is given as below;

\[
x \geq \gamma
\]
If $\gamma$ is set to a constant value, the distribution is called the two parameter Weibull distribution. This also enforces the distribution to begin from the specified constant value. It is advised to set $\gamma$ at a certain value to give a common point of reference if this distribution is to be based on multiple sample data originating from an identical test. [2]

By altering $\alpha$ value, the shape of the distribution can be transformed to imitate other distribution functions. If $\alpha < 1$, the distribution transforms to an inverse function. If $\alpha = 1$, the distribution transforms to a two parameter exponential function. If $\alpha = 2$, the distribution transforms to a Rayleigh distribution. If $\alpha = 3.5$, the distribution very closely imitates the normal distribution.

The value of the scale parameter, $\beta$ is the distance of the centroid of the probability density function from the threshold, $\gamma$.

For the Weibull distribution, there is a reliability factor, $R$ at a certain value of $x$ given by equation 50.

$$R = \exp\left\{ -\left(\frac{x - \gamma}{\beta}\right)^\alpha \right\}$$ (50)

where $x$ is the sample data.

Mathematically, $R$ represents the cumulative density function complementary to unity at a given value of $x$. [2] In other words; $R$ is the ratio of number of events at a given value of $x$ to the total number of events under the distribution envelope.

There are no direct methods to evaluate the three parameters and they have to be determined from plots. The two plots needed to determine all the three parameters are Figure 1.9 and 1.10.
The first plot is illustrated in Figure 1.9 which is needed to determine the $\gamma$ value. The plot will be a logarithmic line where $y'$ and $x'$ are described in Equation 51 and 52 respectively.

$$y' = \ln \left( \ln \frac{1}{R} \right)$$  \hspace{1cm} (51)

$$x' = \ln x$$  \hspace{1cm} (52)
The d value is an arbitrary, fixed distance which usually changes from test to test. For different values of d, γ can also vary, creating a lack of repeatability in the data. Wherever possible it is advisable to fix γ at a given value, to give a common point for each test. [2] Utilizing Figure 1.9, the γ value can be obtained from Equation 53.

\[
γ = γ' - \frac{(γ''' - γ'')γ'' - γ')}{(γ''' - γ'')γ'' - (γ''' - γ')}
\]  

The second plot is illustrated in Figure 1.10. The plot will be a linear line that has a form shown in Equation 46 where y* and x* are described in Equation 54 and 55 respectively.

\[
y^* = \ln(\ln R)
\]  
\[
x^* = \ln(x - γ)
\]  

The equation to evaluate α and β are given in Equation 56 and 57.

\[
α = m
\]  

where m is the slope of the linear plot.

\[
β = \exp\left(-\frac{c}{m}\right)
\]  

where c is the y* axis intercept value. The expression \(-\frac{c}{m}\) is the x* axis intercept value.

At this point, all three shape parameters have been determined and the probability density function for the Weibull distribution can be obtained from Equation 49.

### 1.10 Expected Frequency of Occurrence

Mathematically modeled histograms can be generated by calculating the expected frequency of occurrence by applying Equation 58.

\[
p(x) = nhf(x)
\]
where \( p(x) \) is the expected frequency of occurrence, \( n \) is the total number of data sets and \( h \) is the bin size.

1.11 Chi-square Hypothesis Testing

When data is being mathematically modeled by various statistical methods, the chi-square hypothesis testing can be utilized to test the degree of goodness of the curve fitting. This test is performed by grouping the data into bins, calculating the observed and expected counts for those bins, and computing the chi-square test statistic. [25] The equation for this testing is given below.

\[
\chi^2 = \sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i}
\]

(59)

where \( \chi^2 \) is the chi-square hypothesis testing, \( O_i \) is the observed counts and \( E_i \) is the expected counts. [25] \( E_i \) is similar to \( p(x) \) in Equation 58.

1.12 Back Propagating Neural Network (BPNN)

1.12.1 Background

The human brain is a very powerful and efficient information processing tool. The BPNN is modeled after this and this method belongs between Artificial Intelligence and Approximation Algorithms. Basically, the BPNN is a supervised learning method which takes an input data set from the user and using interconnected weights and mathematical functions (generalization delta rule, GDR), and derives a set of output data which best simulates the possible outcome of the input data set. BPNN is a tool to forecast output data by establishing correlation between the output data and the known input data set.

Figure 1.11 illustrates the BPNN. The input data are multiplied by a weight coefficient. Via a PE, or neuron this new data matrix is then summed and processed through a transfer function. The common transfer functions are sigmoidal (data matrix scaled from 0 to 1) or hyperbolic tangent (data matrix scaled from -1 to 1). Figure 1.12 illustrates this mathematical function. Transfer functions are applied to shrink the input space into a convenient range for data analysis. This is called a nonlinear model. This model is very useful since for example, it can sense changes between 0.01 and 0.02 and also between 100 and 200.
The total number of neurons in the hidden layer are dictated by the number of failure mechanisms predetermined by the user which are illustrated in the equation below. This equation was proposed by Hill et al. [26]

\[
N = 2n + 1
\]

where, \(N\) is number of neurons in the hidden layer and \(n\) is the number of failure mechanisms.

A typical BPNN will consist of one input layer (input slab), one or more hidden layer (hidden slab) and one output layer (output slab).

Figure 1.11: Back Propagation Neural Network [27]
1.12.2 BPNN with MATLAB

MATLAB® is a commercial software created by MathWorks Inc. It is an interactive software package for scientific and engineering numeric computation. [29] MATLAB® is able to compute and undertake highly complex mathematical computations since it has many core routines pre-programmed within its package which is primed for faster and rigorous mathematical analysis. One of its major pre-programmed subroutines is matrix manipulations and since BPNN involves laborious matrix multiplication, MATLAB® is a valuable tool. Furthermore, since the BPNN code is self-generated in MATLAB®, more variables can be controlled in order to develop a competent source code. In a research done, it is proven that BPNN in MATLAB is 4.5 to 7 times faster than BPNN programs written in C language. [30]

To create a back propagating network, the simplified `newff` MATLAB® code can be used as shown in the syntax below:

```matlab
net = newff(PR,[S1 S2...SNl],[TF1 TF2...TFNl],BTF,BLF,PF)
```

where:

- **PR** - Rx2 matrix of min and max value for R input elements
- **Si** - Size of ith layer, for Nl layers.
- **TFi** - Transfer function of ith layer, default = 'tansig'.
- **BTF** - Back propagation network training function, default = 'trainlm'.
- **BLF** - Back propagation weight/bias learning function, default = 'learngdm'.
- **PF** - Performance function, default = 'mse'.

**Figure 1.12: Transfer Functions [28]**
The ‘tansig’ (tangent-sigmoid) transfer function which is also known as the hyperbolic transfer function, is described in Figure 1.12.

The ‘trainlm’ is a network training function that updates weight and bias values according to Levenberg-Marquardt optimization. [31] The parameters and their default values are shown in Table 1.3. This is the first choice supervised algorithm but it takes considerable amount of memory. For lesser memory usage and more efficient algorithm, ‘trainbfg’ can be used. If ‘trainlm’ is still to be used with lesser memory usage, the default value of parameter ‘net.trainParam.mem_reduc’ can be increased to 2 or more.

The ‘learngdm’ is a gradient descent learning function with momentum weights. ‘mse’ is the mean squared normalized error performance function which is the default error regressing function. If the performance function does not achieve the intended performance goal, another performance function can be used, namely the ‘sse’ which is the sum squared error performance function.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>net.trainParam.epochs</td>
<td>100</td>
<td>Maximum epoch number to train</td>
</tr>
<tr>
<td>net.trainParam.goal</td>
<td>0</td>
<td>Performance goal</td>
</tr>
<tr>
<td>net.trainParam.max_fail</td>
<td>5</td>
<td>Maximum validation failures</td>
</tr>
<tr>
<td>net.trainParam.mem_reduc</td>
<td>1</td>
<td>Memory/speed trade-off factor</td>
</tr>
<tr>
<td>net.trainParam.min_grad</td>
<td>1E-10</td>
<td>Minimum performance gradient</td>
</tr>
<tr>
<td>net.trainParam.mu</td>
<td>0.001</td>
<td>Initial mu</td>
</tr>
<tr>
<td>net.trainParam.mu_dec</td>
<td>0.1</td>
<td>Decrease factor of mu</td>
</tr>
<tr>
<td>net.trainParam.mu_inc</td>
<td>10</td>
<td>Increase factor of mu</td>
</tr>
<tr>
<td>net.trainParam.mu_max</td>
<td>1E+10</td>
<td>Maximum mu</td>
</tr>
<tr>
<td>net.trainParam.show</td>
<td>25</td>
<td>Epochs between displays</td>
</tr>
<tr>
<td>net.trainParam.showCommandLine</td>
<td>0</td>
<td>Command line output generator</td>
</tr>
<tr>
<td>net.trainParam.showWindow</td>
<td>1</td>
<td>Training GUI show option</td>
</tr>
<tr>
<td>net.trainParam.time</td>
<td>inf</td>
<td>Maximum train time in seconds</td>
</tr>
</tbody>
</table>
There are three other parameters; learning rate (net.trainParam.lr), learning rate increase (net.trainParam.lr_inc) and momentum coefficient (net.trainParam.mc) which also has to be initiated. The values of these parameters are important to ensure optimum neural network performance. The parameter ‘net.trainParam.showWindow’ can be set to 0 if the user does not want to see the MATLAB® BPNN graphical user interface (GUI) window which is shown in Figure 1.13.

The BPNN analysis by MATLAB stops when the performance goal is met. This is dictated by the parameters in the Progress section in Figure 1.13. When any of the progress parameter values on the left violates the minimum or maximum threshold values indicated on the right (or the maximum amount of time is exceeded), the training iterations grind to a halt and MATLAB proclaims that the performance goal is met as shown at the bottom left of Figure 1.13. When any of the progress parameter bars violates the threshold, the color of the bar changes from blue to green. For example, in this case the violator is the performance parameter.

BPNN divides the input data evenly into three sets of data which are training (trains the network), validation (stops the training if validation checks are met) and testing (checks network predicting ability). The user can set the division manually using special indices by setting the parameter ‘net.divideFcn’ to ‘divideind’ (Data Division in the Algorithm section in Figure 1.13) which has three subroutine codes which are ‘net.divideParam.trainInd’, ‘net.divideParam.valInd’ and ‘net.divideParam.testInd’ which controls the training, validation and testing data respectively.

The entire MATLAB BPNN code used for this thesis with all the above parameters can be found in Appendix F.
1.13 Multiple Linear Regression (MLR)

The MLR is a mathematical predicting algorithm which generates a linear equation which correlates a matrix of $m \times n$ ($m$ is the number of rows and $n$ is the number of columns) dimension of independent (or predictor) variables with a matrix of $m \times 1$ dimension of dependent (or predictand) variables. Each row of values in the dependent variable matrix is associated with its respective row in the independent variable matrix which is also called a data set.
The choice and number of the independent variables is paramount in achieving the best predicting capability by the generated linear equation. This single equation is able to facilitate the regression of every data set, hence it’s known as MLR. The MLR’s goal is to minimize the sum squared error between the independent and dependent values to optimum levels.

The model equation for this analysis is shown in Equation 61.

\[ y_i = b_o + b_1 x_{i,1} + b_2 x_{2,i} + \ldots + b_K x_{K,i} + e_i \]  

(61)

where \( y_i \) is the dependent variables, \( b_o \) is the regression constant, \( i \) is the number of data sets, \( K \) is the total number of independent variables, \( b_K \) is the coefficient associated with the \( K^{th} \) predictor, \( x_{K,i} \) is the value of the \( K^{th} \) independent value and \( e_i \) is the error term.

The term \( b_1, b_2, \ldots, b_K \) is estimated by least squares, which is a method that minimizes the sum of the squares of the errors of the overall solution made in the results of every single equation. [32]

The prediction equation of this analysis is shown in Equation 62.

\[ \hat{y}_i = \hat{b}_o + \hat{b}_1 x_{i,1} + \hat{b}_2 x_{2,i} + \ldots + \hat{b}_K x_{K,i} \]  

(62)

where the variables are similar to Equation 61 except that \(^\wedge\) denotes estimated values.

The error term, \( e_i \) in Equation 61 is unknown since the true model equation to predict the data is unknown. Once the model has been estimated by Equation 62, the regression residuals are defined as in Equation 63.

\[ \hat{e}_i = y_i - \hat{y}_i \]  

(63)

where \( \hat{e}_i \) is the regression residuals, \( y_i \) is the \( i^{th} \) dependent value and \( \hat{y} \) is the \( i^{th} \) predicted value of the dependent value.

The residuals measure the closeness of fit between the predicted values and actual dependent values. [32]

In MLR analysis, the number and choice of variables have to be closely guarded in order not to develop a situation where the number of variables exceeds the number of cases. This can be done
by the stepwise regression, SR method where each variable are added into the prediction equation and tested via analysis of variance (ANOVA) which is explained in section 1.14.

The mathematical process for a forward selection SR begins with adding a constant term (no variables) to the model. The correlation of the prediction equation is tested with the independent data. Then, first order (linear) terms are added and the model is tested again. This process can be repeated until the highest order of variables is accounted for. The process is halted when the addition of variables does not improve the correlation of the model. This method is usually employed when the investigator is interested in constructing a MLR equation by adding one predictor variable, $b_i$ at a time. Here, interest usually focuses in the amount of predictive power that each additional variable contributes and on the search for a small but effective set of predictor variables. [33]

1.14 Analysis Of Variance (ANOVA)

The basis of ANOVA is in the evaluation of the sum of squares, SS of a prediction variable, $y$ or $SS_y$. The total $SS_y$ is the sum of $SS_y$ multiple regression (MR) on the independent variables, $x$ (Equation 64) and $SS_y$ on the residuals, $\hat{e}$ (Equation 65).

$$SS_y\ \text{MR on } x = \sum_i(\hat{y}_i - Y)^2$$  \hspace{1cm} (64)

where $\hat{y}_i$ is the $i^{th}$ predicted value of the dependent value and $Y$ is the mean of the dependent values.

$$SS_y\ \text{Residual} = \sum_i(y_i - \hat{y}_i)^2$$  \hspace{1cm} (65)

where $y_i$ is the $i^{th}$ dependent value and $\hat{y}_i$ is the $i^{th}$ predicted value of the dependent value.

$SS_y$ MR and $SS_y$ residuals have a specific number of degrees of freedom, DF. $SS_y$ MR has $K$ number of DF. $SS_y$ residuals have $N - K - 1$ number of DF. The variables $N$ and $K$ must satisfy the condition $N > K + 1$. If this condition is violated, the number of DF of the residuals will be 0 or less. In this model, the prediction equation might have favorable prediction accuracy but would not necessarily consist of variables that are physically useful or relevant.

Finally, ANOVA utilizes the F distribution and converts it to p values. F distribution is dependent on the degrees of freedom on two ($v_1$ and $v_2$) independent $\chi^2$ variables. In this case, $v_1$
is $K$ and $v_2$ is $N - K - 1$. The $p$-value is a statistical significance testing value. If the $p$ value is lower than the significance level, $\alpha$ (defined by the user) the null hypothesis is rejected and the result is declared to be statistically significant.

$R^2$ is the coefficient of multiple determination which tests the proportion of the dependent variance accounted for by multiple regression on the $K$ predictor variables. In other words, it measures the predictive “goodness” of the prediction model.

Table 1.4 is the basic format of ANOVA testing which summarizes all important parameters undertaken during this analysis.

Table 1.4: ANOVA Results

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>$p (&lt;\alpha)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple Regression</td>
<td>$SS_y MR$</td>
<td>$K$</td>
<td>$\frac{SS_y MR}{K}$</td>
<td>$\frac{MS MR}{MS R}$</td>
<td>From F to p tables</td>
</tr>
<tr>
<td>Residual</td>
<td>$SS_y R$</td>
<td>$N - K - 1$</td>
<td>$SS_y MR$</td>
<td>$\frac{SS_y MR}{N - K - 1}$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$SS_y MR + SS_y R$</td>
<td>$N - 1$</td>
<td>$SS_y MR$</td>
<td>$\frac{SS_y R}{SS_y R}$</td>
<td></td>
</tr>
</tbody>
</table>

$R^2$
CHAPTER 2: EXPERIMENT

2.1 Experimental Setup

A total number of thirty two specimens of A572-G50 steel are subjected to a cyclic loading. This was done to facilitate extensive noise test. The 2-dimensional engineering sketch of these specimens is shown in Figure 2.1. The AE transducers are indicated on the sketch at 102 mm (≈ 4 in) and 229 mm (≈ 9 in) measured from the left-hand end of the specimen.

![Figure 2.1: Engineering Sketch of the Standard S4 x 7.7 I-beam](image)

The transducers used are seen in Figure 2.2. It is the 150 kHz resonant Physical Acoustics Corporation transducers with an operation range of 50-200 kHz.

The steel I-beams used for this experiment are the standard S4 x 7.7. The I-beams are subjected to three-point bending which simulates loading conditions of actual bridge members. Two 150 kHz AE transducers were mounted on the bottom flange of the I-beam.
The experimental setup is shown in Figure 2.3 and Figure 2.4. A 45 kN MTS actuator was used to apply the loading, and the applied load ranged from 1.36 to 17 kN at a frequency of 1 Hz. The beam underwent a maximum deflection of approximately 13 mm (0.5 in). The actual loading was approximately sinusoidal, as the structure was stiff and the actuator would reach its resonant frequency when tasked to match the controller input exactly. The hydraulic pump was located about 1 m behind the setup. A plastic block was placed between the actuator and test specimen to reduce the imminent noise resulting from the actuator since the connections are steel to steel as noise signals will be easily transferred. The Physical Acoustics Corporation Pocket AE analyzer with embedded AEwin software was used to record the AE data from two transducers placed on the beams at different distances from the point of load application. The number of cycles to catastrophic failure is recorded for each steel I-beam. [14]

To initiate cracking, a notch is created below the I-beam. A 3.81 (0.15 in) mm deep 45° angle V-notch was machined on the bottom flange of the beam to ensure that fatigue cracking would initiate on the bottom and not on the top where the stress concentration forms due to the load application. The AE transducers were mounted on each side of the notch using hot melt glue as an adhesive/couplant. This setup allows verification of the location of the source of the AE activity to ensure that the AE data collected are from the known crack location at the center of the I-beam bottom flange. [14]
Figure 2.3: I-Beam Subjected To Cyclic Loading

Figure 2.4: I-Beam Test Structure
2.2 Noise Test

In 2009, extensive noise test was performed by Korcak in thesis regarding cyclic loading on I-beams which are similar to this experiment [14]. Two sources of noise were identified while no loads were subjected to the test specimen. The two sources are rubbing noises from the hydraulic grips holding the specimen and electromagnetic interference from the Pocket AE charger interface. The characteristic of these noises discovered by Korcak is that the frequency of most of the signals lies between 1 kHz and 30 kHz. All the signals also have an average frequency below 5.7 kHz and an amplitude range of 30 to 34 dB. These are illustrated in Figures 2.5 to 2.7.

![Average Frequency Histogram for a Bar Noise Test](image.png)

*Figure 2.5: Average Frequency Histogram for a Bar Noise Test [14]*
Figure 2.6: Duration versus Counts with Noise Overlap [14]

Figure 2.7: Amplitude Histogram for a Bar Noise Test [14]
2.3 Data Acquisition System

Figure 2.8 below shows the Pocket AE data acquisition system. The Pocket AE requires several parameters to be set properly before usage.

![Pocket AE System](image)

**Figure 2.8: Pocket AE System [34]**

The key settings altered from the default values are listed in 2.1. The amplitude threshold for both the channel 1 and 2 were set at 30 dB. This value is based on previous research on isotropic materials. If the threshold is set below this value, unnecessary background noises are added to the analysis, and if the threshold is set higher than this value, there is a high possibility that it could omit valuable fatigue data.

The maximum duration was arbitrarily set at 10 000 $\mu$s (10 ms) in order to ensure that all reasonable duration signals were captured. Multiple hit data (MHD) is a type of data which has long duration signals originating from continuous rubbing between experimental equipment, machine hydraulics and other background noises. It is desirable to limit has much MHD signals as possible from being recorded by the Pocket AE. However, if the maximum duration signal value was set too low, valuable single hit data (SHD) originating from plastic deformation and fatigue cracking from the test specimen could be unintentionally disregarded.
Table 2.1: Pocket AE Setup Parameters

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Timing</th>
<th>Waveform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Threshold = 30</td>
<td>Max Duration = 10 ms</td>
<td>Peak Definition Time (PDT) = 200 µs</td>
</tr>
<tr>
<td>dB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Channel 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Threshold = 30</td>
<td></td>
<td>Hit Definition Time (HDT) = 400 µs</td>
</tr>
<tr>
<td>dB</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hit Lockout Time (HLT) = 900 µs</td>
</tr>
</tbody>
</table>

The waveform parameters are consisted of three parameters which are peak definition time (PDT), hit definition time (HDT), and hit lockout time (HLT). These parameters are preset by the user which is pertinent to separate noise from fatigue data. Figure 2.9 illustrates the range of these parameters based on the waveform of a basic acoustic emission signal.

PDT is the time it takes for the signal to cross its threshold and reach its peak value. This is also called as peak recognition. This parameter is important for rise time measurements. HDT is the time that determines the retention time after a signal has fallen below the threshold to determine the end of a signal hit. This parameter ensures that all SHD signals are recorded as only one hit. HLT terminates the measurement process and stores the AE hit waveform quantification parameters (Table 1.1) in the data acquisition buffer.

The values of HDT and HLT parameters are selected in conjunction with pencil lead break tests (Hsu-Nielsen Source) from various locations on the specimen. Without proper setting of these two timing parameters, the first hit will not be properly stored and closed out before the second hit crosses the threshold and is recorded. This causes two or more distinct SHD to be clustered into a single hit which are known as MHD. This is undesirable since AE parameters for MHD are severely dissimilar than SHD.
2.4 Experimental Results

Catastrophic failure is the condition where the I-beam has surpassed its useful lifespan or in other words it has been so severely damaged that it is unable to withstand anymore substantial loads. Figure 2.10 shows an I-beam which has been severely fatigue cracked through its web. The speed of the fatigue crack propagation increased very rapidly as the I-beam approached the end of its life.

The number of cycles to catastrophic failure for each beam is tabulated in Table 2.2.

Figure 2.10: Fatigue Cracked I-Beam
Table 2.2: Experimental Number of Cycles to Catastrophic Failure

<table>
<thead>
<tr>
<th>Beam</th>
<th>Cycles To Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11809</td>
</tr>
<tr>
<td>2</td>
<td>16584</td>
</tr>
<tr>
<td>3</td>
<td>16847</td>
</tr>
<tr>
<td>4</td>
<td>19654</td>
</tr>
<tr>
<td>5</td>
<td>16994</td>
</tr>
<tr>
<td>6</td>
<td>19189</td>
</tr>
<tr>
<td>7</td>
<td>13573</td>
</tr>
<tr>
<td>8</td>
<td>15833</td>
</tr>
<tr>
<td>9</td>
<td>16084</td>
</tr>
</tbody>
</table>
CHAPTER 3: KOHONEN SELF-ORGANIZING MAP (KSOM) ANALYSIS

3.1 Overview

The main objective of KSOM analysis is to classify the raw AE data acquired from fatigue cycle testing of the steel I-beams and extract the plastic deformation data. This plastic deformation data is later utilized on to be mathematically modeled using bounded Johnson distribution and Weibull distribution. However, before KSOM analysis can be performed on the data, the AE data has to be properly stored, filtered and managed before it can be successfully classified.

3.2 Data Storing

All the AE data acquired from the cyclic loading is stored in Microsoft Excel for easy data management and mathematical analysis. Figure 3.1 shows a sample of an excel sheet that contains all the information.

<table>
<thead>
<tr>
<th>ID</th>
<th>N</th>
<th>CH</th>
<th>RISE</th>
<th>COUN</th>
<th>ENER</th>
<th>DURATION</th>
<th>AMP</th>
<th>A.FREQ</th>
<th>RMS</th>
<th>ASL</th>
<th>AMS ENERGY</th>
<th>RA_VALUE</th>
<th>CUM ENERGY</th>
<th>CUM COUNTS</th>
<th>TIME</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>0.430305</td>
<td>1</td>
<td>1499</td>
<td>35</td>
<td>21</td>
<td>1776</td>
<td>50</td>
<td>20</td>
<td>0.020</td>
<td>32</td>
<td>1.1E+03</td>
<td>0.0005</td>
<td>1574</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.430305</td>
<td>2</td>
<td>411</td>
<td>18</td>
<td>5</td>
<td>971</td>
<td>46</td>
<td>19</td>
<td>0.024</td>
<td>62</td>
<td>3.3E+02</td>
<td>0.0104</td>
<td>1933</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.377124</td>
<td>1</td>
<td>493</td>
<td>15</td>
<td>4</td>
<td>662</td>
<td>47</td>
<td>20</td>
<td>0.024</td>
<td>20</td>
<td>2.1E+02</td>
<td>0.0104</td>
<td>2193</td>
<td>250</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0.009053</td>
<td>2</td>
<td>113</td>
<td>11</td>
<td>5</td>
<td>662</td>
<td>48</td>
<td>18</td>
<td>0.024</td>
<td>50</td>
<td>2.8E+02</td>
<td>0.0104</td>
<td>2453</td>
<td>415</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0.571949</td>
<td>2</td>
<td>1399</td>
<td>35</td>
<td>15</td>
<td>1889</td>
<td>45</td>
<td>16</td>
<td>0.020</td>
<td>50</td>
<td>7.7E+02</td>
<td>0.0104</td>
<td>2669</td>
<td>329</td>
</tr>
</tbody>
</table>

Figure 3.1: Partial Data Sample of Beam 1 in Microsoft Excel

Column B to I represents the number of cycles to failure, transducer channel (1 or 2 since it is a two channel system), rise time, count, energy, duration, amplitude and average frequency respectively.
3.3 Data Filtering

To eliminate as much meaningless data as possible such as noise or numerical outliers, two methods are undertaken which are tabulated in Table 3.1.

Table 3.1: AE Data Filtration Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Duration ≥ 1900 μs</td>
</tr>
<tr>
<td>2</td>
<td>Energy = 0</td>
</tr>
<tr>
<td>3</td>
<td>Average Frequency ≤ 15 kHz</td>
</tr>
</tbody>
</table>

Noise has the longest duration in AE data. MHD is type of noise where two or more AE hits are clustered into one AE signal due to time overlap causing this data to skew normal statistics of single hit data, SHD. Rubbing or frictional are classical examples of MHD. These signals normally have very high duration. Hence, the chosen cut-off point for duration is 1900 μs. It is a value which eliminates a lot of meaningless data without the risk of omitting a huge number of meaningful data which could corrupt KSOM classification analysis.

Method 2 involves the omission of zero energy hit data. These data indicate that no energy has been imparted on the test specimen, hence all other parameters that are attached to this data carry no physical significance and are considered meaningless AE data and are consequently discarded.

Method 3 involves the omission of data with an average frequency below 15 kHz since very low frequencies are mechanical noises. The limit is not set higher than 15 kHz to avoid omitting too much data which could cause the elimination of many significant AE data. No maximum limit of frequency was set since there is not much data more than 50 kHz to warrant the need of such filtration method.

Hence, filtering methods were only placed on three parameters which are duration, energy and average frequency.
Figure 3.2 shows the topology of the filtered data for Beam 1. The topology figures of all beams can be found in Appendix A.

Figure 3.2: Filtered Data of Beam 1

3.4 Data Pre-Analysis in KSOM

After sorting the data, the four input parameter; energy, duration, amplitude and average frequency (column F to I) are selected. The four columns; F to I are stored in a Notepad file which has a ‘.txt’ extension. This is an important step before the input data can be transferred in the KSOM for data classification. Figure 3.3 shows a sample of a Notepad that contains the selected information.

The final task before the data can be ideally classified is to determine the number of clusters that are existent within the data. This is accomplished by the verification criterion methods.
3.5 Verification Criterion Analysis

The three verification criterion (VC) methods used are the DB Criterion, SW Criterion and T Criterion. These methods are important to safeguard against inaccurate choice of number of clusters (output parameters). Too many output parameters will cause similar data to be unnecessarily broken down into multiple clusters. Too few parameters will cause multiple unique data clusters to be clustered into one.

The most common number of clusters in an AE data set is usually 3 to 6. To establish a more comprehensive VC analysis, the iteration range of the number of clusters is increased by 1 in both directions. Therefore, all three VC methods are iterated on all nine beam data sets from 2 to 7 number of clusters. Each criterion method will establish the optimum number of clusters for each data set as indicated by its highest index value. There is a possibility that each criterion will suggest a different optimum number of clusters. Hence, a voting value system is designed to identify the best number of clusters for each beam data.
Each VC index value is assigned a certain number of votes dependent upon its rank. This system is described below:

- **Highest VC Index** = 100 Votes
- 2nd Highest VC Index = 40 Votes
- 3rd Highest VC Index = 20 Votes
- All Other VC Index = 0 Votes

The number of votes assigned to each rank illustrates its relative weight. The highest VC has more than twice the number of votes than the second highest VC. This is to assert a commanding dominance for the highest rank. The second highest rank has two times more votes than the third highest rank. Lastly, all other ranks hold no weight; hence no voting value is assigned to them.

![Graph of Verification Criterion Analysis for Beam 1](image)

**Figure 3.4: Verification Criterion Analysis for Beam 1**
The total vote is simply the sum of all the voting value for each number of clusters. Figure 3.4 shows the voting value results for Beam 1. All criterion values and voting values are normalized within the range of -1 to 1. This is done in order to collate all VC and VV plots into a single figure for better visual comparison. The highest voting value tally (Total Votes = 1) for Beam 1, is 3. This indicates that the best number of classifications for Beam 1 data is 3. The figures for all other beam data set can be found in Appendix B.1. The final voting value results for all data sets are summarized in Table 3.2.

### Table 3.2: Voting Value Results

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Optimum Number of Clusters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>Average</td>
<td>4.44</td>
</tr>
<tr>
<td>Mode</td>
<td>5</td>
</tr>
</tbody>
</table>

### 3.6 Data Classification

Five output classifications or parameters are chosen for KSOM analysis. The KSOM neural network classification can be utilize by using the NeuralWorks® Professional II/Plus software. The start-up page of this software is shown in Figure 3.5. All KSOM five classifications figures for all beams can be found in Appendix B.3. Since there are two data sets (Beam 1 and Beam 7) that indicate that three classifications are superior, KSOM analysis with three output classifications are also undertaken. All KSOM three classifications figures for all beams can be found in Appendix B.2.
From visual inspection, KSOM five classifications are the best. The classified data is plotted using MATLAB. Figure 3.6 shows the classified data for Beam 1. The main goal is to identify and extract the plastic deformation data. From the same figure, the color magenta represents plastic deformation since it has the lowest amplitude and duration compared to other classifications. The other colors (yellow, blue, green, red) are shaded with their proposed mechanisms in the ‘Legend’ table in Figure 3.6.
Figure 3.6: KSOM EDAF with 5 Classifications for Beam 1
3.7 Data Validation

Statistical analysis is done on each of the five classifications to validate the conclusion of the proposed mechanisms. The statistical analysis done for each parameter which is; minimum value, maximum value, mean value, standard deviation and number of hits. This is done separately for energy, duration and amplitude data as shown in Table 3.3 to 3.5 for Beam 1.

Mechanisms 1 to 3 are Multiple Hit Data (MHD) since they have much higher duration compared to Mechanism 4 and 5 as can be seen in Table 3.4. This indicates that there are only two true failure mechanisms present which are plastic deformation and fatigue cracking.

Mechanism 4 is concluded to be fatigue cracking data since it has the highest average frequency from inspection in Figure 3.5 (Amplitude versus Average Frequency plot).

From Table 3.3, mechanism 5 has the lowest mean energy value with 5.79. From Table 3.4, mechanism 5 also has the lowest mean duration value with 654.01. Again, from Table 3.5, mechanism 5 has the lowest mean amplitude with 45.31. Hence, mechanism 5 is plastic deformation data.

Similar statistical tables for all beams are shown in Appendix C.

Now, plastic deformation data can be extracted from each nine data sets for mathematical modeling.
### Table 3.3: Energy Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>35</td>
<td>21.20</td>
<td>2.13</td>
<td>4084</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>22</td>
<td>17.35</td>
<td>2.28</td>
<td>4014</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>19</td>
<td>11.55</td>
<td>2.13</td>
<td>2401</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>14</td>
<td>7.05</td>
<td>1.73</td>
<td>2956</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>10</td>
<td>5.79</td>
<td>1.54</td>
<td>4122</td>
</tr>
</tbody>
</table>

### Table 3.4: Duration Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1286</td>
<td>1899</td>
<td>1841.84</td>
<td>63.50</td>
<td>4084</td>
</tr>
<tr>
<td>2</td>
<td>1383</td>
<td>1899</td>
<td>1803.81</td>
<td>97.48</td>
<td>4014</td>
</tr>
<tr>
<td>3</td>
<td>851</td>
<td>1554</td>
<td>1130.49</td>
<td>159.82</td>
<td>2401</td>
</tr>
<tr>
<td>4</td>
<td>128</td>
<td>1003</td>
<td>687.62</td>
<td>131.26</td>
<td>2956</td>
</tr>
<tr>
<td>5</td>
<td>105</td>
<td>1093</td>
<td>654.01</td>
<td>136.03</td>
<td>4122</td>
</tr>
</tbody>
</table>

### Table 3.5: Amplitude Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47</td>
<td>55</td>
<td>48.93</td>
<td>0.94</td>
<td>4084</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
<td>50</td>
<td>47.19</td>
<td>0.86</td>
<td>4014</td>
</tr>
<tr>
<td>3</td>
<td>43</td>
<td>52</td>
<td>47.10</td>
<td>1.23</td>
<td>2401</td>
</tr>
<tr>
<td>4</td>
<td>47</td>
<td>54</td>
<td>47.74</td>
<td>0.96</td>
<td>2956</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
<td>47</td>
<td>45.31</td>
<td>0.78</td>
<td>4122</td>
</tr>
</tbody>
</table>
CHAPTER 4: MATHEMATICAL MODELING ANALYSIS

4.1 Overview

Before the process of mathematical modeling is undertaken, the appropriate bin size for the plastic deformation data is selected to generate its amplitude histograms. This data is required for mathematical modeling and BPNN analysis. Then, the plastic deformation data is subjected to mathematical modeling via three bounded Johnson distribution methods namely; Slifker and Shapiro, Mage and Linearization method.

4.2 Bin Size Selection

It is known that square-root choice (Equation 2), Scott’s normal reference rule (Equation 3) and engineering choice will cause the bin size to be larger than, lesser than and exactly 1 respectively. Hence for simplicity, the bin size of 2, 0.5 and 1 are selected and compared. These are shown in Figure 4.1, 4.2 and 4.3.

Figure 4.1: Plastic Deformation Histogram with Bin Size of 2 for Beam 1
Figure 4.2: Plastic Deformation Histogram with Bin Size of 0.5 for Beam 1

Figure 4.3: Plastic Deformation Histogram with Bin Size of 1 for Beam 1
From Figure 4.1, by selecting a bin size higher than 1, the data precision is severely compromised. From Figure 4.2, by selecting a bin size lesser than 1, the data precision does not change. This is due to the fact that the amplitude data are integers. The widths of the bars become smaller to maintain the same frequency density. Finally, Figure 4.3 indicates that the best choice of bin size is 1 where it maintains data precision and there are no extreme values or outlier that calls for the need to alter the engineering bin size choice.

Amplitude histogram figures with bin choice of 2, 0.5 and 1 for all beams are shown in Appendix D.1, D.2 and D.3 respectively.

4.3 Plastic Deformation Amplitude Data

The amplitude histogram with bin size of 1 is extracted for the plastic deformation data. This data is required for mathematical modeling and BPNN analysis later on. This data will be mathematically modeled using three bounded Johnson distribution methods as discussed in section 4.4 to 4.6. Table 4.1 shows the non-mathematically modeled plastic deformation data for all beams.
## Table 4.1: Plastic Deformation Data

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Amplitude Distribution (40 to 60 dB) With Increments of 1 dB</th>
<th>Experimental Cycles To Failure Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 0 3 75 548 1524 1953 19 0 0 0</td>
<td>11809</td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0 0 11 347 1896 3576 3475 2039 788 195</td>
<td>16584</td>
</tr>
<tr>
<td></td>
<td>32 15 1 1 2 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0 0 0 130 799 1748 2045 1330 569 148</td>
<td>16847</td>
</tr>
<tr>
<td></td>
<td>27 6 1 0 1 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0 0 6 122 734 1544 1617 984 397 98</td>
<td>19654</td>
</tr>
<tr>
<td></td>
<td>4 2 1 1 2 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0 0 5 65 541 1111 1130 678 260 82 17</td>
<td>16994</td>
</tr>
<tr>
<td></td>
<td>4 0 0 1 0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0 0 5 141 657 1196 1283 0 0 0 0</td>
<td>19189</td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0 0 2 58 398 1077 1305 950 472 128</td>
<td>13573</td>
</tr>
<tr>
<td></td>
<td>37 9 2 2 0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0 0 1 107 657 1601 1905 1366 657 192</td>
<td>15833</td>
</tr>
<tr>
<td></td>
<td>47 16 0 0 0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0 0 14 287 1426 2187 1590 627 175 33</td>
<td>16084</td>
</tr>
<tr>
<td></td>
<td>11 3 2 0 2 0 1 0 0 0 0</td>
<td></td>
</tr>
</tbody>
</table>

### 4.4 Bounded Johnson Distribution Mathematical Modelling

#### 4.4.1 Bounded Johnson (SB) Distribution by Slifker and Shapiro’s Method

To find the best curve fitting model which has the lowest chi-square value via Slifker and Shapiro’s method, the standard normal variate, ‘z’ is iterated from 0.5 to 1.5 with an increment of 0.01. An increment of 0.01 is chosen because it gives reasonable accuracy without exhausting too much computational run time.

Table 4.2 shows the best configuration parameters for each beam. Figure 4.4 shows the curve fitting of Slifker and Shapiro’s method for beam 1. The curve fitting figures via Slifker and Shapiro’s method for all beams are attached in Appendix E.1. The mathematically modeled plastic deformation data using Slifker and Shapiro’s method for each beam is shown in Table 4.3.
Table 4.2: Best Slifker and Shapiro’s Method Configuration

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>$\gamma$</th>
<th>$\eta$</th>
<th>$\varepsilon$</th>
<th>$\lambda$</th>
<th>$z$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1129</td>
<td>1.1589</td>
<td>47.27</td>
<td>-4.7220</td>
<td>1.05</td>
<td>9895</td>
</tr>
<tr>
<td>2</td>
<td>-4.1700</td>
<td>2.8886</td>
<td>67.42</td>
<td>-26.8328</td>
<td>1.39</td>
<td>221900</td>
</tr>
<tr>
<td>3</td>
<td>0.0000</td>
<td>0.6442</td>
<td>48.24</td>
<td>-4.4721</td>
<td>0.62</td>
<td>17088</td>
</tr>
<tr>
<td>4</td>
<td>0.0000</td>
<td>0.6442</td>
<td>48.24</td>
<td>-4.4721</td>
<td>0.62</td>
<td>13896</td>
</tr>
<tr>
<td>5</td>
<td>0.0000</td>
<td>0.6546</td>
<td>48.24</td>
<td>-4.4721</td>
<td>0.63</td>
<td>9811</td>
</tr>
<tr>
<td>6</td>
<td>-0.7728</td>
<td>0.8784</td>
<td>42.29</td>
<td>-4.2046</td>
<td>0.65</td>
<td>2025</td>
</tr>
<tr>
<td>7</td>
<td>0.0000</td>
<td>0.6026</td>
<td>48.24</td>
<td>-4.4721</td>
<td>0.58</td>
<td>11210</td>
</tr>
<tr>
<td>8</td>
<td>0.0000</td>
<td>0.6026</td>
<td>48.24</td>
<td>-4.4721</td>
<td>0.58</td>
<td>16495</td>
</tr>
<tr>
<td>9</td>
<td>-0.6100</td>
<td>0.9264</td>
<td>49.46</td>
<td>-6.9282</td>
<td>0.61</td>
<td>26447</td>
</tr>
</tbody>
</table>

Figure 4.4: SB Distribution by Slifker and Shapiro’s Method for Beam 1
Table 4.3: Mathematical Modeled Plastic Deformation Data (Slifker and Shapiro)

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Amplitude Distribution (40 to 60 dB) With Increments of 1 dB</th>
<th>Experimental Cycles To Failure Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>11809</td>
</tr>
<tr>
<td></td>
<td>1184.71 53.03 0 0 118.99 1090.78 1616.60</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>16584</td>
</tr>
<tr>
<td></td>
<td>3234.77 2076.23 1.78 1011.56 277.19 1820.81 3373.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.32 1.97 0.36 0.05 0.01 0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>16847</td>
</tr>
<tr>
<td></td>
<td>1564.03 1613.18 1386.67 0 0 1386.67 1613.18</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>19654</td>
</tr>
<tr>
<td></td>
<td>1274.39 1314.44 1129.88 0 0 1129.88 1314.44</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>16994</td>
</tr>
<tr>
<td></td>
<td>909.55 932.29 762.30 0 0 762.30 932.29</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>19189</td>
</tr>
<tr>
<td></td>
<td>1603.01 0 0 0 0 183.97 615.92 1157.07</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>13573</td>
</tr>
<tr>
<td></td>
<td>954.77 1008.70 1050.59 0 0 1050.59 1008.70</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>15833</td>
</tr>
<tr>
<td></td>
<td>1408.72 1488.28 1550.10 0 0 1550.10 1488.28</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>16084</td>
</tr>
<tr>
<td></td>
<td>1126.31 754.75 381.37 1016.98 1689.46 1477.33</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0 0 0</td>
<td></td>
</tr>
</tbody>
</table>
4.4.2 Bounded Johnson (SB) Distribution by Mage’s Method

To find the best curve fitting model which has the lowest chi-square value via Mage’s method, the quantile, $z_1$ is iterated from -1.5 to -3.0 with decrements of 0.01 and the quantile, $z_2$ is iterated from -0.5 to -1.5 with similar decrements of 0.01. At over twenty-two thousand iterations, these values generate reasonable accuracy for the shape parameters without committing too much into superfluous computations.

Table 4.4 shows the best configuration parameters for each beam. Figure 4.5 shows the curve fitting of Mage’s method for beam 1. The curve fitting figures via Mage’s method for all beams are attached in Appendix E.2. The mathematically modeled plastic deformation data using Mage’s method for each beam is shown in Table 4.5.

**Table 4.4: Best Mage’s Method Configuration**

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>$\gamma$</th>
<th>$\eta$</th>
<th>$\varepsilon$</th>
<th>$\lambda$</th>
<th>$z_1$</th>
<th>$z_2$</th>
<th>$z_3$</th>
<th>$z_4$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.500</td>
<td>0.8066</td>
<td>43.66</td>
<td>2.6858</td>
<td>-2.05</td>
<td>-0.50</td>
<td>1.05</td>
<td>2.60</td>
<td>4797</td>
</tr>
<tr>
<td>2</td>
<td>-0.345</td>
<td>0.5974</td>
<td>42.76</td>
<td>4.4721</td>
<td>2.07</td>
<td>-0.92</td>
<td>0.23</td>
<td>1.38</td>
<td>29954</td>
</tr>
<tr>
<td>3</td>
<td>-0.520</td>
<td>0.6026</td>
<td>42.76</td>
<td>4.4721</td>
<td>-2.26</td>
<td>-1.10</td>
<td>0.06</td>
<td>1.22</td>
<td>16109</td>
</tr>
<tr>
<td>4</td>
<td>-0.450</td>
<td>0.6026</td>
<td>42.76</td>
<td>4.4721</td>
<td>-2.19</td>
<td>-1.03</td>
<td>0.13</td>
<td>-0.45</td>
<td>13322</td>
</tr>
<tr>
<td>5</td>
<td>-0.800</td>
<td>0.4173</td>
<td>43.55</td>
<td>2.8925</td>
<td>-1.51</td>
<td>-0.80</td>
<td>-0.09</td>
<td>0.62</td>
<td>8247</td>
</tr>
<tr>
<td>6</td>
<td>0.120</td>
<td>0.4288</td>
<td>43.73</td>
<td>2.5443</td>
<td>-1.70</td>
<td>-0.79</td>
<td>0.12</td>
<td>1.03</td>
<td>3769</td>
</tr>
<tr>
<td>7</td>
<td>-0.510</td>
<td>0.4763</td>
<td>43.60</td>
<td>2.8069</td>
<td>-2.21</td>
<td>-1.36</td>
<td>-0.51</td>
<td>0.34</td>
<td>4328</td>
</tr>
<tr>
<td>8</td>
<td>-0.630</td>
<td>0.5923</td>
<td>42.76</td>
<td>4.4721</td>
<td>-2.34</td>
<td>-1.20</td>
<td>-0.06</td>
<td>1.08</td>
<td>15859</td>
</tr>
<tr>
<td>9</td>
<td>-0.120</td>
<td>0.6650</td>
<td>42.76</td>
<td>4.4721</td>
<td>-2.04</td>
<td>-0.76</td>
<td>0.52</td>
<td>1.80</td>
<td>15646</td>
</tr>
</tbody>
</table>
Figure 4.5: SB Distribution by Mage’s Method for Beam 1
Table 4.5: Mathematical Modeled Plastic Deformation Data (Mage Method)

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Amplitude Distribution (40 to 60 dB) With Increments of 1 dB</th>
<th>Experimental Cycles To Failure Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 0 0 0 0 542.31 1743.27</td>
<td>11809</td>
</tr>
<tr>
<td></td>
<td>2555.08 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0 0 0 0 1548.54 2160.35 2486.34</td>
<td>16584</td>
</tr>
<tr>
<td></td>
<td>3212.40 5091.46 0 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0 0 0 0 569.05 998.72 1278.10</td>
<td>16847</td>
</tr>
<tr>
<td></td>
<td>1825.62 3475.76 0 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0 0 0 0 541.81 876.76 1077.38</td>
<td>19654</td>
</tr>
<tr>
<td></td>
<td>1477.68 2593.91 0 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0 0 0 0 549.33 650.97</td>
<td>16994</td>
</tr>
<tr>
<td></td>
<td>1710.76 0 0 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0 0 0 0 1690.96 876.35</td>
<td>19189</td>
</tr>
<tr>
<td></td>
<td>1359.20 0 0 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0 0 0 0 968.56 1055.67</td>
<td>13573</td>
</tr>
<tr>
<td></td>
<td>2304.94 0 0 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0 0 0 0 447.95 842.34 1135.23</td>
<td>15833</td>
</tr>
<tr>
<td></td>
<td>1727.43 3863.32 0 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0 0 0 0 941.63 1412.78 1497.83</td>
<td>16084</td>
</tr>
<tr>
<td></td>
<td>1647.34 1492.80 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0</td>
<td></td>
</tr>
</tbody>
</table>
4.4.3 Bounded Johnson (SB) Distribution by Linearization Method

For the Linearization method, to find the best curve fitting model where the linear line has the coefficient of determination, $R^2$ value closest to 1, the shape parameter $\lambda$ and $\epsilon$ are iterated from 0 to 100 with increments of 1. This double loop computation undertaken by MATLAB has ten thousand iterations. Increments of 1 were chosen due to the fact that finer increments caused no changes to the $R^2$ values to four significant digits (beyond the necessary accuracy of $\epsilon$ and $\lambda$), hence exponentially increasing the computational time were regarded pointless.

The first step is to generate the linear line plot to obtain all the four shape parameters. The linear plot for beam 1 is shown in Figure 4.6. Figure 4.6, was generated with $\epsilon$ and $\lambda$ equals to 20 and 50 respectively since this generated a linear line with the best $R^2$ value. The linear equation is attached on the top left of the figure. The gradient of the line is 16.02, which indicates that $\eta$ is 16.02. The z-axis interception value or $\gamma$ is -0.3994. After acquiring all the four shape parameters, the probability density function or curve fitting model can be generated.

![Figure 4.6: Linear Line Plot for Beam 1](image)
The linear line plot for all beams is attached in Appendix E.3. Table 4.6 shows the best Linearization method shape parameter configuration for all beams. The curve fitting model for beam 1 is shown in Figure 4.7. The curve fitting model via Linearization method for all beams is attached in Appendix E.4. The mathematically modeled plastic deformation data using Linearization method for each beam is shown in Table 4.7.

Table 4.6: Best Linearization Method Configuration

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>$\gamma$</th>
<th>$\eta$</th>
<th>$\varepsilon$</th>
<th>$\lambda$</th>
<th>$R^2$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.3994</td>
<td>16.02</td>
<td>20</td>
<td>50</td>
<td>0.999999516501796</td>
<td>1772</td>
</tr>
<tr>
<td>2</td>
<td>0.6296</td>
<td>9.33</td>
<td>22</td>
<td>49</td>
<td>0.999987478003354</td>
<td>7696</td>
</tr>
<tr>
<td>3</td>
<td>0.4568</td>
<td>9.32</td>
<td>22</td>
<td>49</td>
<td>0.999990005695765</td>
<td>4237</td>
</tr>
<tr>
<td>4</td>
<td>0.9020</td>
<td>9.33</td>
<td>22</td>
<td>50</td>
<td>0.999973406909009</td>
<td>3464</td>
</tr>
<tr>
<td>5</td>
<td>0.5529</td>
<td>9.35</td>
<td>22</td>
<td>49</td>
<td>0.999986475241873</td>
<td>2413</td>
</tr>
<tr>
<td>6</td>
<td>-0.6854</td>
<td>13.97</td>
<td>20</td>
<td>49</td>
<td>0.999999732836616</td>
<td>1596</td>
</tr>
<tr>
<td>7</td>
<td>0.6635</td>
<td>9.19</td>
<td>22</td>
<td>50</td>
<td>0.999989480549116</td>
<td>2807</td>
</tr>
<tr>
<td>8</td>
<td>0.6956</td>
<td>9.09</td>
<td>22</td>
<td>50</td>
<td>0.999968275558022</td>
<td>4161</td>
</tr>
<tr>
<td>9</td>
<td>1.4459</td>
<td>10.28</td>
<td>22</td>
<td>50</td>
<td>0.999965775884429</td>
<td>3745</td>
</tr>
</tbody>
</table>
Figure 4.7: SB Distribution by Linearization Method for Beam 1
Table 4.7: Mathematical Modeled Plastic Deformation Data (Linearization Method)

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Amplitude Distribution (40 to 60 dB) With Increments of 1 dB</th>
<th>Experimental Cycles To Failure Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0 1429.56 0 201.58 0.24 5.29 25.89 0.02 0 0</td>
<td>0 1946.20 0 0 0 0</td>
</tr>
<tr>
<td>2</td>
<td>0.21 3645.99 5.45 2257.61 69.87 783.61 466.31 151.16 0.89 0.03</td>
<td>0 0 0 0 0 0</td>
</tr>
<tr>
<td>3</td>
<td>0.05 2060.72 1.58 1455.50 23.23 576.70 177.32 127.04 726.95 15.30</td>
<td>0 0 0 0 0 0</td>
</tr>
<tr>
<td>4</td>
<td>0.08 1633.11 2.13 1098.79 27.19 424.77 183.40 93.81 675.55 1392.66</td>
<td>0 0 0 0 0 0</td>
</tr>
<tr>
<td>5</td>
<td>0.04 1168.61 1.25 766.33 17.26 281.01 123.57 57.09 474.19 6.32</td>
<td>0 0 0 0 0 0</td>
</tr>
<tr>
<td>6</td>
<td>0 884.07 0.02 141.39 2.80 5.87 84.46 0.06 0 0</td>
<td>0 0 0 0 0 0</td>
</tr>
<tr>
<td>7</td>
<td>0.03 1300.35 0.89 1044.42 12.53 490.14 94.57 133.73 396.26 20.96</td>
<td>0 0 0 0 0 0</td>
</tr>
<tr>
<td>8</td>
<td>0.07 1903.08 1.75 1492.48 22.60 691.39 159.04 188.24 628.90 29.76</td>
<td>0 0 0 0 0 0</td>
</tr>
<tr>
<td>9</td>
<td>0.10 733.18 3.58 159.36 53.58 17.52 1241.58 0.96 2056.53 1720.03</td>
<td>0 0 0 0 0 0</td>
</tr>
</tbody>
</table>

81
4.4.4 Chi-square Hypothesis Testing Comparison

The degree of goodness of fit for the plastic deformation data between all three bounded Johnson distributions is analysed. This is done as an indicator of which method is best suited in generating the best probability density function that can best describe the disposition of the plastic deformation data. Table 4.8 recaptures the chi-square, $\chi^2$ values of all three bounded Johnson distribution methods for all beams for easy comparison. Statistical analysis is performed on the $\chi^2$ values of each distribution method which are mean value, minimum value, maximum value and standard deviation.

**Table 4.8: Statistical Analysis of Chi-square ($\chi^2$) Values**

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>$\chi^2$ values</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slifker and Shapiro</td>
<td>Mage</td>
<td>Linearization</td>
</tr>
<tr>
<td>1</td>
<td>9895</td>
<td>4797</td>
<td>1772</td>
</tr>
<tr>
<td>2</td>
<td>221900</td>
<td>29954</td>
<td>7696</td>
</tr>
<tr>
<td>3</td>
<td>17088</td>
<td>16109</td>
<td>4237</td>
</tr>
<tr>
<td>4</td>
<td>13896</td>
<td>13322</td>
<td>3464</td>
</tr>
<tr>
<td>5</td>
<td>9811</td>
<td>8247</td>
<td>2413</td>
</tr>
<tr>
<td>6</td>
<td>2025</td>
<td>3769</td>
<td>1596</td>
</tr>
<tr>
<td>7</td>
<td>11210</td>
<td>4328</td>
<td>2807</td>
</tr>
<tr>
<td>8</td>
<td>16495</td>
<td>15859</td>
<td>4161</td>
</tr>
<tr>
<td>9</td>
<td>26447</td>
<td>15646</td>
<td>3745</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statistical Analysis</th>
<th>Slifker and Shapiro</th>
<th>Mage</th>
<th>Linearization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>36530</td>
<td>12448</td>
<td>3543</td>
</tr>
<tr>
<td>Min</td>
<td>2025</td>
<td>3769</td>
<td>1596</td>
</tr>
<tr>
<td>Max</td>
<td>221900</td>
<td>29954</td>
<td>7696</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>69831</td>
<td>8366</td>
<td>1834</td>
</tr>
</tbody>
</table>

From Table 4.8, the Linearization method has the least $\chi^2$ values for each beam indicating it is the best bounded Johnson distribution method in curve fitting the plastic deformation data.
4.5 Weibull Distribution Mathematical Modelling

To successfully model the plastic deformation (tabulated in Table 4.1) via Weibull Distribution, all the three Weibull parameters ($\alpha$, $\beta$ and $\gamma$) have to be determined. However, the $\gamma$ value is set at 42 because the minimum amplitude value for all nine sets of beam data is 42 dB.

The $\alpha$ and $\beta$ parameters are iterated from 2 to 5 with increments of 0.0001. Such high accuracy is recommended because the shape of the Weibull curve is very sensitive to slight changes in its parameter values. This double loop computation has nine hundred million iterations so the parameter values were first generated with the help of EasyFit® software created by MathWave Technologies to generate ballpark numbers. Then the iteration range was shrunk around these ballpark numbers and the computation reperformed with MATLAB®.

To find the best curve fitting model the linear line in the Weibull linear plot should have the coefficient of determination, $R^2$ value closest to 1. The linear plot for beam 1 is shown in Figure 4.8. The linear equation is attached on the top left of the figure. By using Equation 53 and 54, $\alpha$ is 4.0710 (slope of the line) and $\beta$ is 3.6791.

![Weibull Linear Plot for Beam 1](image)

Figure 4.8: Weibull Linear Plot for Beam 1
The linear line plot for all beams is attached in Appendix E.5. Table 4.9 shows the best Weibull shape parameter configuration for all beams.

Table 4.9: Weibull Distribution Parameters (Fixed $\gamma$)

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.0710</td>
<td>3.6791</td>
<td></td>
<td>0.984769907247008</td>
</tr>
<tr>
<td>2</td>
<td>3.0759</td>
<td>4.1231</td>
<td></td>
<td>0.988513084920737</td>
</tr>
<tr>
<td>3</td>
<td>3.3096</td>
<td>4.3581</td>
<td></td>
<td>1.000000000000000</td>
</tr>
<tr>
<td>4</td>
<td>3.1445</td>
<td>4.2512</td>
<td></td>
<td>0.984633930996672</td>
</tr>
<tr>
<td>5</td>
<td>3.2081</td>
<td>4.2260</td>
<td></td>
<td>0.980908134811515</td>
</tr>
<tr>
<td>6</td>
<td>3.3588</td>
<td>3.4878</td>
<td></td>
<td>0.979997961513432</td>
</tr>
<tr>
<td>7</td>
<td>3.3791</td>
<td>4.5731</td>
<td></td>
<td>0.992069827165424</td>
</tr>
<tr>
<td>8</td>
<td>3.3108</td>
<td>4.5182</td>
<td></td>
<td>0.997502686473216</td>
</tr>
<tr>
<td>9</td>
<td>2.9328</td>
<td>3.6542</td>
<td></td>
<td>0.977097829440674</td>
</tr>
</tbody>
</table>

Figure 4.9: Weibull Distribution (Fixed $\gamma$) for Beam 1
The curve fitting model for beam 1 is shown in Figure 4.9. The curve fitting model via Weibull distribution for all beams is attached in Appendix E.6. The mathematically modeled plastic deformation data using Weibull distribution for each beam is shown in Table 4.10.

**Table 4.10: Mathematical Modeled Plastic Deformation Data (Weibull)**

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Amplitude Distribution (40 to 60 dB) With Increments of 1 dB</th>
<th>Experimental Cycles To Failure Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1446.01 358.18 13.51 83.08 0.04 645.38 1576.43 11809</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3486.89 2255.93 844.55 481.61 169.92 1846.05 3276.49 16584</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1996.38 1467.96 606.34 171.16 127.21 792.50 1631.12 16847</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1575.94 1098.05 447.17 182.15 98.54 741.36 1390.22 19654</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1132.19 771.05 295.02 121.43 57.86 517.49 994.16 16994</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>895.41 258.66 23.41 163.48 0.51 729.45 1212.29 19189</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1262.95 1049.61 512.09 87.64 133.50 431.42 945.99 13573</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1857.25 1498.17 716.40 146.16 186.29 682.59 1440.04 15833</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1650.19 761.50 183.90 407.70 21.43 1341.93 1989.34 16084</td>
<td></td>
</tr>
</tbody>
</table>
From Table 4.9, the average $\alpha$ value is 3.3 which is in close proximity to the value 3.5, indicating the shape of the Weibull probability distribution function of all data sets closely imitate the normal distribution probability density function.
CHAPTER 5: RESULTS

5.1 Backpropagation Neural Network (BPNN) Results

5.1.1 Overview

All five sets of plastic deformation data as tabulated in Table 4.1 (non-mathematically modeled), Table 4.3 (bounded Johnson distribution model by Slifker and Shapiro’s method), Table 4.5 (bounded Johnson distribution model by Mage’s method), Table 4.7 (bounded Johnson distribution model by Linearization method) and Table 4.10 (Weibull distribution), are fed into the MATLAB based BPNN for fatigue life prediction.

To achieve the best prediction capability of the BPNN, the BPNN parameters are optimized until the percentage error between the predicted fatigue life cycle and experimental fatigue life cycle are the lowest.

5.1.2 BPNN Parameters

Based on Equation 51, since there are five predetermined failure mechanisms, the number of neurons in the hidden layer is set to 11. The values of the learning rate (‘net.trainParam.lr’), learning rate increase (‘net.trainParam.lr_inc’), momentum coefficient (‘net.trainParam.mc’) and ‘trainlm’ function parameters used are summarized in Table 5.1.

For all BPNN analysis, beam 1 to 5 is used as training data, beam 6 and 7 is used as validation data, and beam 8 and 9 is used as testing data.
Table 5.1: Neural Network Function Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>neurons (user named function)</td>
<td>11</td>
</tr>
<tr>
<td>net.trainParam.lr</td>
<td>0.01</td>
</tr>
<tr>
<td>net.trainParam.lr_inc</td>
<td>1.05</td>
</tr>
<tr>
<td>net.trainParam.mc</td>
<td>0.9</td>
</tr>
<tr>
<td>net.trainParam.epochs</td>
<td>100</td>
</tr>
<tr>
<td>net.trainParam.goal</td>
<td>1E-15</td>
</tr>
<tr>
<td>net.trainParam.max_fail</td>
<td>100</td>
</tr>
<tr>
<td>net.trainParam.mem_reduc</td>
<td>2</td>
</tr>
<tr>
<td>net.trainParam.min_grad</td>
<td>1e-10</td>
</tr>
<tr>
<td>net.trainParam.mu</td>
<td>0.001</td>
</tr>
<tr>
<td>net.trainParam.mu_dec</td>
<td>0.1</td>
</tr>
<tr>
<td>net.trainParam.mu_inc</td>
<td>10</td>
</tr>
<tr>
<td>net.trainParam.mu_max</td>
<td>1E+11</td>
</tr>
<tr>
<td>net.trainParam.show</td>
<td>25</td>
</tr>
<tr>
<td>net.trainParam.showCommandLine</td>
<td>0</td>
</tr>
<tr>
<td>net.trainParam.showWindow</td>
<td>1</td>
</tr>
<tr>
<td>net.trainParam.time</td>
<td>inf</td>
</tr>
</tbody>
</table>

5.1.3 BPNN Prediction Results

The BPNN error predictions for all five plastic deformation data are tabulated in Table 5.2 to 5.6. The absolute error is also computed and statistical analysis is performed on it. Statistical analysis involving mean value, minimum value, maximum value and standard deviation are performed on the absolute error. The results of this analysis for all sets of data are summarized in Table 5.7 for easy comparison.
<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Experimental Results</th>
<th>Predicted Results</th>
<th>Error (%)</th>
<th>Absolute Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11809</td>
<td>12025</td>
<td>1.83</td>
<td>1.83</td>
</tr>
<tr>
<td>2</td>
<td>16584</td>
<td>14087</td>
<td>-15.06</td>
<td>15.06</td>
</tr>
<tr>
<td>3</td>
<td>16847</td>
<td>16747</td>
<td>-0.59</td>
<td>0.59</td>
</tr>
<tr>
<td>4</td>
<td>19654</td>
<td>19633</td>
<td>-0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>5</td>
<td>16994</td>
<td>19116</td>
<td>12.48</td>
<td>12.48</td>
</tr>
<tr>
<td>6</td>
<td>19189</td>
<td>18073</td>
<td>-5.82</td>
<td>5.82</td>
</tr>
<tr>
<td>7</td>
<td>13573</td>
<td>13591</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>8</td>
<td>15833</td>
<td>17171</td>
<td>8.45</td>
<td>8.45</td>
</tr>
<tr>
<td>9</td>
<td>16084</td>
<td>21696</td>
<td>34.89</td>
<td>34.89</td>
</tr>
</tbody>
</table>

Table 5.3: BPNN Prediction Results (Slifker and Shapiro’s Method)

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Experimental Results</th>
<th>Predicted Results</th>
<th>Error (%)</th>
<th>Absolute Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11809</td>
<td>12296</td>
<td>4.13</td>
<td>4.13</td>
</tr>
<tr>
<td>2</td>
<td>16584</td>
<td>16653</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td>3</td>
<td>16847</td>
<td>17296</td>
<td>2.67</td>
<td>2.67</td>
</tr>
<tr>
<td>4</td>
<td>19654</td>
<td>17219</td>
<td>-12.39</td>
<td>12.39</td>
</tr>
<tr>
<td>5</td>
<td>16994</td>
<td>17185</td>
<td>1.12</td>
<td>1.12</td>
</tr>
<tr>
<td>6</td>
<td>19189</td>
<td>18880</td>
<td>-1.61</td>
<td>1.61</td>
</tr>
<tr>
<td>7</td>
<td>13573</td>
<td>15385</td>
<td>13.35</td>
<td>13.35</td>
</tr>
<tr>
<td>8</td>
<td>15833</td>
<td>15534</td>
<td>-1.89</td>
<td>1.89</td>
</tr>
<tr>
<td>9</td>
<td>16084</td>
<td>10080</td>
<td>-37.33</td>
<td>37.33</td>
</tr>
</tbody>
</table>
Table 5.4: BPNN Prediction Results (Mage’s Method)

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Experimental Results</th>
<th>Predicted Results</th>
<th>Error (%)</th>
<th>Absolute Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11809</td>
<td>11961</td>
<td>1.29</td>
<td>1.29</td>
</tr>
<tr>
<td>2</td>
<td>16584</td>
<td>7880</td>
<td>-52.48</td>
<td>52.48</td>
</tr>
<tr>
<td>3</td>
<td>16847</td>
<td>17012</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>4</td>
<td>19654</td>
<td>19555</td>
<td>-0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>5</td>
<td>16994</td>
<td>18688</td>
<td>9.97</td>
<td>9.97</td>
</tr>
<tr>
<td>6</td>
<td>19189</td>
<td>19634</td>
<td>2.32</td>
<td>2.32</td>
</tr>
<tr>
<td>7</td>
<td>13573</td>
<td>13581</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>8</td>
<td>15833</td>
<td>20167</td>
<td>27.37</td>
<td>27.37</td>
</tr>
<tr>
<td>9</td>
<td>16084</td>
<td>12013</td>
<td>-25.31</td>
<td>25.31</td>
</tr>
</tbody>
</table>

Table 5.5: BPNN Prediction Results (Linearization Method)

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Experimental Results</th>
<th>Predicted Results</th>
<th>Error (%)</th>
<th>Absolute Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11809</td>
<td>11739</td>
<td>-0.59</td>
<td>0.59</td>
</tr>
<tr>
<td>2</td>
<td>16584</td>
<td>14743</td>
<td>-11.10</td>
<td>11.10</td>
</tr>
<tr>
<td>3</td>
<td>16847</td>
<td>16008</td>
<td>-4.98</td>
<td>4.98</td>
</tr>
<tr>
<td>4</td>
<td>19654</td>
<td>18721</td>
<td>-4.75</td>
<td>4.75</td>
</tr>
<tr>
<td>5</td>
<td>16994</td>
<td>19110</td>
<td>12.45</td>
<td>12.45</td>
</tr>
<tr>
<td>6</td>
<td>19189</td>
<td>15980</td>
<td>-16.72</td>
<td>16.72</td>
</tr>
<tr>
<td>7</td>
<td>13573</td>
<td>13394</td>
<td>-1.32</td>
<td>1.32</td>
</tr>
<tr>
<td>8</td>
<td>15833</td>
<td>14924</td>
<td>-5.74</td>
<td>5.74</td>
</tr>
<tr>
<td>9</td>
<td>16084</td>
<td>17512</td>
<td>8.88</td>
<td>8.88</td>
</tr>
</tbody>
</table>
Table 5.6: BPNN Prediction Results (Weibull Distribution – Fixed γ)

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Experimental Results</th>
<th>Predicted Results</th>
<th>Error (%)</th>
<th>Absolute Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11809</td>
<td>10293</td>
<td>-12.84</td>
<td>12.84</td>
</tr>
<tr>
<td>2</td>
<td>16584</td>
<td>17648</td>
<td>6.42</td>
<td>6.42</td>
</tr>
<tr>
<td>3</td>
<td>16847</td>
<td>13680</td>
<td>-18.80</td>
<td>18.80</td>
</tr>
<tr>
<td>4</td>
<td>19654</td>
<td>19367</td>
<td>-1.46</td>
<td>1.46</td>
</tr>
<tr>
<td>5</td>
<td>16994</td>
<td>19573</td>
<td>15.17</td>
<td>15.17</td>
</tr>
<tr>
<td>6</td>
<td>19189</td>
<td>19889</td>
<td>3.65</td>
<td>3.65</td>
</tr>
<tr>
<td>7</td>
<td>13573</td>
<td>15278</td>
<td>12.56</td>
<td>12.56</td>
</tr>
<tr>
<td>8</td>
<td>15833</td>
<td>16052</td>
<td>1.38</td>
<td>1.38</td>
</tr>
<tr>
<td>9</td>
<td>16084</td>
<td>16371</td>
<td>1.78</td>
<td>1.78</td>
</tr>
</tbody>
</table>

5.1.4 BPNN Prediction Summary

Table 5.7: Statistical Analysis of the Absolute Errors

<table>
<thead>
<tr>
<th>Statistical Analysis</th>
<th>No Model</th>
<th>Johnson Distribution</th>
<th>Weibull Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>8.82</td>
<td>8.32</td>
<td>13.37</td>
</tr>
<tr>
<td>Min</td>
<td>0.11</td>
<td>0.42</td>
<td>0.06</td>
</tr>
<tr>
<td>Max</td>
<td>34.89</td>
<td>37.33</td>
<td>52.48</td>
</tr>
<tr>
<td>StdDev</td>
<td>11.23</td>
<td>11.90</td>
<td>18.18</td>
</tr>
</tbody>
</table>

The non-mathematically modeled data is also fed into the BPNN to establish the effectiveness of the mathematical data modeling process. From Table 5.7, it can be seen that all four mathematical modeling methods compare well with the non modeled data.
Furthermore, from Table 5.7, bounded Johnson distribution via Linearization method proves to be the best mathematically modeling method since it gives the lowest BPNN prediction error. The highest percentage error is about 17%.

5.2 Multiple Linear Regression (MLR) Results

5.2.1 Overview

MLR analysis is performed on the bounded Johnson shape parameters as tabulated in Table 4.2 (bounded Johnson distribution model by Slifker and Shapiro’s method), Table 4.4 (bounded Johnson distribution model by Mage’s method) and Table 4.6 (bounded Johnson distribution model by Linearization method).

MLR analysis is also performed on the Weibull shape parameters as tabulated in Table 4.9.

5.2.2 ANOVA p-Value Rejection Criteria

The maximum value of number of fatigue cycles to failure is 19654. The mean value is 16285.22. The standard deviation is 2457.78. The percentage error between the maximum value and the mean value is 20.69%. Using Equation 44, this data is normalized based on the normal distribution and the z value calculated for the maximum number of fatigue cycles to failure is 1.3707. The z value is converted to the two-tailed p-value which is 0.1705.

Hence, the \( \alpha \) value set for the p-value test in ANOVA is 0.20 whereby if the p-value of a variable is higher than \( \alpha = 0.20 \), the variable is rejected from the model. The selected value is higher than 0.1705 to ensure that the strength of test is elevated. By making \( \alpha \) value higher, the probability of Type 1 error (tested hypothesis is wrongly rejected) increases which produces a more powerful test. [33] For this research, it is more costly to make a Type 2 error (failing to reject tested hypothesis when it is false) and not very costly to commit Type 1 error. This means that it is vital to reject any shape parameters that do not fit into the prediction model since the ultimate goal is to establish prediction models with the most optimum (or least) number of independent variables.

ANOVA results generated by MATLAB® for all Johnson and Weibull shape parameters are attached in Appendix G.
5.2.3 Multiple Linear Regression (MLR) Results for Johnson Distribution

5.2.3.1 MLR Variables

The independent variables for regression analysis are chosen from the four bounded Johnson shape parameters which are; \( \gamma \), \( \eta \), \( \varepsilon \) and \( \lambda \). The independent variables are broken down into orders and analyzed via forward selection stepwise regression method. This method is ideal to compare the effectiveness of the analysis depending on the complexity of the independent variables. Table 5.8 tabulates the independent variables. The dependent variables are the experimental fatigue life cycle values as tabulated in Table 2.1.

Table 5.8: Independent MLR Variables

<table>
<thead>
<tr>
<th>Order</th>
<th>Independent Variable</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>( \gamma ) ( \eta ) ( \varepsilon ) ( \lambda )</td>
<td>4</td>
</tr>
<tr>
<td>2nd</td>
<td>( \gamma \eta ) ( \gamma \varepsilon ) ( \gamma \lambda ) ( \eta \varepsilon ) ( \eta \lambda ) ( \varepsilon \lambda )</td>
<td>6</td>
</tr>
<tr>
<td>3rd</td>
<td>( \gamma \eta \varepsilon ) ( \gamma \eta \lambda ) ( \gamma \varepsilon \lambda ) ( \eta \varepsilon \lambda )</td>
<td>4</td>
</tr>
<tr>
<td>4th</td>
<td>( \gamma \eta \varepsilon \lambda )</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>15</td>
</tr>
</tbody>
</table>

Adding 3\(^{rd}\) and 4\(^{th}\) order independent variables into the analysis are superfluous. By adding these higher order variables there is a possibility where the prediction model equations do not truly describe the real physical significance of the bounded Johnson shape parameters. From ANOVA, since there are nine sets of beam data, only a maximum of seven predictor variables can be included into the prediction model. Since there are four 1\(^{st}\) order terms, this indicates that only a maximum of three of the six 2\(^{nd}\) order terms can be included into the model. Hence, stepwise regression with the aid of ANOVA is performed to determine which 2\(^{nd}\) order terms can be included.

5.2.3.2 Stepwise Regression (SR) for Slifker and Shapiro’s Method

The SR analysis is performed on the variables generated from the mathematically modeled plastic deformation data via Slifker and Shapiro’s method. The prediction model generated is listed in Equation 66. The final ANOVA results are tabulated in Table 5.9.
Fatigue Life Cycle Prediction = $-1637219.2 - 21703.4\gamma + 63956.4\eta + 35628.0\epsilon + 23516.3\lambda + 32600.7\gamma\eta$ \hspace{1cm} (66)

Table 5.9: ANOVA Results for Stepwise Regression (Slifker and Shapiro’s Method)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>5</td>
<td>9.67E+06</td>
<td>3.21</td>
<td>0.1830</td>
</tr>
<tr>
<td>Residual</td>
<td>9.03E+06</td>
<td>3</td>
<td>3.01E+06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5.74E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R$^2$</td>
<td>0.8425</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.10 tabulates the percentage errors between the MLR prediction values from the target (experimental) values.

Table 5.10: MLR (SR) Prediction Results (Slifker and Shapiro’s Method)

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Experimental Results</th>
<th>Predicted Results</th>
<th>Error (%)</th>
<th>Absolute Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11809</td>
<td>11809</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>16584</td>
<td>16771</td>
<td>1.13</td>
<td>1.13</td>
</tr>
<tr>
<td>3</td>
<td>16847</td>
<td>17511</td>
<td>3.94</td>
<td>3.94</td>
</tr>
<tr>
<td>4</td>
<td>19654</td>
<td>17511</td>
<td>-10.90</td>
<td>10.90</td>
</tr>
<tr>
<td>5</td>
<td>16994</td>
<td>18177</td>
<td>6.96</td>
<td>6.96</td>
</tr>
<tr>
<td>6</td>
<td>19189</td>
<td>19189</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>13573</td>
<td>14851</td>
<td>9.41</td>
<td>9.41</td>
</tr>
<tr>
<td>8</td>
<td>15833</td>
<td>14851</td>
<td>-6.20</td>
<td>6.20</td>
</tr>
<tr>
<td>9</td>
<td>16084</td>
<td>16084</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

5.2.3.3 Stepwise Regression (SR) for Mage’s Method

The SR analysis is performed on the variables generated from the mathematically modeled plastic deformation data via Mage’s method. The prediction model generated is listed in Equation 67. The final ANOVA results are tabulated in Table 5.11.
Fatigue Life Cycle Prediction = 32594748.5 + 20828.6\gamma - 23086.5\eta -
723631.2\varepsilon - 362390.7\lambda - 2570.4\gamma\eta \tag{67}

Table 5.1: ANOVA Results for Stepwise Regression (Mage’s Method)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>5</td>
<td>9.67E+06</td>
<td>3.52</td>
<td>0.1646</td>
</tr>
<tr>
<td>Residual</td>
<td>8.23E+06</td>
<td>3</td>
<td>2.74E+06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5.32E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.8544</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.12 tabulates the percentage errors between the MLR prediction values from the target (experimental) values.

Table 5.12: MLR (SR) Prediction Results (Mage’s Method)

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Experimental Results</th>
<th>Predicted Results</th>
<th>Error (%)</th>
<th>Absolute Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11809</td>
<td>11803</td>
<td>-0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>16584</td>
<td>17368</td>
<td>4.72</td>
<td>4.72</td>
</tr>
<tr>
<td>3</td>
<td>16847</td>
<td>17096</td>
<td>1.48</td>
<td>1.48</td>
</tr>
<tr>
<td>4</td>
<td>19654</td>
<td>17180</td>
<td>-12.59</td>
<td>12.59</td>
</tr>
<tr>
<td>5</td>
<td>16994</td>
<td>16973</td>
<td>-0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>6</td>
<td>19189</td>
<td>19251</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>7</td>
<td>13573</td>
<td>13528</td>
<td>-0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>8</td>
<td>15833</td>
<td>16990</td>
<td>7.31</td>
<td>7.31</td>
</tr>
<tr>
<td>9</td>
<td>16084</td>
<td>16380</td>
<td>1.84</td>
<td>1.84</td>
</tr>
</tbody>
</table>

5.2.3.4 Stepwise Regression (SR) for Linearization Method

The SR analysis is performed on the variables generated from the mathematically modeled plastic deformation data via Linearization method. The prediction model generated is listed in Equation 68. The final ANOVA results are tabulated in Table 5.13.
\[ \text{Fatigue Life Cycle Prediction} = -2167377.4 - 1435260.4\gamma + 211272.0\eta + 156842.9\varepsilon - 25799.4\lambda - 14841.3\gamma\eta + 32098.4\gamma\lambda - 611.3\eta\varepsilon \] (68)

Table 5.13: ANOVA Results for Stepwise Regression (Linearization Method)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>7</td>
<td>6.90E+06</td>
<td>40.35</td>
<td>0.1206</td>
</tr>
<tr>
<td>Residual</td>
<td>1.71E+05</td>
<td>1</td>
<td>1.71E+05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>4.85E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.9965</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.14 tabulates the percentage errors between the MLR prediction values from the target (experimental) values.

Table 5.14: MLR (SR) Prediction Results (Linearization Method)

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Experimental Results</th>
<th>Predicted Results</th>
<th>Error (%)</th>
<th>Absolute Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11809</td>
<td>11809</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>16584</td>
<td>16766</td>
<td>1.09</td>
<td>1.09</td>
</tr>
<tr>
<td>3</td>
<td>16847</td>
<td>16992</td>
<td>0.86</td>
<td>0.86</td>
</tr>
<tr>
<td>4</td>
<td>19654</td>
<td>19672</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>5</td>
<td>16994</td>
<td>16668</td>
<td>-1.92</td>
<td>1.92</td>
</tr>
<tr>
<td>6</td>
<td>19189</td>
<td>19189</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>13573</td>
<td>13636</td>
<td>0.47</td>
<td>0.47</td>
</tr>
<tr>
<td>8</td>
<td>15833</td>
<td>15754</td>
<td>-0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>9</td>
<td>16084</td>
<td>16082</td>
<td>-0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

5.2.3.5 Johnson Shape Parameters MLR Prediction Model Summary

The \(R^2\) value from ANOVA testing for the Linearization method is the highest. This indicates that the bounded Johnson shape parameters generated via Linearization method is able to generate the best prediction model.
Table 5.15 lists the statistical analysis performed on the absolute errors of all three methods of the bounded Johnson distribution. The Linearization method has the best predictive ability with the lowest maximum error value of 1.92%.

**Table 5.15: Statistical Analysis on the Absolute Errors of all Bounded Johnson Methods**

<table>
<thead>
<tr>
<th>Statistical Analysis</th>
<th>Slifker and Shapiro</th>
<th>Mage</th>
<th>Linearization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>4.28</td>
<td>3.20</td>
<td>0.55</td>
</tr>
<tr>
<td>Min</td>
<td>0.00</td>
<td>0.05</td>
<td>0.00</td>
</tr>
<tr>
<td>Max</td>
<td>10.90</td>
<td>12.59</td>
<td>1.92</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.27</td>
<td>4.30</td>
<td>0.65</td>
</tr>
</tbody>
</table>

From Equation 66 to 69, the shape parameters $\gamma$ and $\eta$ play a much larger role in the prediction models compared to the scale parameter, $\lambda$ and the location parameter, $\varepsilon$. This indicates that the prediction model is heavily dependent on the shape of the distribution.

### 5.2.4 Multiple Linear Regression (MLR) Results for Weibull Distribution

#### 5.2.4.1 MLR Variables

The independent variables for regression analysis are chosen from the Weibull shape parameters which are; $\alpha$ and $\beta$. The shape parameter $\gamma$ is not included since this parameter is a fixed quantity. Hence, there are only two independent variables. The independent variables are broken down into orders and analyzed via forward selection stepwise regression method. Table 5.16 tabulates the independent variables. The dependent variables are the experimental fatigue life cycle values as tabulated in Table 2.1.

**Table 5.16: Independent MLR Variables**

<table>
<thead>
<tr>
<th>Order</th>
<th>Independent Variable</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>$\alpha$ $\beta$</td>
<td>2</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>$\alpha \beta$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>3</td>
</tr>
</tbody>
</table>
5.2.4.2 Stepwise Regression (SR) for Weibull Distribution

From ANOVA, the parameter $\beta$ individually did not contribute significantly in generating the optimum prediction model. Instead, $\alpha$ and the product of the two shape parameters, $\alpha\beta$ produced the highest $R^2$ value of 0.6500 with a $p$ value of 0.1561 as tabulated in Table 5.17.

This indicates that the prediction model is heavily dependent on the shape of the distribution since $\alpha$ is the shape parameter. The prediction model generated is listed in Equation 69. Table 5.18 tabulates the percentage errors between the MLR prediction values from the target (experimental) values.

\[
\text{Fatigue Life Cycle Prediction} = 34187.62 - 3802.28\alpha - 392.68\alpha\beta
\]  

(69)

Table 5.17: ANOVA Results for Stepwise Regression (Weibull Distribution)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>2</td>
<td>2.42E+07</td>
<td>5.57</td>
<td>0.1561</td>
</tr>
<tr>
<td>Residual</td>
<td>2.60E+07</td>
<td>6</td>
<td>4.34E+06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>7.43E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.6500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.18: MLR (SR) Prediction Results (Weibull Distribution)

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Experimental Results</th>
<th>Predicted Results</th>
<th>Error (%)</th>
<th>Absolute Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11809</td>
<td>12827</td>
<td>8.62</td>
<td>8.62</td>
</tr>
<tr>
<td>2</td>
<td>16584</td>
<td>17512</td>
<td>5.60</td>
<td>5.60</td>
</tr>
<tr>
<td>3</td>
<td>16847</td>
<td>15940</td>
<td>-5.39</td>
<td>5.39</td>
</tr>
<tr>
<td>4</td>
<td>19654</td>
<td>16982</td>
<td>-13.60</td>
<td>13.60</td>
</tr>
<tr>
<td>5</td>
<td>16994</td>
<td>16666</td>
<td>-1.93</td>
<td>1.93</td>
</tr>
<tr>
<td>6</td>
<td>19189</td>
<td>16816</td>
<td>-12.36</td>
<td>12.36</td>
</tr>
<tr>
<td>7</td>
<td>13573</td>
<td>15271</td>
<td>12.51</td>
<td>12.51</td>
</tr>
<tr>
<td>8</td>
<td>15833</td>
<td>15725</td>
<td>-3.55</td>
<td>3.55</td>
</tr>
<tr>
<td>9</td>
<td>16084</td>
<td>18828</td>
<td>-2.23</td>
<td>2.23</td>
</tr>
</tbody>
</table>
5.2.4.3 Weibull Shape Parameters MLR Prediction Model Summary

Table 5.19 lists the statistical analysis performed on the absolute errors of Weibull distribution.

Table 5.19: Statistical Analysis on the Absolute Errors of Weibull Distribution

<table>
<thead>
<tr>
<th>Statistical Analysis</th>
<th>Weibull Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>7.31</td>
</tr>
<tr>
<td>Min</td>
<td>1.93</td>
</tr>
<tr>
<td>Max</td>
<td>13.60</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.60</td>
</tr>
</tbody>
</table>

5.2.5 MLR Prediction Summary

Table 5.20: Statistical Analysis of the Absolute Errors

<table>
<thead>
<tr>
<th>Statistical Analysis</th>
<th>Johnson Distribution</th>
<th>Weibull Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slifker and Shapiro</td>
<td>Mage</td>
</tr>
<tr>
<td>Mean</td>
<td>4.28</td>
<td>3.20</td>
</tr>
<tr>
<td>Min</td>
<td>0.00</td>
<td>0.05</td>
</tr>
<tr>
<td>Max</td>
<td>10.90</td>
<td>12.59</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.27</td>
<td>4.30</td>
</tr>
</tbody>
</table>

The prediction model generated from bounded Johnson distribution shape parameters via Linearization method is the best. The maximum error of this method is less than 2 %. However, the maximum percentage errors for all methods are below 15 %.
CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

Acoustic Emission (AE) Nondestructive Testing (NDT) is a powerful tool in collecting data to facilitate the analysis of multiple failure mechanisms undergoing in a structure or specimen which is being subjected to any types of loading. AE data collected from the nine steel I-beams subjected to fatigue cycle loading is filtered to remove multiple hit data (MHD) and outliers. The three filtering techniques employed are omission of any data with duration more than 1900 μs, 0 energy and average frequency less than 15 kHz.

The filtered data are classified using Kohonen Self Organizing Map (KSOM) with five mechanisms. The input parameters used during this classification are energy, duration, amplitude and average frequency. After classification, the amplitude data are identified and extracted among the five mechanisms via key plots and statistical analysis employing the fact that plastic deformation data have the lowest energy, duration and amplitude.

The plastic deformation data are mathematically modeled via bounded Johnson distribution. Three methods of the bounded Johnson distribution are utilized. The data is also modeled via Weibull distribution. These two methods are chosen since they are capable of curve fitting bounded models which imitate the normal distribution probability density function. For the bounded Johnson distribution, four shape parameters; η, γ, ε and λ are needed to define its probability density function. Similarly, for the Weibull distribution, three shape parameters; α, β and γ are needed to define its probability density function.

The mathematical modeled plastic deformation data are subjected to Backpropagating Neural Network (BPNN) prediction analysis. The Linearization method of the bounded Johnson distribution has the best prediction capability with a maximum percentage error of 16.72%. The Weibull distribution comes in second with a maximum percentage error of 18.80%.

Multiple Linear Regression (MLR) analysis is also undertaken where the shape parameters of the bounded Johnson distribution and Weibull distribution are subjected to forward selection stepwise regression analysis. The Linearization method of the bounded Johnson distribution has the best prediction model with a maximum percentage error of 1.92%. For the Weibull
distribution, only $\alpha$ and $\beta$ perimeters are subjected to MLR analysis since the $\gamma$ perimeter is set at 42 dB which is the minimum amplitude threshold of all nine plastic deformation data sets. The prediction model developed has a maximum percentage error of 13.60%.

The figure below summarizes the five main steps undertaken in this research.

![Flow Chart of Major Processes Undertaken](image)
6.2 Recommendations

Two recommendations are proposed here to enhance BPNN and MLR prediction results.

Different classifying techniques other than KSOM can be used to identify and extract plastic deformation data. Other types of unsupervised clustering techniques are K-Means and Gaussian Mixture Model (GMM). Supervised clustering techniques can also be used such as Kth Nearest Neighbor (KNN), Learning Vector Quantization (LVQ), or Support Vector Machine (SVM)

Another mathematical modeling method to model the plastic deformation data can be used which is the Phased Bi-Weibull distribution. This distribution utilizes 6 parameters compared to only 4 parameters for Johnson distribution and 3 parameters for Weibull distribution. This distribution is recommended since it is a mixture of two Weibull distributions: 2-parameter Weibull and 3-parameter Weibull which makes it a very flexible distribution.
REFERENCES


30. Santosa, Budi. Introduction To Matlab Neural Network Toolbox. West Lafayette, Indiana: Purdue University, 1 Sept. 1992. PDF.


APPENDIX A: TOPOLOGY FIGURES FOR AE FILTERED DATA

Figure A.1: Filtered Data of Beam 1

Figure A.2: Filtered Data of Beam 2
Figure A.3: Filtered Data of Beam 3

Figure A.4: Filtered Data of Beam 4
Figure A.5: Filtered Data of Beam 5

Figure A.6: Filtered Data of Beam 6
Figure A.7: Filtered Data of Beam 7

Figure A.8: Filtered Data of Beam 8
Figure A.9: Filtered Data of Beam 9
APPENDIX B: KSOM CLASSIFICATIONS

B.1 Verification Criterion

![Verification Criterion Analysis for Beam 1](image1)

**Figure B.1:** Verification Criterion Analysis for Beam 1

![Verification Criterion Analysis for Beam 2](image2)

**Figure B.2:** Verification Criterion Analysis for Beam 2
Figure B.3: Verification Criterion Analysis for Beam 3

Figure B.4: Verification Criterion Analysis for Beam 4
Figure B.5: Verification Criterion Analysis for Beam 5

Figure B.6: Verification Criterion Analysis for Beam 6
Figure B.7: Verification Criterion Analysis for Beam 7

Figure B.8: Verification Criterion Analysis for Beam 8
B.2 Three Classifications

Figure B.9: Verification Criterion Analysis for Beam 9

Figure B.10: KSOM EDAF with 3 Classifications for Beam 1
Figure B.11: KSOM EDAF with 3 Classifications for Beam 2

Figure B.12: KSOM EDAF with 3 Classifications for Beam 3
Figure B.13: KSOM EDAF with 3 Classifications for Beam 4

Figure B.14: KSOM EDAF with 3 Classifications for Beam 5
Figure B.15: KSOM EDAF with 3 Classifications for Beam 6

Figure B.16: KSOM EDAF with 3 Classifications for Beam 7
Figure B.17: KSOM EDAF with 3 Classifications for Beam 8

Figure B.18: KSOM EDAF with 3 Classifications for Beam 9
Table B.1: Legend for 3 Classifications

<table>
<thead>
<tr>
<th>Legend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanism 1</td>
</tr>
<tr>
<td>Mechanism 2</td>
</tr>
<tr>
<td>Mechanism 3</td>
</tr>
</tbody>
</table>

B.3 Five Classifications

Figure B.19: KSOM EDAF with 5 Classifications for Beam 1
Figure B.20: KSOM EDAF with 5 Classifications for Beam 2

Figure B.21: KSOM EDAF with 5 Classifications for Beam 3
Figure B.22: KSOM EDAF with 5 Classifications for Beam 4

Figure B.23: KSOM EDAF with 5 Classifications for Beam 5
Figure B.24: KSOM EDAF with 5 Classifications for Beam 6

Figure B.25: KSOM EDAF with 5 Classifications for Beam 7
**Figure B.26:** KSOM EDAF with 5 Classifications for Beam 8

**Figure B.27:** KSOM EDAF with 5 Classifications for Beam 9
Table B.2: Legend for 5 Classifications

<table>
<thead>
<tr>
<th>Legend</th>
</tr>
</thead>
<tbody>
<tr>
<td>MHD Type 3 (Mechanism 1)</td>
</tr>
<tr>
<td>MHD Type 2 (Mechanism 2)</td>
</tr>
<tr>
<td>MHD Type 1 (Mechanism 3)</td>
</tr>
<tr>
<td>Fatigue Cracking (Mechanism 4)</td>
</tr>
<tr>
<td>Plastic Deformation (Mechanism 5)</td>
</tr>
</tbody>
</table>
APPENDIX C: STATISTICAL ANALYSIS OF KSOM 5 MECHANISMS

C.1 Statistical Analysis of Beam 1

Table C.1: Energy Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>35</td>
<td>21.20</td>
<td>2.13</td>
<td>4084</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>22</td>
<td>17.35</td>
<td>2.28</td>
<td>4014</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>19</td>
<td>11.55</td>
<td>2.13</td>
<td>2401</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>14</td>
<td>7.05</td>
<td>1.73</td>
<td>2956</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>10</td>
<td>5.79</td>
<td>1.54</td>
<td>4122</td>
</tr>
</tbody>
</table>

Table C.2: Duration Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1286</td>
<td>1899</td>
<td>1841.84</td>
<td>63.50</td>
<td>4084</td>
</tr>
<tr>
<td>2</td>
<td>1383</td>
<td>1899</td>
<td>1803.81</td>
<td>97.48</td>
<td>4014</td>
</tr>
<tr>
<td>3</td>
<td>851</td>
<td>1554</td>
<td>1130.49</td>
<td>159.82</td>
<td>2401</td>
</tr>
<tr>
<td>4</td>
<td>128</td>
<td>1003</td>
<td>687.62</td>
<td>131.26</td>
<td>2956</td>
</tr>
<tr>
<td>5</td>
<td>105</td>
<td>1093</td>
<td>654.01</td>
<td>136.03</td>
<td>4122</td>
</tr>
</tbody>
</table>

Table C.3: Amplitude Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47</td>
<td>55</td>
<td>48.93</td>
<td>0.94</td>
<td>4084</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
<td>50</td>
<td>47.19</td>
<td>0.86</td>
<td>4014</td>
</tr>
<tr>
<td>3</td>
<td>43</td>
<td>52</td>
<td>47.10</td>
<td>1.23</td>
<td>2401</td>
</tr>
<tr>
<td>4</td>
<td>47</td>
<td>54</td>
<td>47.74</td>
<td>0.96</td>
<td>2956</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
<td>47</td>
<td>45.31</td>
<td>0.78</td>
<td>4122</td>
</tr>
</tbody>
</table>
C.2 Statistical Analysis of Beam 2

Table C.4: Energy Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>35</td>
<td>21.07</td>
<td>2.46</td>
<td>6569</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>24</td>
<td>17.56</td>
<td>1.80</td>
<td>15808</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>21</td>
<td>14.87</td>
<td>1.48</td>
<td>12183</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>20</td>
<td>9.92</td>
<td>1.98</td>
<td>6930</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>10</td>
<td>5.61</td>
<td>1.37</td>
<td>12378</td>
</tr>
</tbody>
</table>

Table C.5: Duration Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1314</td>
<td>1899</td>
<td>1851.69</td>
<td>57.62</td>
<td>6569</td>
</tr>
<tr>
<td>2</td>
<td>1318</td>
<td>1899</td>
<td>1840.96</td>
<td>67.19</td>
<td>15808</td>
</tr>
<tr>
<td>3</td>
<td>1376</td>
<td>1899</td>
<td>1798.46</td>
<td>105.43</td>
<td>12183</td>
</tr>
<tr>
<td>4</td>
<td>509</td>
<td>1475</td>
<td>1088.40</td>
<td>174.67</td>
<td>6930</td>
</tr>
<tr>
<td>5</td>
<td>84</td>
<td>1049</td>
<td>651.78</td>
<td>132.19</td>
<td>12378</td>
</tr>
</tbody>
</table>

Table C.6: Amplitude Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47</td>
<td>55</td>
<td>49.46</td>
<td>0.89</td>
<td>6569</td>
</tr>
<tr>
<td>2</td>
<td>46</td>
<td>50</td>
<td>47.88</td>
<td>0.66</td>
<td>15808</td>
</tr>
<tr>
<td>3</td>
<td>44</td>
<td>48</td>
<td>46.33</td>
<td>0.72</td>
<td>12183</td>
</tr>
<tr>
<td>4</td>
<td>43</td>
<td>55</td>
<td>46.70</td>
<td>1.30</td>
<td>6930</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
<td>54</td>
<td>45.68</td>
<td>1.31</td>
<td>12378</td>
</tr>
</tbody>
</table>
C.3 Statistical Analysis of Beam 3

**Table C.7: Energy Analysis**

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>34</td>
<td>22.61</td>
<td>2.26</td>
<td>3732</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>25</td>
<td>19.45</td>
<td>2.28</td>
<td>5179</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>22</td>
<td>15.80</td>
<td>2.00</td>
<td>3690</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>18</td>
<td>10.60</td>
<td>2.08</td>
<td>1610</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>11</td>
<td>6.11</td>
<td>1.48</td>
<td>3366</td>
</tr>
</tbody>
</table>

**Table C.8: Duration Analysis**

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1326</td>
<td>1899</td>
<td>1858.92</td>
<td>44.87</td>
<td>3732</td>
</tr>
<tr>
<td>2</td>
<td>1363</td>
<td>1899</td>
<td>1848.88</td>
<td>60.51</td>
<td>5179</td>
</tr>
<tr>
<td>3</td>
<td>1418</td>
<td>1899</td>
<td>1819.25</td>
<td>89.63</td>
<td>3690</td>
</tr>
<tr>
<td>4</td>
<td>746</td>
<td>1483</td>
<td>1110.63</td>
<td>172.23</td>
<td>1610</td>
</tr>
<tr>
<td>5</td>
<td>218</td>
<td>1029</td>
<td>669.78</td>
<td>122.47</td>
<td>3366</td>
</tr>
</tbody>
</table>

**Table C.9: Amplitude Analysis**

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48</td>
<td>55</td>
<td>49.55</td>
<td>0.80</td>
<td>3732</td>
</tr>
<tr>
<td>2</td>
<td>46</td>
<td>50</td>
<td>47.85</td>
<td>0.59</td>
<td>5179</td>
</tr>
<tr>
<td>3</td>
<td>43</td>
<td>48</td>
<td>46.25</td>
<td>0.81</td>
<td>3690</td>
</tr>
<tr>
<td>4</td>
<td>43</td>
<td>52</td>
<td>46.80</td>
<td>1.34</td>
<td>1610</td>
</tr>
<tr>
<td>5</td>
<td>43</td>
<td>51</td>
<td>45.89</td>
<td>1.32</td>
<td>3366</td>
</tr>
</tbody>
</table>
C.4 Statistical Analysis of Beam 4

Table C.10: Energy Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>34</td>
<td>20.28</td>
<td>1.88</td>
<td>5863</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>21</td>
<td>16.90</td>
<td>1.42</td>
<td>5915</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>19</td>
<td>12.39</td>
<td>1.83</td>
<td>1115</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>12</td>
<td>8.34</td>
<td>1.30</td>
<td>2058</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>9</td>
<td>5.47</td>
<td>1.07</td>
<td>2626</td>
</tr>
</tbody>
</table>

Table C.11: Duration Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1465</td>
<td>1900</td>
<td>1855.32</td>
<td>44.85</td>
<td>5863</td>
</tr>
<tr>
<td>2</td>
<td>1492</td>
<td>1900</td>
<td>1828.61</td>
<td>69.80</td>
<td>5915</td>
</tr>
<tr>
<td>3</td>
<td>963</td>
<td>1579</td>
<td>1272.62</td>
<td>137.58</td>
<td>1115</td>
</tr>
<tr>
<td>4</td>
<td>627</td>
<td>1156</td>
<td>876.37</td>
<td>104.33</td>
<td>2058</td>
</tr>
<tr>
<td>5</td>
<td>155</td>
<td>835</td>
<td>608.69</td>
<td>92.26</td>
<td>2626</td>
</tr>
</tbody>
</table>

Table C.12: Amplitude Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>46</td>
<td>55</td>
<td>48.75</td>
<td>1.02</td>
<td>5863</td>
</tr>
<tr>
<td>2</td>
<td>43</td>
<td>50</td>
<td>47.078</td>
<td>0.88</td>
<td>5915</td>
</tr>
<tr>
<td>3</td>
<td>43</td>
<td>52</td>
<td>47.17</td>
<td>1.20</td>
<td>1115</td>
</tr>
<tr>
<td>4</td>
<td>42</td>
<td>52</td>
<td>46.61</td>
<td>1.24</td>
<td>2058</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
<td>53</td>
<td>45.83</td>
<td>1.31</td>
<td>2626</td>
</tr>
</tbody>
</table>
### C.5 Statistical Analysis of Beam 5

#### Table C.13: Energy Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>41</td>
<td>23.23</td>
<td>1.91</td>
<td>3927</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>26</td>
<td>20.57</td>
<td>1.56</td>
<td>6646</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>23</td>
<td>18.44</td>
<td>2.04</td>
<td>3737</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>17</td>
<td>10.62</td>
<td>2.16</td>
<td>893</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>10</td>
<td>5.95</td>
<td>1.26</td>
<td>2374</td>
</tr>
</tbody>
</table>

#### Table C.14: Duration Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1445</td>
<td>1899</td>
<td>1866.57</td>
<td>34.02</td>
<td>3927</td>
</tr>
<tr>
<td>2</td>
<td>1387</td>
<td>1899</td>
<td>1856.47</td>
<td>46.66</td>
<td>6646</td>
</tr>
<tr>
<td>3</td>
<td>1422</td>
<td>1899</td>
<td>1846.24</td>
<td>64.58</td>
<td>3737</td>
</tr>
<tr>
<td>4</td>
<td>709</td>
<td>1524</td>
<td>1080.03</td>
<td>173.82</td>
<td>893</td>
</tr>
<tr>
<td>5</td>
<td>113</td>
<td>963</td>
<td>664.41</td>
<td>108.45</td>
<td>2374</td>
</tr>
</tbody>
</table>

#### Table C.15: Amplitude Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48</td>
<td>56</td>
<td>49.92</td>
<td>0.81</td>
<td>3927</td>
</tr>
<tr>
<td>2</td>
<td>47</td>
<td>51</td>
<td>48.43</td>
<td>0.53</td>
<td>6646</td>
</tr>
<tr>
<td>3</td>
<td>43</td>
<td>48</td>
<td>46.49</td>
<td>0.95</td>
<td>3737</td>
</tr>
<tr>
<td>4</td>
<td>43</td>
<td>51</td>
<td>46.96</td>
<td>1.32</td>
<td>893</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
<td>51</td>
<td>45.78</td>
<td>1.28</td>
<td>2374</td>
</tr>
</tbody>
</table>
C.6 Statistical Analysis of Beam 6

Table C.16: Energy Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>41</td>
<td>20.28</td>
<td>2.85</td>
<td>2394</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>22</td>
<td>15.72</td>
<td>2.45</td>
<td>2660</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>17</td>
<td>9.92</td>
<td>1.98</td>
<td>1497</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>14</td>
<td>6.35</td>
<td>1.81</td>
<td>1580</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>9</td>
<td>5.19</td>
<td>1.38</td>
<td>3282</td>
</tr>
</tbody>
</table>

Table C.17: Duration Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1313</td>
<td>1899</td>
<td>1845.59</td>
<td>65.02</td>
<td>2394</td>
</tr>
<tr>
<td>2</td>
<td>1373</td>
<td>1899</td>
<td>1796.19</td>
<td>103.79</td>
<td>2660</td>
</tr>
<tr>
<td>3</td>
<td>795</td>
<td>1480</td>
<td>1057.72</td>
<td>154.11</td>
<td>1497</td>
</tr>
<tr>
<td>4</td>
<td>85</td>
<td>937</td>
<td>652.22</td>
<td>139.89</td>
<td>1580</td>
</tr>
<tr>
<td>5</td>
<td>58</td>
<td>979</td>
<td>628.20</td>
<td>125.91</td>
<td>3282</td>
</tr>
</tbody>
</table>

Table C.18: Amplitude Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47</td>
<td>55</td>
<td>49.06</td>
<td>1.11</td>
<td>2394</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
<td>49</td>
<td>46.90</td>
<td>0.87</td>
<td>2660</td>
</tr>
<tr>
<td>3</td>
<td>44</td>
<td>50</td>
<td>46.60</td>
<td>1.17</td>
<td>1497</td>
</tr>
<tr>
<td>4</td>
<td>47</td>
<td>53</td>
<td>47.65</td>
<td>0.94</td>
<td>1580</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
<td>46</td>
<td>45.10</td>
<td>0.88</td>
<td>3282</td>
</tr>
</tbody>
</table>
C.7 Statistical Analysis of Beam 7

Table C.19: Energy Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>42</td>
<td>22.37</td>
<td>2.37</td>
<td>1994</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>27</td>
<td>20.33</td>
<td>2.09</td>
<td>6879</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>24</td>
<td>18.06</td>
<td>2.06</td>
<td>3875</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>19</td>
<td>11.21</td>
<td>2.11</td>
<td>1521</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>11</td>
<td>6.38</td>
<td>1.41</td>
<td>3308</td>
</tr>
</tbody>
</table>

Table C.20: Duration Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1405</td>
<td>1899</td>
<td>1853.86</td>
<td>50.03</td>
<td>1994</td>
</tr>
<tr>
<td>2</td>
<td>1377</td>
<td>1899</td>
<td>1846.39</td>
<td>57.34</td>
<td>6879</td>
</tr>
<tr>
<td>3</td>
<td>1453</td>
<td>1899</td>
<td>1832.31</td>
<td>72.85</td>
<td>3875</td>
</tr>
<tr>
<td>4</td>
<td>734</td>
<td>1511</td>
<td>1117.15</td>
<td>164.53</td>
<td>1521</td>
</tr>
<tr>
<td>5</td>
<td>95</td>
<td>995</td>
<td>674.39</td>
<td>115.03</td>
<td>3308</td>
</tr>
</tbody>
</table>

Table C.21: Amplitude Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49</td>
<td>54</td>
<td>50.29</td>
<td>0.60</td>
<td>1994</td>
</tr>
<tr>
<td>2</td>
<td>48</td>
<td>50</td>
<td>48.46</td>
<td>0.50</td>
<td>6879</td>
</tr>
<tr>
<td>3</td>
<td>43</td>
<td>48</td>
<td>46.54</td>
<td>0.76</td>
<td>3875</td>
</tr>
<tr>
<td>4</td>
<td>43</td>
<td>53</td>
<td>47.07</td>
<td>1.28</td>
<td>1521</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
<td>52</td>
<td>46.10</td>
<td>1.34</td>
<td>3308</td>
</tr>
</tbody>
</table>
C.8 Statistical Analysis of Beam 8

Table C.22: Energy Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
<td>45</td>
<td>23.10</td>
<td>2.30</td>
<td>2944</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>27</td>
<td>20.44</td>
<td>1.65</td>
<td>6689</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>22</td>
<td>17.67</td>
<td>1.92</td>
<td>3529</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>17</td>
<td>10.86</td>
<td>2.13</td>
<td>1550</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>11</td>
<td>6.15</td>
<td>1.31</td>
<td>2865</td>
</tr>
</tbody>
</table>

Table C.23: Duration Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1327</td>
<td>1899</td>
<td>1857.81</td>
<td>45.36</td>
<td>2944</td>
</tr>
<tr>
<td>2</td>
<td>1391</td>
<td>1899</td>
<td>1849.34</td>
<td>53.81</td>
<td>6689</td>
</tr>
<tr>
<td>3</td>
<td>1400</td>
<td>1899</td>
<td>1820.85</td>
<td>90.01</td>
<td>3529</td>
</tr>
<tr>
<td>4</td>
<td>737</td>
<td>1475</td>
<td>1082.02</td>
<td>167.24</td>
<td>1550</td>
</tr>
<tr>
<td>5</td>
<td>126</td>
<td>946</td>
<td>664.47</td>
<td>110.62</td>
<td>2865</td>
</tr>
</tbody>
</table>

Table C.24: Amplitude Analysis

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49</td>
<td>55</td>
<td>50.14</td>
<td>0.75</td>
<td>2944</td>
</tr>
<tr>
<td>2</td>
<td>47</td>
<td>50</td>
<td>48.48</td>
<td>0.53</td>
<td>6689</td>
</tr>
<tr>
<td>3</td>
<td>43</td>
<td>49</td>
<td>46.86</td>
<td>0.70</td>
<td>3529</td>
</tr>
<tr>
<td>4</td>
<td>43</td>
<td>52</td>
<td>47.17</td>
<td>1.29</td>
<td>1550</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
<td>51</td>
<td>46.10</td>
<td>1.35</td>
<td>2865</td>
</tr>
</tbody>
</table>
### C.9 Statistical Analysis of Beam 9

**Table C.25: Energy Analysis**

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>31</td>
<td>20.06</td>
<td>1.87</td>
<td>5785</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>21</td>
<td>16.42</td>
<td>1.75</td>
<td>4052</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>18</td>
<td>11.54</td>
<td>1.86</td>
<td>1448</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>12</td>
<td>7.45</td>
<td>1.31</td>
<td>2938</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>8</td>
<td>4.84</td>
<td>0.96</td>
<td>3354</td>
</tr>
</tbody>
</table>

**Table C.26: Duration Analysis**

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1355</td>
<td>1899</td>
<td>1852.43</td>
<td>47.62</td>
<td>5785</td>
</tr>
<tr>
<td>2</td>
<td>1406</td>
<td>1899</td>
<td>1811.21</td>
<td>84.07</td>
<td>4052</td>
</tr>
<tr>
<td>3</td>
<td>907</td>
<td>1589</td>
<td>1221.64</td>
<td>149.36</td>
<td>1448</td>
</tr>
<tr>
<td>4</td>
<td>346</td>
<td>1089</td>
<td>806.81</td>
<td>114.28</td>
<td>2938</td>
</tr>
<tr>
<td>5</td>
<td>128</td>
<td>895</td>
<td>583.29</td>
<td>101.34</td>
<td>3354</td>
</tr>
</tbody>
</table>

**Table C.27: Amplitude Analysis**

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Number of Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47</td>
<td>57</td>
<td>48.74</td>
<td>1.02</td>
<td>5785</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
<td>50</td>
<td>46.96</td>
<td>0.88</td>
<td>4052</td>
</tr>
<tr>
<td>3</td>
<td>44</td>
<td>52</td>
<td>46.93</td>
<td>1.31</td>
<td>1448</td>
</tr>
<tr>
<td>4</td>
<td>43</td>
<td>52</td>
<td>46.65</td>
<td>1.32</td>
<td>2938</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
<td>50</td>
<td>45.26</td>
<td>1.16</td>
<td>3354</td>
</tr>
</tbody>
</table>
APPENDIX D: PLASTIC DEFORMATION HISTOGRAM

D.1 Histograms with Bin Size of 2

Figure D.1: Plastic Deformation Histogram with Bin Size of 2 for Beam 1

Figure D.2: Plastic Deformation Histogram with Bin Size of 2 for Beam 2
Figure D.3: Plastic Deformation Histogram with Bin Size of 2 for Beam 3

Figure D.4: Plastic Deformation Histogram with Bin Size of 2 for Beam 4
Figure D.5: Plastic Deformation Histogram with Bin Size of 2 for Beam 5

Figure D.6: Plastic Deformation Histogram with Bin Size of 2 for Beam 6
Figure D.7: Plastic Deformation Histogram with Bin Size of 2 for Beam 7

Figure D.8: Plastic Deformation Histogram with Bin Size of 2 for Beam 8
Figure D.9: Plastic Deformation Histogram with Bin Size of 2 for Beam 9

D.2 Histograms with Bin Size of 0.5

Figure D.10: Plastic Deformation Histogram with Bin Size of 0.5 for Beam 1
Figure D.11: Plastic Deformation Histogram with Bin Size of 0.5 for Beam 2

Figure D.12: Plastic Deformation Histogram with Bin Size of 0.5 for Beam 3
Figure D.13: Plastic Deformation Histogram with Bin Size of 0.5 for Beam 4

Figure D.14: Plastic Deformation Histogram with Bin Size of 0.5 for Beam 5
Figure D.15: Plastic Deformation Histogram with Bin Size of 0.5 for Beam 6

Figure D.16: Plastic Deformation Histogram with Bin Size of 0.5 for Beam 7
Figure D.17: Plastic Deformation Histogram with Bin Size of 0.5 for Beam 8

Figure D.18: Plastic Deformation Histogram with Bin Size of 0.5 for Beam 9
D.3 Histograms with Bin Size of 1

Figure D.19: Plastic Deformation Histogram with Bin Size of 1 for Beam 1

Figure D.20: Plastic Deformation Histogram with Bin Size of 1 for Beam 2
Figure D.21: Plastic Deformation Histogram with Bin Size of 1 for Beam 3

Figure D.22: Plastic Deformation Histogram with Bin Size of 1 for Beam 4
Figure D.23: Plastic Deformation Histogram with Bin Size of 1 for Beam 5

Figure D.24: Plastic Deformation Histogram with Bin Size of 1 for Beam 6
Figure D.25: Plastic Deformation Histogram with Bin Size of 1 for Beam 7

Figure D.26: Plastic Deformation Histogram with Bin Size of 1 for Beam 8
Figure D.27: Plastic Deformation Histogram with Bin Size of 1 for Beam 9
APPENDIX E: MATHEMATICALLY MODELED PLASTIC DEFORMATION DATA

E.1 Mathematically Modeled by Johnson Distribution Slifker and Shapiro’s Method

Figure E.1: SB Distribution by Slifker and Shapiro’s Method for Beam 1

Figure E.2: SB Distribution by Slifker and Shapiro’s Method for Beam 2
Figure E.3: SB Distribution by Slifker and Shapiro’s Method for Beam 3

Figure E.4: SB Distribution by Slifker and Shapiro’s Method for Beam 4
Figure E.5: SB Distribution by Slifker and Shapiro’s Method for Beam 5

Figure E.6: SB Distribution by Slifker and Shapiro’s Method for Beam 6
Figure E.7: SB Distribution by Slifker and Shapiro’s Method for Beam 7

Figure E.8: SB Distribution by Slifker and Shapiro’s Method for Beam 8
Figure E.9: SB Distribution by Slifker and Shapiro’s Method for Beam 9

E.2 Mathematically Modeled by Johnson Distribution Mage’s Method

Figure E.10: SB Distribution by Mage’s Method for Beam 1
Figure E.11: SB Distribution by Mage’s Method for Beam 2

Figure E.12: SB Distribution by Mage’s Method for Beam 3
Figure E.13: SB Distribution by Mage’s Method for Beam 4

Figure E.14: SB Distribution by Mage’s Method for Beam 5
Figure E.15: SB Distribution by Mage’s Method for Beam 6

Figure E.16: SB Distribution by Mage’s Method for Beam 7
Figure E.17: SB Distribution by Mage’s Method for Beam 8

Figure E.18: SB Distribution by Mage’s Method for Beam 9
E.3 Linear Line Plots for Johnson Distribution Linearization Method

Figure E.19: Linear Line Plot for Beam 1

Figure E.20: Linear Line Plot for Beam 2
Figure E.21: Linear Line Plot for Beam 3

Figure E.22: Linear Line Plot for Beam 4
Figure E.23: Linear Line Plot for Beam 5

Figure E.24: Linear Line Plot for Beam 6
Figure E.25: Linear Line Plot for Beam 7

Figure E.26: Linear Line Plot for Beam 8
Figure E.27: Linear Line Plot for Beam 9

E.4 Mathematically Modeled by Johnson Distribution Linearization Method

Figure E.28: SB Distribution by Linearization Method for Beam 1
Figure E.29: SB Distribution by Linearization Method for Beam 2

Figure E.30: SB Distribution by Linearization Method for Beam 3
Figure E.31: SB Distribution by Linearization Method for Beam 4

Figure E.32: SB Distribution by Linearization Method for Beam 5
Figure E.33: SB Distribution by Linearization Method for Beam 6

Figure E.34: SB Distribution by Linearization Method for Beam 7
Figure E.35: SB Distribution by Linearization Method for Beam 8

Figure E.36: SB Distribution by Linearization Method for Beam 9
E.5 Linear Line Plots for Weibull Distribution

Figure E.37: Weibull Linear Plot for Beam 1

Figure E.38: Weibull Linear Plot for Beam 2
Figure E.39: Weibull Linear Plot for Beam 3

Figure E.40: Weibull Linear Plot for Beam 4
Figure E.41: Weibull Linear Plot for Beam 5

Figure E.42: Weibull Linear Plot for Beam 6
Figure E.43: Weibull Linear Plot for Beam 7

Figure E.44: Weibull Linear Plot for Beam 8
Figure E.45: Weibull Linear Plot for Beam 9

E.6 Mathematically Modeled by Weibull Distribution

Figure E.46: Weibull Distribution for Beam 1
Figure E.47: Weibull Distribution for Beam 2

Figure E.48: Weibull Distribution for Beam 3
Figure E.49: Weibull Distribution for Beam 4

Figure E.50: Weibull Distribution for Beam 5
Figure E.51: Weibull Distribution for Beam 6

Figure E.52: Weibull Distribution for Beam 7
Figure E.53: Weibull Distribution for Beam 8

Figure E.54: Weibull Distribution for Beam 9
APPENDIX F: MATLAB CODE

F.1 VerificationCriterion.m

% Verification Criterion

fileNameInput = importdata('fileName.xlsx');
[a1,b]=size(fileNameInput.Sheet1);

for fileInc = 1:a1
    inputM = fileNameInput.Sheet1(fileInc,1);
    inputD = fileNameInput.Sheet1(fileInc,2);
    inputT = fileNameInput.Sheet1(fileInc,3);
    T = str2num(cell2mat(inputT));
    rawInput = strcat(inputM,'_',inputD,'_',inputT,'_txtN.xlsx');
    fileNameEdit = strcat(inputM,'-',inputD,'-',inputT,'-Ksom');

    trainFile = xlsread(str2mat(rawInput));
    dataextract = trainFile(:,(4:7));
    dataextract(logical(sum(dataextract~=dataextract,2)),:)=[];

    q=2;p=1;k=7;

    for num = 2:k
        classification = num;
        kohonen = [1 classification];
        TFCN = 'hextop';
        DFCN = 'linkdist';
        STEPS = 100;
        IN = 70;

        net = newsom(input,kohonen,TFCN,DFCN,STEPS,IN);
        net.trainParam.epochs = (length(input))/100;
        net.trainParam.show = 25;
        net.trainParam.showCommandLine = false;
        net.trainParam.showWindow = true;
        net.trainParam.time = inf;

        [net_updated,training_record,net_output] = train(net,input);
        y = net(input);
        clusternum = vec2ind(y);
output = [input; clusternum];
nntraintool('close');

idx = clusternum';

tracker = [max(idx) num]
idxSave(:,num-1) = idx;

% DB Criterion
for i=1:num
    N0=0;
    A=size(dataextract(idx==i,:));
    Txr(i,1)=A(1,1);
    D=dataextract(idx==i,:);
    ctrs(i,:)=mean(D);
    for l=1:Txr(i,1)
        Nr=(norm(D(l,:)-ctrs(i,:)))^q;
        N0=N0+Nr;
    end
    Nl(i,1)=N0;
    S(i,1)=(N0/Txr(i,1))^(1/q);
end

S;
for i=1:num
    for j=1:num
        M(i,j)=(sum(((ctrs(i,:)-ctrs(j,:)).^2).^(1/2)).^p)^(1/p);
    end
end
M;
for i=1:num
    for j=1:num
        R1(i,j)=(S(i,1)+S(j,1))/M(i,j);
    end
    R1(i,i)=0;
end
R1;
for i=1:num
    C(i,1)=max(R1(i,:));
end
C;
R(num-1,1)=(sum(C(:,1))./num);
cluster_num(num-1,1)=num;

% SW Criterion
Sx1 = 0;
for i=1:num
    D=dataextract(idx==i,:);
    ctrs(i,:)=mean(D);
end
for i=1:num
    D=dataextract(idx==i,:);
    ctrs(i,:)=mean(D);
    A=size(dataextract(idx==i,:));
    T1(i,1)=A(1,1);
    for g=1:T1(i,1)
        Nl=0;
Ns = (norm(D(g,:)-ctrs(i,:)))/(num-1);  
Nsum(i,g) = Ns;  
Nsum2(i,g) = inf;  
for h = 1:num  
    if (h ~= i)  
        N3 = 0;  
        N2 = (norm(D(g,:)-ctrs(h,:)))/(num-1);  
        N3 = N2 + N3;  
        Nsum2(i,g) = min(Nsum2(i,g), N3);  
    end  
end  
Sx = (Nsum2(i,g) - Nsum(i,g))/max(Nsum2(i,g), Nsum(i,g));  
Sx1 = Sx1 + Sx;  
end  
end  
SWc(num-1,1) = Sx1/sum(T1(:,1));  
cluster_num(num-1,1) = num;  

% T Criterion  
for i = 1:num  
    N0 = 0;  
    A = size(dataextract(idx==i,:));  
    T2(i,1) = A(1,1);  
    D = dataextract(idx==i,:);  
    ctrs(i,:) = mean(D);  
    for l = 1:T2(i,1)  
        Nt = (norm(D(l,:)-ctrs(i,:))).^2;  
        N0 = N0 + Nt;  
    end  
    N1(i,1) = N0;  
    S(i,1) = (2*(N0/T2(i,1))).^0.5;  
end  
S;  
S1 = max(S);  
for i = 1:num  
    for j = 1:num  
        Mx(i,j) = norm(ctrs(i,:)-ctrs(j,:));  
    end  
end  
Mx;  
min(Mx);  
N = min(min(Mx));  
To(num-1,1) = N/S1;  
cluster_num(num-1,1) = num;  
end  

%%  

fileNameExIDX = strcat(fileNameEdit, 'IDX.xlsx');  
xlsWrite(str2mat(fileNameExIDX), idxSave);  
dataR = R.^-1;  
dataS = SWc;  
dataT = To;  
constructR = dataR;
constructS = dataS;
constructT = dataT;

newR = constructR/max(abs(constructR));
newS = constructS/max(abs(constructS));
newT = constructT/max(abs(constructT));

[dsR, viR] = sort(newR);
xR, vrR = sort(viR);
[dsS, viS] = sort(newS);
xS, vrS = sort(viS);
[dsT, viT] = sort(newT);
xT, vrT = sort(viT);

for i=1:length(vrR)
    if vrR(i,:) == 6
        vrR(i,:) = 100;
    elseif vrR(i,:) == 5
        vrR(i,:) = 40;
    elseif vrR(i,:) == 4
        vrR(i,:) = 20;
    elseif vrR(i,:) ~= 6 && vrR(i,:) ~= 5 && vrR(i,:) ~= 4
        vrR(i,:) = 0;
    end
end
for i=1:length(vrS)
    if vrS(i,:) == 6
        vrS(i,:) = 100;
    elseif vrS(i,:) == 5
        vrS(i,:) = 40;
    elseif vrS(i,:) == 4
        vrS(i,:) = 20;
    elseif vrS(i,:) ~= 6 && vrS(i,:) ~= 5 && vrS(i,:) ~= 4
        vrS(i,:) = 0;
    end
end
for i=1:length(vrT)
    if vrT(i,:) == 6
        vrT(i,:) = 100;
    elseif vrT(i,:) == 5
        vrT(i,:) = 40;
    elseif vrT(i,:) == 4
        vrT(i,:) = 20;
    elseif vrT(i,:) ~= 6 && vrT(i,:) ~= 5 && vrT(i,:) ~= 4
        vrT(i,:) = 0;
    end
end
VotingValue = (vrR + vrS + vrT);
newVotingValue = VotingValue/max(abs(VotingValue));

figure(3)
plot(cluster_num,newR,'r','LineWidth', 3)
hold on
plot(cluster_num,newS,'g','LineWidth', 3)
hold on
plot(cluster_num,newT,'b','LineWidth', 3)
hold on
plot(cluster_num,newVotingValue,'k','LineWidth', 5)
axis([2 k -1 1])
set(gca,'xtick',2:k)
set(gca,'FontWeight','bold')
legend('DB','SW','T','Total Votes','Location','SouthWest')
title([fileNameEdit,' ', 'Verification Criterion'],'fontWeight','bold')
xlabel('Number of Clusters', 'fontWeight','bold')
ylabel('Criterion Value', 'fontWeight','bold')

filename2 = strcat(fileNameEdit,'-', 'AllVerification.jpg');
saveas(3,str2mat(filename2));

clearvars num i idx idxSave normdata C M R S SWc Txr To;
close all;
end
close all;
F.2 KSOMPlot.m

figure(1);
subplot(2,2,1)
hist(a,0:100);
title([fileNameEdit,'Amplitude Histogram'],'fontsize',12,...
'fontweight','b')
xlabel('Amplitude (dB)');
ylabel('Frequency of Occurrence');

subplot(2,2,2)
scatter(c,d,2);
title('Duration (µsec) vs. Counts','fontsize',12,'fontweight','b')
xlabel('Counts');
ylabel('Duration (µsec)');

subplot(2,2,3)
hist(f,0:150);
title('Frequency Histogram','fontsize',12,'fontweight','b')
xlabel('Average Frequency (kHz)');
ylabel('Frequency of Occurrence');
filename = strcat(fileNameEdit,'-Filtered Raw Data.jpg');
saveas(1,str2mat(filename))
end
% KSOM Plot

k = 5; % Specify number of classifications

for fileInc = 1:a1
    inputM = fileNameInput.Sheet1(fileInc,1);
    inputD = fileNameInput.Sheet1(fileInc,2);
    inputT = fileNameInput.Sheet1(fileInc,3);
    T = str2num(cell2mat(inputT));
    rawInput = strcat(inputM,'_',inputD,'_',inputT,'_txt.xlsx');
    fileNameEdit = strcat(inputM,'-',inputD,'-',inputT,'-EDAF');
    k1 = k-1;
    net_output = xlsread(str2mat(rawInput));
    binaryindex = (net_output(:,6:6+k1))';
    clusternum = vec2ind(binaryindex);
    idx = clusternum';
    kstring = num2str(k);
    [r c] = size(idx);
    c1count=1; c2count=1; c3count=1; c4count=1; c5count=1; c6count=1; c7count= 1;
    num = k;
    for inc = 1:r
        if num>=2
            if idx(inc,1) == 1
                class1(c1count,:) = [net_output(inc,1),...
                                    net_output(inc,2), net_output(inc,3),...
                                    net_output(inc,4), net_output(inc,5), idx(inc,1)];
                c1count = c1count+1;
            elseif idx(inc,1) == 2
                class2(c2count,:) = [net_output(inc,1),...
                                    net_output(inc,2), net_output(inc,3),...
                                    net_output(inc,4), net_output(inc,5), idx(inc,1)];
                c2count = c2count+1;
            end
        end
        if num>=3
            if idx(inc,1) == 3
                class3(c3count,:) = [net_output(inc,1),...
                                    net_output(inc,2), net_output(inc,3),...
                                    net_output(inc,4), net_output(inc,5), idx(inc,1)];
                c3count = c3count+1;
            end
        end
        if num>=4
            if idx(inc,1) == 4
                class4(c4count,:) = [net_output(inc,1),...
                                    net_output(inc,2), net_output(inc,3),...
                                    net_output(inc,4), net_output(inc,5), idx(inc,1)];
                c4count = c4count+1;
            end
        end
    end
end
if num >= 5
    if idx(inc,1) == 5
        class5(c5count,:) = [net_output(inc,1),...
                        net_output(inc,2), net_output(inc,3),...
                        net_output(inc,4), net_output(inc,5), idx(inc,1)];
        c5count = c5count+1;
    end
end
if num >= 6
    if idx(inc,1) == 6
        class6(c6count,:) = [net_output(inc,1),...
                        net_output(inc,2), net_output(inc,3),...
                        net_output(inc,4), net_output(inc,5), idx(inc,1)];
        c6count = c6count+1;
    end
end
if num == 7
    if idx(inc,1) == 7
        class7(c7count,:) = [net_output(inc,1),...
                        net_output(inc,2), net_output(inc,3),...
                        net_output(inc,4), net_output(inc,5), idx(inc,1)];
        c7count = c7count+1;
    end
end

figure(2)
subplot(2,2,1)  % Duration vs. Counts
if num >= 2
    plot(class1(:,1),class1(:,3),'xr')
    hold on
    plot(class2(:,1),class2(:,3),'xg')
    hold on
end
if num >= 3
    plot(class3(:,1),class3(:,3),'xb')
    hold on
end
if num >= 4
    plot(class4(:,1),class4(:,3),'xy')
    hold on
end
if num >= 5
    plot(class5(:,1),class5(:,3),'xm')
    hold on
end
if num >= 6
    plot(class6(:,1),class6(:,3),'xc')
    hold on
end
if num >= 7
    plot(class7(:,1),class7(:,3),'xk')
    hold on
end
title(['fileNameEdit', ',', kstring, ' Counts vs. Duration'],...
    'fontsize', 12, 'fontweight', 'b')
xlabel('Counts', 'fontsize', 12, 'fontweight', 'b')
ylabel('Duration (µs)', 'fontsize', 12, 'fontweight', 'b')

subplot(2,2,2) % Energy vs. Amplitude
if num>=2
    plot(class1(:,4),class1(:,2),'.r')
    hold on
    plot(class2(:,4),class2(:,2),'.g')
    hold on
end
if num>=3
    plot(class3(:,4),class3(:,2),'.b')
    hold on
end
if num>=4
    plot(class4(:,4),class4(:,2),'.y')
    hold on
end
if num>=5
    plot(class5(:,4),class5(:,2),'.m')
    hold on
end
if num>=6
    plot(class6(:,4),class6(:,2),'.c')
    hold on
end
if num==7
    plot(class7(:,4),class7(:,2),'.k')
    hold on
end

title(['Energy vs. Amplitude'], 'fontsize', 12, 'fontweight', 'b')
xlabel('Amplitude (dB)', 'fontsize', 12, 'fontweight', 'b')
ylabel('Energy', 'fontsize', 12, 'fontweight', 'b')

subplot(2,2,3) % Duration vs. Amplitude
if num>=2
    plot(class1(:,4),class1(:,3),'.r')
    hold on
    plot(class2(:,4),class2(:,3),'.g')
    hold on
end
if num>=3
    plot(class3(:,4),class3(:,3),'.b')
    hold on
end
if num>=4
    plot(class4(:,4),class4(:,3),'.y')
    hold on
end
if num>=5
    plot(class5(:,4),class5(:,3),'.m')
    hold on
end
if num>=6
    plot(class6(:,4),class6(:,3),'.c')
    hold on
end
end
if num==7  
    plot(class7(:,4),class7(:,3),'.k')  
    hold on  
end  
title(["Duration vs. Amplitude'\],'fontsize',12,'fontweight','b')  
xlabel('Duration (µs)',"fontsize",12,'fontweight','b')  
ylabel('Amplitude (dB)',"fontsize",12,'fontweight','b')  

subplot(2,2,4) % Amplitude vs. Average Frequency  
if num>=2  
    plot(class1(:,5),class1(:,4),'.r')  
    hold on  
    plot(class2(:,5),class2(:,4),'.g')  
    hold on  
end  
if num>=3  
    plot(class3(:,5),class3(:,4),'.b')  
    hold on  
end  
if num>=4  
    plot(class4(:,5),class4(:,4),'.y')  
    hold on  
end  
if num>=5  
    plot(class5(:,5),class5(:,4),'.m')  
    hold on  
end  
if num>=6  
    plot(class6(:,5),class6(:,4),'.c')  
    hold on  
end  
if num==7  
    plot(class7(:,5),class7(:,4),'.k')  
    hold on  
end  
title(["Amplitude vs. Average Frequency'\],'fontsize',12,'fontweight','b')  
xlabel('Average Frequency (kHz)',"fontsize",12,'fontweight','b')  
ylabel('Amplitude (dB)',"fontsize",12,'fontweight','b')  

filename = strcat(fileNameEdit,'_',kstring,'_Subplot.jpg');  
saveas(2,str2mat(filename))  
clearvars inc class1 class2 class3 class4 class5 class6 class7 figure(2)  
close all  
c1count =1;c2count=1;c3count=1;c4count=1;c5count=1;c6count=1;c7count=1;  
end  

fprintf('\nProgram terminated.')  
close all;
F.3 K-MeanAndGMM

```matlab
clc;
clear all;
close all;

k=5; % Specify number of classifications

fileNameInput = importdata('fileName2.xlsx');
[a,b]=size(fileNameInput.Sheet1);

for fileInc = 1:a
    inputM = fileNameInput.Sheet1(fileInc,1);
    inputD = fileNameInput.Sheet1(fileInc,2);
    inputT = fileNameInput.Sheet1(fileInc,3);
    T = str2num(cell2mat(inputT));
    rawInput = strcat(inputM,'_',inputD,'_',inputT,'_txtN.xlsx');
    fileNameEdit = strcat(inputM,'-',inputD,'-',inputT,'-Kmean');
    trainFile = xlsread (str2mat(rawInput));
    readOFile = xlsread (str2mat(rawInput));

    for i=4:7
        trainFile(:,i)=(trainFile(:,i)-min(trainFile(:,i)))/
        (max(trainFile(:,i))-min(trainFile(:,i)));
    end
    train = [trainFile(:,4),trainFile(:,5), trainFile(:,6), trainFile(:,7)];
    num = k;
    [idxKM,ctrs,sumd] = kmeans (train,num,'replicates',25,'display',...
    'final','maxiter',300);
end

options = statset('Display','final','maxiter',700);
for fileInc = 1:a
    inputM = fileNameInput.Sheet1(fileInc,1);
    inputD = fileNameInput.Sheet1(fileInc,2);
```
inputT = fileNameInput.Sheet1(fileInc,3);
T = str2num(cell2mat(inputT));

rawInput = strcat(inputM,'_ ',inputD,'_ ',inputT,'_ txtN.xlsx');
fileNameEdit = strcat(inputM,'- ',inputD,'- ',inputT,'- GMM');

trainFile = xlsread (str2mat(rawInput));
readOFile = xlsread (str2mat(rawInput));
train = [trainFile(:,4),trainFile(:,5), trainFile(:,6), trainFile(:,7)];
num = k;
gm = gmdistribution.fit(train,num,'Options',options,...
    'Regularize',0.01);
idxGMM = cluster(gm,train);
end

idxtarget = [idxKM idxGMM];
% idxKM is the vector column indicating the K-Mean classifications
% idxGMM is the vector column indicating the GMM classifications

close all;
F.4 StatisticalAnalysis.m

```matlab
clc;
clear all;
close all;

filename1 = '01_01_01.xlsx';
filename2 = 'StatisticalAnalysis.xlsx';
net_output = xlsread(filename1);

rowC1 = 1; rowC2 = 1; rowC3 = 1; rowC4 = 1; rowC5 = 1;

for i = 1:length(net_output)
    hold on;
    if net_output(i,6) == 1
        E1(rowC1,:) = net_output(i,3);
        D1(rowC1,:) = net_output(i,4);
        A1(rowC1,:) = net_output(i,5);
        rowC1 = rowC1+1;
    elseif net_output(i,6) == 2
        E2(rowC2,:) = net_output(i,3);
        D2(rowC2,:) = net_output(i,4);
        A2(rowC2,:) = net_output(i,5);
        rowC2 = rowC2+1;
    elseif net_output(i,6) == 3
        E3(rowC3,:) = net_output(i,3);
        D3(rowC3,:) = net_output(i,4);
        A3(rowC3,:) = net_output(i,5);
        rowC3 = rowC3+1;
    elseif net_output(i,6) == 4
        E4(rowC4,:) = net_output(i,4);
        D4(rowC4,:) = net_output(i,5);
        A4(rowC4,:) = net_output(i,6);
        rowC4 = rowC4+1;
    elseif net_output(i,6) == 5
        E5(rowC5,:) = net_output(i,4);
        D5(rowC5,:) = net_output(i,5);
        A5(rowC5,:) = net_output(i,6);
        rowC5 = rowC5+1;
    end
end
```

E1(isnan(E1(:,1)),:)=[]; E2(isnan(E2(:,1)),:)=[]; E3(isnan(E3(:,1)),:)=[];
E4(isnan(E4(:,1)),:)=[]; E5(isnan(E5(:,1)),:)=[];
E1(:,~any(E1,1)) =[]; E2(:,~any(E2,1)) =[]; E3(:,~any(E3,1)) =[];
E4(:,~any(E4,1)) =[]; E5(:,~any(E5,1)) =[];
D1(isnan(D1(:,1)),:)=[]; D2(isnan(D2(:,1)),:)=[]; D3(isnan(D3(:,1)),:)=[];
D4(isnan(D4(:,1)),:)=[]; D5(isnan(D5(:,1)),:)=[];
D1(:,~any(D1,1)) =[]; D2(:,~any(D2,1)) =[]; D3(:,~any(D3,1)) =[];
```
D4(:,~any(D4,1)) =[]; D5(:,~any(D5,1)) =[];
A1(isnan(A1(:,1)),:)=[]; A2(isnan(A2(:,1)),:)=[]; A3(isnan(A3(:,1)),:)=[];
A4(isnan(A4(:,1)),:)=[]; E5(isnan(A5(:,1)),:)=[];
A1(:,~any(A1,1)) =[]; A2(:,~any(A2,1)) =[]; A3(:,~any(A3,1)) =[];
A4(:,~any(A4,1)) =[]; A5(:,~any(A5,1)) =[];

StatE1 = [min(E1); max(E1); mean(E1); std2(E1); length(E1)];
StatE2 = [min(E2); max(E2); mean(E2); std2(E2); length(E2)];
StatE3 = [min(E3); max(E3); mean(E3); std2(E3); length(E3)];
StatE4 = [min(E4); max(E4); mean(E4); std2(E4); length(E4)];
StatE5 = [min(E5); max(E5); mean(E5); std2(E5); length(E5)];

StatD1 = [min(D1); max(D1); mean(D1); std2(D1); length(D1)];
StatD2 = [min(D2); max(D2); mean(D2); std2(D2); length(D2)];
StatD3 = [min(D3); max(D3); mean(D3); std2(D3); length(D3)];
StatD4 = [min(D4); max(D4); mean(D4); std2(D4); length(D4)];
StatD5 = [min(D5); max(D5); mean(D5); std2(D5); length(D5)];

StatA1 = [min(A1); max(A1); mean(A1); std2(A1); length(A1)];
StatA2 = [min(A2); max(A2); mean(A2); std2(A2); length(A2)];
StatA3 = [min(A3); max(A3); mean(A3); std2(A3); length(A3)];
StatA4 = [min(A4); max(A4); mean(A4); std2(A4); length(A4)];
StatA5 = [min(A5); max(A5); mean(A5); std2(A5); length(A5)];

StatD = [StatD1'; StatD2'; StatD3'; StatD4'; StatD5'];

xlswrite(filename2,StatE,1,'A1');
xlswrite(filename2,StatD,1,'A7');
xlswrite(filename2,StatA,1,'A13');

close all;
clc; clear all; close all;

filename     = '01_01_01_txtN.xlsx';
filenameedit = '01-01-01';
net_output   = xlsread(filename);

i = 1;
for i = 1:21
    hold on;
    pdamphist(i,:) = net_output(i,35);
    i = i + 1;
end

i = 1;
for i = 1:length(net_output)
    hold on;
    amphistplot(i,:) = net_output(i,6);
    i = i + 1;
end

i = 1;
for i = 1:length(net_output)
    hold on;
    if net_output(i,12) == 1;
        pdamphistplot(i,:) = net_output(i,6);
    end
    i = i + 1;
end

pdamphistplot(pdamphistplot==0,:) = [];
num = numel(pdamphistplot);

y = pdamphistplot;
ystack = histc(pdamphistplot,(40:1:60));

counter = 1;
for z = 0.50:0.01:1.50;
    zl = 3*z;
    z2 = z;
    z3 = -z;
    z4 = -3*z;

    percentage(1,1) = normcdf(zl,0,1)*100.000;
    percentage(2,1) = normcdf(z2,0,1)*100.000;
    percentage(3,1) = normcdf(z3,0,1)*100.000;
    percentage(4,1) = normcdf(z4,0,1)*100.000;
for prc = 1 : 4
    percentile(prc,1) = prctile(y,percentage(prc,1));
end

p = percentile(3,1) - percentile(2,1);
m = percentile(4,1) - percentile(3,1);
n = percentile(2,1) - percentile(1,1);
mnp = (m * n)/(p^2);

mp = m/p;
np = n/p;
pm = p/m;
pn = p/n;

if mnp < 0.999
    type = 'bounded';
    type_num = 1;
elseif mnp > 1.001
    type = 'unbounded';
    type_num = 2;
elseif 0.999 < mnp && mnp < 1.001
    type = 'lognormal';
    type_num = 3;
end

if type_num == 1
    eta = z/(acosh((1/2)*((1+pm)*(1+pn))^(1/2)));
    gamma = eta*asinh(((pn-pm)*((1+pm)*(1+pn)-4)^(1/2))/(2*(pm*pn-1)));
    lamda = (p*(((1+pm)*(1+pn)-2)^(2-4)^(1/2))/((pm*pn-1)));
    epsilon = ((percentile(2,1)+percentile(3,1))/2)-(lamda/2)+... 
               ((p*(pn-pm))/(2*(pm*pn-1)));
    i = 1;
    for x = 40:1:60
        ss_pdf(i,:) = (eta/(sqrt(2*pi)))*(lamda/((x-... 
                                 epsilon)*(lamda-x+epsilon)))*... 
                      exp((-1/2)*(gamma+eta*log((x-... 
                                 epsilon)/(lamda-x+epsilon)))^2);
        test1 = isnan(ss_pdf(i,1));
        test2 = isreal(ss_pdf(i,1));
        if test1 == 1 || test2 == 0
            ss_pdf(i,:) = 0;
        end
        E(i,:) = ss_pdf(i,:)*ystack(i,:);
        chi(i,:) = ((ystack(i,:)-E(i,:))^2)/E(i,:);
        test3 = isnan(chi(i,:));
        test4 = isinf(chi(i,:));
        test5 = lt(E(i,:),0);
        if test3 == 1 || test4 == 1 || test5 == 1
            chi(i,:) = 0;
        end
        i = i + 1;
    end
    chisquared(counter,:) = sum(chi);
end

if type_num == 2
eta = (2*z)/acosh((1/2)*(mp + np));
gamma = eta*asinh((np-mp)/(2*(mp*np-1)^((1/2))));
lamda = (2*p*(mp*np-1)^((1/2))/((mp+np-2)*(mp+np+2)^((1/2))));
epsilon = ((percentile(2,1)+percentile(3,1))/2)+
((p*(np-mp))/(2*(mp+np-2)));
i = 1;
for x = 40:1:60
    ss_pdf(i,:) = (eta/(sqrt(2*pi)))*(1/sqrt((x-epsilon)^2+
    lamda^2))*exp((-1/2)*(gamma+eta*log((x-epsilon...)
    )/lamda^2))/lamda^2);
test1 = isnan(ss_pdf(i,1));
test2 = isreal(ss_pdf(i,1));
if test1 == 1 || test2 == 0
    ss_pdf(i,:) = 0;
end
E(i,:) = ss_pdf(i,:)*ystack(i,:);
chi(i,:) = ((ystack(i,:)-E(i,:))^2)/E(i,:);
test3 = isnan(chi(i,:));
test4 = isinf(chi(i,:));
if test3 == 1 || test4 == 1
    chi(i,:) = 0;
end
i = i + 1;
end
chisquared(counter,:) = 0;
end

if type_num == 3
    eta = (2*z)/log(mp);
    gamma = eta*log((mp-1)/(p*(mp)^((1/2))));
    lamda = gamma;
    epsilon = ((percentile(2,1)+percentile(3,1))/2)-
    (p/2)*((mp+1)/(mp-1));
i = 1;
for x = 40:1:60
    ss_pdf(i,:) = (eta/((sqrt(2*pi))*(x-epsilon)))*exp((-1/2)*
    (eta^2)*((gamma/eta)+eta*log(x-epsilon...)^2));
test1 = isnan(ss_pdf(i,1));
test2 = isreal(ss_pdf(i,1));
if test1 == 1 || test2 == 0
    ss_pdf(i,:) = 0;
end
E(i,:) = ss_pdf(i,:)*ystack(i,:);
chi(i,:) = ((ystack(i,:)-E(i,:))^2)/E(i,:);
test3 = isnan(chi(i,:));
test4 = isinf(chi(i,:));
if test3 == 1 || test4 == 1
    chi(i,:) = 0;
end
i = i + 1;
end
chisquared(counter,:) = 0;
end

zstorage(counter,:) = z;
gammastorage(counter,:) = gamma;
etastorage(counter,:) = eta;
epsilonstorage(counter,:) = epsilon;
lamdastorage(counter,:) = lamda;

counter = counter + 1;
end
datastorage = [chisquared, zstorage, gammastorage, etastorage, epsilonstorage, lamdastorage];
datastorage(datastorage(:,1)==0,:)=[];
[minVal, minInd] = min(datastorage(:,1));
datatarget = datastorage(minInd,:)
gamma = datatarget(1,3);
etta = datatarget(1,4);
epsilon = datatarget(1,5);
lamda = datatarget(1,6);

i = 1;
for x = 40:0.1:60
    ss_pdf(i,:) = (eta/(sqrt(2*pi)))*(lamda/((x-epsilon)*(lamda-x+epsilon)))^2)*exp((-1/2)*(gamma+eta*log((x-epsilon)/(lamda-x+epsilon)))^2);
    test1 = isnan(ss_pdf(i,1));
    test2 = isreal(ss_pdf(i,1));
    if test1 == 1 || test2 == 0
        ss_pdf(i,:) = 0;
    end
    i = i + 1;
end
figure(1);
hist(pdamphistplot,40:60);
xlabel('Amplitude (dB)');
ylabel('Frequency of Occurrence');
legend('Amplitude Histogram');
h1 = gca;
h2 = axes('Position',get(h1,'Position'));
plot(40:0.1:60,ss_pdf,'color','r','LineWidth',3);
set(h2,'YAxisLocation','right','Color','none','XTickLabel',[]);
set(h2,'XLim',get(h1,'XLim'),'Layer','top');
ylabel('Probability Distribution Function');
legend('Johnson SB Curve','location','East');
title([filenameedit,...
    '-Amplitude Histogram - Johnson SB (Slifker and Shapiro)'...,...
    'fontsize',10,'fontweight','b']);
filenamesavel = strcat(filenameedit,...
    '-SB Amplitude Histogram (Slifker and Shapiro).jpg');
saveas(1,str2mat(filenamesavel));

i = 1;
for x = 40:1:60
    hold on;
    ssamphist1(i,:) = (eta/(sqrt(2*pi)))*(lamda/((x-epsilon)*(lamda-x+epsilon)))^2)*exp((-1/2)*(gamma+eta*log((x-epsilon)/(lamda-x+epsilon)))^2);
    test1 = isnan(ssamphist1(i,1));
    test2 = isreal(ssamphist1(i,1));
    if test1 == 1 || test2 == 0
        ssamphist1(i,:) = 0;
    end
    i = i + 1;
end
\( \text{test} = \text{isreal}(\text{ssamphist}1(i,:)); \)

\[
\text{if test} == 0 \\
\quad \text{ssamphist}1(i,:) = 0; \\
\text{end} \\
\text{i} = i + 1; \\
\text{end}
\]

\text{i} = 1; 
\text{for x} = 40:1:60 
\text{hold on;} 
\quad \text{ssamphist}2(i,:) = (\eta/(\sqrt{2\pi}))*((\lambda/((x-\epsilon)*(\lambda-x+\epsilon)))*exp((-1/2)*(\gamma+\eta*\log((x-\epsilon)/(\lambda-x+\epsilon)))^2)*\text{num}; 
\text{test} = \text{isreal}(\text{ssamphist}2(i,:)); 
\text{if test} == 0 
\quad \text{ssamphist}2(i,:) = 0; 
\text{end} 
\text{i} = i + 1; 
\text{end}
\]

\text{xlswrite(filename,datatarget',1,'AK21')} \\
\text{xlswrite(filename,ssamphist1,1,'AS1')} \\
\text{xlswrite(filename,ssamphist2,1,'AT1')} \\
\text{close all;} \\
\text{beep;
F.6 JohnsonMage.m

```
clc;
clc all;
close all;

filename     = '01_01_01_txtN.xlsx';
filenameedit = '01-01-01';
net_output   = xlsread(filename);

i = 1;
for i = 1:21
    hold on;
    pdamphist(i,:) = net_output(i,35);
    i = i + 1;
end

i = 1;
for i = 1:length(net_output)
    hold on;
    amphistplot(i,:) = net_output(i,6);
    i = i + 1;
end

i = 1;
for i = 1:length(net_output)
    hold on;
    if net_output(i,12) == 1;
        pdamphistplot(i,:) = net_output(i,6);
    end
    i = i + 1;
end

pdamphistplot(pdamphistplot==0,:) = [];
num = numel(pdamphistplot);

y = pdamphistplot;
ystack = histc(pdamphistplot,(40:1:60));

counter = 1;
for z1 = -1.5:-0.01:-3.0;
    for z2 = -0.50:-0.01:-1.49;
        z3 = 2*z2 - z1;
        z4 = 3*z2 - 2*z1;
        percentage(1,1) = normcdf(z1,0,1)*100.000;
        percentage(2,1) = normcdf(z2,0,1)*100.000;
```
percentage(3,1) = normcdf(z3,0,1)*100.000;
percentage(4,1) = normcdf(z4,0,1)*100.000;

for prc = 1 : 4
    percentile(prc,1) = prctile(y,percentage(prc,1));
end

p = percentile(3,1) - percentile(2,1);

m = percentile(4,1) - percentile(3,1);
n = percentile(2,1) - percentile(1,1);
mnp = (m * n)/(p^2);

mp = m/p;
np = n/p;

if mnp < 0.999
    type = 'bounded';
    type_num = 1;
elseif mnp > 1.001
    type = 'unbounded';
    type_num = 2;
elseif 0.999 < mnp && mnp < 1.001
    type = 'lognormal';
    type_num = 3;
end

if type_num == 1
    mage_a = percentile(2,1)+percentile(4,1)-(2*percentile(3,1));
    mage_b = percentile(3,1)^2-percentile(2,1)*percentile(4,1);
    mage_c = 2*percentile(2,1)*percentile(3,1)*percentile(4,1)-... 
             (percentile(2,1)+percentile(4,1))*percentile(3,1)^2;
    mage_d = percentile(1,1)+percentile(3,1)-2*percentile(2,1);
    mage_e = percentile(2,1)^2-percentile(1,1)*percentile(3,1);
    mage_f = 2*percentile(1,1)*percentile(2,1)*percentile(3,1)-... 
             (percentile(1,1)+percentile(3,1))*percentile(2,1)^2;

    mage_phi = (mage_c*mage_d - mage_a*mage_f)/... 
              (mage_b*mage_d-mage_a*mage_e);
    mage_theta = (mage_c*mage_e - mage_b*mage_f)/... 
                 (mage_b*mage_d - mage_a*mage_e);
    mage_tau = (-mage_phi/2) + sqrt((mage_phi^2)/4 - mage_theta);

    epsilon = (-mage_phi/2)-sqrt((mage_phi^2)/4-mage_theta);
    lamda = 2*sqrt((mage_phi)^2/4-mage_theta);
    eta = (z2-z1)/log(((percentile(2,1)-epsilon)*(mage_tau... 
                      -percentile(1,1)))/(mage_tau-percentile(2,1))*... 
                      (percentile(1,1)-epsilon));
    gamma = z1-eta*log((percentile(1,1)-epsilon)/... 
                       (mage_tau-percentile(1,1)));

    z1storage(counter,:) = z1;
    z2storage(counter,:) = z2;
    z3storage(counter,:) = z3;
z4storage(counter,:) = z4;

gammastorage(counter,:)   = gamma;
etastorage(counter,:)     = eta;
epsilonstorage(counter,:) = epsilon;
lamdastorage(counter,:)   = lamda;

i = 1;
for x = 40:1:60
    mage_pdf(i,:) = (eta/(sqrt(2*pi)))*(lamda/((x -
    ...epsilon)*(lamda - x + epsilon)))*
    ...exp((-1/2)*(gamma + eta*log((x -
    ...epsilon)/(lamda - x + epsilon )))^2);
    test1 = isnan(mage_pdf(i,1));
    test2 = isreal(mage_pdf(i,1));
    if test1 == 1 || test2 == 0
        mage_pdf(i,:) = 0;
    end
    E(i,:) = mage_pdf(i,:) * ystack(i,:);
    chi(i,:) = ((ystack(i,:) - E(i,:))^2)/E(i,:);
    test3 = isnan(chi(i,:));
    test4 = isinf(chi(i,:));
    if test3 == 1 || test4 == 1
        chi(i,:) = 0;
    end
    i = i + 1;
end
chisquared(counter,:) = sum(chi);
end

if type_num == 2 || type_num == 3
    z1storage(counter,:) = 0;
    z2storage(counter,:) = 0;
    z3storage(counter,:) = 0;
    z4storage(counter,:) = 0;
    gammastorage(counter,:)   = 0;
etastorage(counter,:)     = 0;
epsilonstorage(counter,:) = 0;
lamdastorage(counter,:)   = 0;
    chisquared(counter,:) = 0;
end
datastorage = [chisquared, z1storage, z2storage, z3storage,...
    z4storage, gammastorage, etastorage,...
    epsilonstorage, lamdastorage];
    counter = counter + 1;
end
    counter = counter + 1;
end
datastorage(datastorage(:,1)==0,:)=[];
[minVal, minInd] = min(datastorage(:,1));
datatarget = datastorage(minInd,:);
gamma  = datatarget(1,6);
etta     = datatarget(1,7);
epsilon = datatarget(1,8);
lamda   = datatarget(1,9);

i = 1;
for x = 40:0.1:60
    mage_pdf(i,:) = (eta/(sqrt(2*pi)))*(lamda/((x-epsilon)*(lamda-x+...epsilon)))*exp((-1/2)*(gamma+eta*log((x-epsilon).../(lamda-x+epsilon)))^2);
    test1 = isnan(mage_pdf(i,1));
    test2 = isreal(mage_pdf(i,1));
    if test1 == 1 || test2 == 0
        mage_pdf(i,:) = 0;
    end
    i = i + 1;
end

figure(1);
hist(pdamphistplot,40:60);
xlabel('Amplitude (dB)');
ylabel('Frequency of Occurrence');
legend('Amplitude Histogram');
hl = gca;
h2 = axes('Position',get(hl,'Position'));
plot(40:0.1:60,mage_pdf,'color','r','LineWidth',3);
set(h2,'YAxisLocation','right','Color','none','XTickLabel',[]);
set(h2,'XLim',get(hl,'XLim'),'Layer','top');
ylabel('Probability Distribution Function');
legend('Johnson SB Curve','Location','East');
title([filenameedit,'-Amplitude Histogram - Johnson SB (Mage)'],$...'
'fontsize',10,'fontweight','b');
filenamesave1 = strcat(filenameedit,'-SB Amplitude Histogram (Mage).jpg');
saveas(1,str2mat(filenamesave1));

i = 1;
for x = 40:1:60
    hold on;
    mageamphist1(i,:) = (eta/(sqrt(2*pi)))*(lamda/((x-epsilon)*(lamda-x+...epsilon)))*exp((-1/2)*(gamma+eta*log((x-epsilon).../(lamda-x+epsilon)))^2)*pdamphist(i);
    test = isreal(mageamphist1(i,:));
    if test == 0
        mageamphist1(i,:) = 0;
    end
    i = i + 1;
end

i = 1;
for x = 40:1:60
    hold on;
    mageamphist2(i,:) = (eta/(sqrt(2*pi)))*(lamda/((x-epsilon)*(lamda-x+...epsilon)))*exp((-1/2)*(gamma+eta*log((x-epsilon).../(lamda-x+epsilon)))^2)*num;
    test = isreal(mageamphist2(i,:));
    if test == 0
mageamphist2(i,:) = 0;
end
i = i + 1;
end
xlswrite(filename,datatarget',1,'AK11');
xlswrite(filename,mageamphist1,1,'AP1');
xlswrite(filename,mageamphist2,1,'AQ1');
close all;
beep;
F.7 JohnsonLinear.m

```matlab
%%
%% Johnson Distribution (Linearized Method)
%%
clc;
clear all;
close all;

filename     = '01_01_01_txtN.xlsx';
filenameedit = '01-01-01';
net_output   = xlsread(filename);

i = 1;
for i = 1:21
    hold on;
    pdamphist(i,:) = net_output(i,35);
    i = i + 1;
end

i = 1;
for i = 1:length(net_output)
    hold on;
    amphistplot(i,:) = net_output(i,6);
    i = i + 1;
end

i = 1;
for i = 1:length(net_output)
    hold on;
    if net_output(i,12) == 1;
        pdamphistplot(i,:) = net_output(i,6);
    end
    i = i + 1;
end

pdamphistplot(pdamphistplot==0,:) = [];
num = numel(pdamphistplot);

x = pdamphistplot;
micro_x = mean(x);
sigma_x = std(x);

counter = 1;
for epsilon = 0:1:50
    for lamda = 0:1:50
        i = 1:length(x)
        X(i,:) = log((x(i,:)-epsilon)/(lamda+epsilon-x(i,:)));
        Y(i,:) = (x(i,:)-micro_x)/sigma_x;
        end
        lin = polyfit(X,Y,1);
        reg = regstats(Y,X,'linear',{'rsquare','r'});
```
rsquare = reg.rsquare;
test1 = isnan(rsquare);
test2 = isreal(rsquare);
if test1 == 1 || test2 == 0
    rsquarestorage(counter,:) = 0;
elseif test1 == 0
    rsquarestorage(counter,:) = rsquare;
end
gradientstorage(counter,:) = lin(1);
interceptstorage(counter,:) = lin(2);
epsilonstorage(counter,:) = epsilon;
lamdastorage(counter,:) = lamda;
Ymaxstorage(counter,:) = max(Y);
Yminstorage(counter,:) = min(Y);
counter = counter + 1
end
datastorage = [rsquarestorage interceptstorage gradientstorage...
                 epsilonstorage lamdastorage Ymaxstorage Yminstorage];
datastorage(datastorage(:,1)==0,:)=[];
[maxVal, maxInd] = max(datastorage(:,1));
datatarget = datastorage(maxInd,:)
gamma = datatarget(1,2);
etta = datatarget(1,3);
epsilon = datatarget(1,4);
lamda = datatarget(1,5);
for i = 1:length(x)
    X(i,:) = log((x(i,:)-epsilon)/(lamda+epsilon-
x(i,:)));
    Y(i,:) = (x(i,:)-micro_x)/sigma_x;
end
lin = polyfit(X,Y,1);
plot(X,Y);
xL = get(gca,'XLim');
line(xL,[0 0],'Color','k');
yL = get(gca,'YLim');
line([0 0],yL,'Color','k');
xlabel('X');
ylabel('Z');
legend('Z = (lin(1))X + lin(2)', 'location', 'northwest');
filenamesavel = strcat(filenameedit,'-Linearization Plot.jpg');
saveas(1,str2mat(filenamesavel));
i = 1;
for x = 40:0.1:60
    linear_pdf(i,:) = (eta/(sqrt(2*pi)))*(lamda/((x-epsilon)*(lamda-x+...
                                epsilon)))*exp((-1/2)*(gamma+eta*log((x-epsilon)...
                                /(lamda-x+epsilon)))^2);
    test1 = isnan(linear_pdf(i,1));
    test2 = isreal(linear_pdf(i,1));
    if test1 == 1 || test2 == 0
        linear_pdf(i,:) = 0;
    end
    i = i + 1;
figure(1);
hist(pdamphistplot,40:60);
xlabel('Amplitude (dB)');
ylabel('Frequency of Occurrence');
legend('Amplitude Histogram');
h1 = gca;
h2 = axes('Position',get(h1,'Position'),3);
plot(40:0.1:60,linear_pdf,'color','r','LineWidth',3);
set(h2,'YAxisLocation','right','Color','none','XTickLabel',[]);
set(h2,'XLim',get(h1,'XLim'),'Layer','top');
ylabel('Probability Distribution Function');
legend('Johnson SB Curve','location','East');
title(fullfile(edit,...
   '-Amplitude Histogram - Johnson SB (Linear)'...,
   'fontweight','b');
filenamesave1 = strcat(fullfile,...
   '-SB Amplitude Histogram (Linear).jpg');
saveas(1,str2mat(filenamesave1));

i = 1;
for x = 40:1:60
    hold on;
    linearamphist1(i,:) = (eta/(sqrt(2*pi)))*(lamda/((x-epsilon)*...;
       (lamda-x+epsilon))*exp((-1/2)*(gamma+eta*log...
       ((x-epsilon)/(lamda-x+epsilon)))^2)*pdamphist(i);;
    test = isreal(linearamphist1(i,:));
    if test == 0
        linearamphist1(i,:) = 0;
    end
    i = i + 1;
end

i = 1;
for x = 40:1:60
    hold on;
    linearamphist2(i,:) = (eta/(sqrt(2*pi)))*(lamda/((x-epsilon)*...;
       (lamda-x+epsilon))*exp((-1/2)*(gamma+eta*log...
       ((x-epsilon)/(lamda-x+epsilon)))^2)*num;
    test = isreal(linearamphist2(i,:));
    if test == 0
        linearamphist2(i,:) = 0;
    end
    i = i + 1;
end

datatarget2 = [gamma eta epsilon lamda];
xlswrite(fullfile,...
   datatarget2',1,'AK28');
xlswrite(fullfile,...
   linearamphist1,1,'AV1');
xlswrite(fullfile,...
   linearamphist2,1,'AW1');
close all;
close all
beep;
clc; clear all; close all;

filename = '01_01_01_txtN.xlsx';
filenameedit = '01-01-01';
net_output = xlsread(filename);

i = 1;
for i = 1:length(net_output)
    hold on;
    if net_output(i,12) == 1;
        pdamphistplot(i,:) = net_output(i,6);
    end
    i = i + 1;
end

pdamphistplot(pdamphistplot==0,:) = [];
um = numel(pdamphistplot);
x = pdamphistplot;
ystack = histc(pdamphistplot,(40:1:60));

counter = 1;
gamma = 42;
for alpha = 2:0.0001:5
    for beta = 2:0.0001:5
        if (alpha/beta)*(((40-gamma)/beta)^(alpha-1))*... exp(-(((40-gamma)/beta)^alpha))>0
            break
        end
        for i = 1:length(x)
            R(i,:) = real(exp(-((x(i,:)-gamma)/(beta))^alpha));
            X(i,:) = log(x(i,:)-gamma);
            Y(i,:) = log(log(1/R(i,:)));
            test1 = isnan(X(i,:));
            test2 = isnan(Y(i,:));
            if test1 == 1 || test2 == 1
                X(i,:) = [];
                Y(i,:) = [];
            end
        end
        lin = polyfit(X,Y,1);
        reg = regstats(Y,X,'linear','rsquare');
        rsquare = reg.rsquare;
        test3 = isnan(rsquare);
        test4 = isreal(rsquare);
        if test3 == 1 || test4 == 0

    end
end
rsquarestorage(counter,:) = 0;
elseif test3 == 0
    rsquarestorage(counter,:) = rsquare;
end
alphastorage(counter,:) = alpha;
betastorage(counter,:) = beta;
counter = counter + 1
end

datastorage = [rsquarestorage alphastorage betastorage];
datastorage(datastorage(:,1)==0,:)=[];
[maxVal, maxInd] = max(datastorage(:,1));
datatarget = datastorage(maxInd,:)

alpha = datatarget(1,2);
beta = datatarget(1,3);

for i = 1:length(x)
    R(i,:) = real(exp(-(x(i,:)-gamma)/beta)^alpha));
    X(i,:) = log(x(i,:)-gamma);
    Y(i,:) = log(log(1/R(i,:)));
end
NewX = X(~isinf(X));
NewY = Y(~isinf(Y));
lin = polyfit(NewX,NewY,1);
reg = regstats(Y,X,'linear',{rsquare','r'});
rsquare = reg.rsquare;
plot(NewX,NewY);
xL = get(gca,'XLim');
line(xL,[0 0],'Color','k');
yL = get(gca,'YLim');
line([0 0],yL,'Color','k');
xlabel('X');
ylabel('Y');
legend('Y = (lin(1))X - lin(2)','location','northwest');
filenamesavel = strcat(filenameedit,'-Weibull Logarithmic Plot.jpg');
saveas(1,str2mat(filenamesavel));
real(lin(1));
real(lin(2));
i = 1;
for x = 42:1:60
   weibull_pdf(i,:) = (alpha/beta)*(((x-gamma)/beta)^(alpha-1))*...
         exp(-(x-gamma)/beta)^alpha));
   test1 = isnan(weibull_pdf(i,1));
   test2 = isreal(weibull_pdf(i,1));
   if test1 == 1 || test2 == 0
       weibull_pdf(i,:) = 0;
   end
   E(i,:) = weibull_pdf(i,:)*ystack(i,:);
   chi(i,:) = ((ystack(i,:)-E(i,:))^2)/E(i,:);
   test3 = isnan(chi(i,:));
   test4 = isinf(chi(i,:));
   test5 = lt(E(i,:),0);
   if test3 == 1 || test4 == 1 || test5 == 1
       weibull_pdf(i,:) = 0;
   end
end
chi(i,:) = 0;
end
    i = i + 1;
end
chisquared = sum(chi);

i = 1;
for x = 40:0.1:60
    weibull_pdf(i,:) = (alpha/beta)*(((x-gamma)/beta)^(alpha-1))*...
                        exp(-(((x-gamma)/beta)^alpha));
    test1 = isnan(weibull_pdf(i,1));
    test2 = isreal(weibull_pdf(i,1));
    if test1 == 1 || test2 == 0
        weibull_pdf(i,:) = 0;
    end
    i = i + 1;
end
weibull_pdf(weibull_pdf < 0) = 0;
figure(1);
hist(pdamphistplot,40:60);
xlabel('Amplitude (dB)');
ylabel('Frequency of Occurrence');
legend('Amplitude Histogram');

hl = gca;
h2 = axes('Position',get(h1,'Position'));
plot(40:0.1:60,weibull_pdf,'color','r','LineWidth',3);
set(h2,'YAxisLocation','right','Color','none','XTickLabel',[]);
set(h2,'XLim',get(h1,'XLim')','Layer','top');
ylabel('Probability Distribution Function');
legend('Weibull Curve','location','East');
title([filenameedit,...
      '-Amplitude Histogram - Weibull (Fixed Gamma)'],...
      'fontsize',10,'fontweight','b');
filenamesavel = strcat(filenameedit,...
    '-SB Amplitude Histogram (Weibull - Fixed Gamma).jpg');
saveas(1,str2mat(filenamesavel));

i = 1;
for x = 40:1:60
    hold on;
    weibullamphist(i,:) = (alpha/beta)*(((x-gamma)/beta)^(alpha-1))*...
                          exp(-(((x-gamma)/beta)^alpha))*num;
    test = isreal(weibullamphist(i,:));
    if test == 0
        weibullamphist(i,:) = 0;
    end
    i = i + 1;
end
weibulldata = chisquared
xlswrite(filename,weibulldata,1,'BP5');
xlswrite(filename,weibullamphist,1,'BQ1');
close all;
beep;
clc;
clear all;
close all;

filename = '01_01_01_txtN.xlsx';
net_output = xlsread(filename);

row = 1;
for i = 1:length(net_output)
    hold on;
    a2(i,:) = [net_output(i,35)];
    row = row + 1;
end
a2 = a2(~isnan(a2));
close all;

Tracker = 'Phase 1 complete'

filename = '01_01_02_txtN.xlsx';
net_output = xlsread(filename);

row = 1;
for i = 1:length(net_output)
    hold on;
    b2(i,:) = [net_output(i,35)];
    row = row + 1;
end
b2 = b2(~isnan(b2));
close all;

Tracker = 'Phase 2 complete'

filename = '01_01_03_txtN.xlsx';
net_output = xlsread(filename);

row = 1;
for i = 1:length(net_output)
    hold on;
    c2(i,:) = [net_output(i,35)];
    row = row + 1;
end
c2 = c2(~isnan(c2));
close all;

Tracker = 'Phase 3 complete'
filename = '01_01_04_txtN.xlsx';
net_output = xlsread(filename);
row = 1;
for i = 1:length(net_output)
    hold on;
    d2(i,:) = [net_output(i,35)];
    row = row + 1;
end
d2 = d2(~isnan(d2));
close all;
Tracker = 'Phase 4 complete'

filename = '01_01_05_txtN.xlsx';
net_output = xlsread(filename);
row = 1;
for i = 1:length(net_output)
    hold on;
    e2(i,:) = [net_output(i,35)];
    row = row + 1;
end
e2 = e2(~isnan(e2));
close all;
Tracker = 'Phase 5 complete'

filename = '01_01_06_txtN.xlsx';
net_output = xlsread(filename);
row = 1;
for i = 1:length(net_output)
    hold on;
    f2(i,:) = [net_output(i,35)];
    row = row + 1;
end
f2 = f2(~isnan(f2));
close all;
Tracker = 'Phase 6 complete'

filename = '01_01_07_txtN.xlsx';
net_output = xlsread(filename);
row = 1;
for i = 1:length(net_output)
    hold on;
    g2(i,:) = [net_output(i,35)];
    row = row + 1;
end
g2 = g2(~isnan(g2));
close all;
Tracker = 'Phase 7 complete'
filename = '01_01_08_txtN.xlsx';
net_output = xlsread(filename);

row = 1;
for i = 1:length(net_output)
    hold on;
    h2(i,:) = [net_output(i,35)];
    row = row + 1;
end
h2 = h2(~isnan(h2));
close all;
Tracker = 'Phase 8 complete'

filename = '01_01_09_txtN.xlsx';
net_output = xlsread(filename);

row = 1;
for i = 1:length(net_output)
    hold on;
    i2(i,:) = [net_output(i,35)];
    row = row + 1;
end
i2 = i2(~isnan(i2));
close all;
Tracker = 'Phase 9 complete'
%
%---------------------------------------------------------------------
% Backpropagating Neural Network
%---------------------------------------------------------------------

InputData = [a2 b2 c2 d2 e2 f2 g2 h2 i2];
TargetData = [11809, 16584, 16847, 19654, 16994, 19189, 13573, 15833, 16084];
umsamples = 9;

%---------------------------------------------------------------------
% Neural Network
%---------------------------------------------------------------------
i = 1;
for neurons = 1:1:20;
    net = newff(InputData,TargetData,{},'trainlm');
    net.trainParam.lr = 0.01;
    net.trainParam.lr_inc = 1.05;
    net.trainParam.mc = 0.9;
    net.trainParam.epochs = 100;
    net.trainParam.goal = 1e-10;
    net.trainParam.max_fail = 100;
    net.trainParam.mem_reduc = 2;
    net.trainParam.min_grad = 1e-15;
    net.trainParam.mu = 0.001;
    net.trainParam.mu_dec = 0.1;
net.trainParam.mu_inc = 10;
net.trainParam.mu_max = 1e11;
net.trainParam.show = 25;
net.trainParam.showCommandLine = 0;
net.trainParam.showWindow = 1;
net.trainParam.time = inf;
net.divideFcn = 'divideind';
net.divideParam.trainInd = [1 2 3 4 5];
net.divideParam.valInd = [6 7];
net.divideParam.testInd = [8 9];

net = train(net,InputData,TargetData);

y = sim(net,InputData);

for j = 1:numsamples
    error(j) = ((y(j)-TargetData(j))/TargetData(j))*100;
    abserror(j) = abs(error(j));
end

avgerror = sum(abserror)/numsamples;
datastorage{i,:} = [y' TargetData' error'];
construct(i,:) = [avgerror neurons];
nntraintool('close');
i = i + 1;
end

[minVal, minInd] = min(construct(:,1));
constructtarget = construct(minInd,:)
datatarget = datastorage(minInd,:)

filename     = 'ZTest.xlsx'
xlswrite(filename,constructtarget,1,'A3');
xlswrite(filename,datatarget,1,'A5');

```
% MLR Results - Slifker And Shapiro
clc; clear all; close all;

y = [11809 16584 16847 19654 16994 19189 13573 15833 16084]';
gamma = [-0.5 -0.345 -0.520 -0.450 -0.8 0.12 -0.510 -0.630 -0.12]';
eta = [0.8066 0.5974 0.6026 0.6026 0.4173 0.4288 0.4763 0.5923 0.6650]';
epsilon = [43.66 42.76 42.76 42.76 43.55 43.73 43.60 42.76 42.76]';
lamda = [2.6858 4.4721 4.4721 4.4721 2.8925 2.5443 2.8069 4.4721 4.4721]';

x1 = [ones(size(gamma)) gamma eta epsilon lamda... gamma.*eta gamma.*epsilon gamma.*lamda... eta.*epsilon eta.*lamda... epsilon.*lamda... gamma.*eta.*epsilon gamma.*eta.*lamda gamma.*epsilon.*lamda... eta.*epsilon.*lamda... gamma.*eta.*epsilon.*lamda];

b1 = regress(y,x1)
X = [gamma eta epsilon lamda... gamma.*eta gamma.*epsilon gamma.*lamda... eta.*epsilon eta.*lamda... epsilon.*lamda... gamma.*eta.*epsilon.*lamda];
inmodel = [true true true true false false false false false false];
[b, sel, pval, inmodel, stats] = stepwisefit(X,y,'penter',0.99,'premove',0.99);
mean(stats.PVAL)

x2 = [ones(size(gamma)) gamma eta epsilon lamda... gamma.*eta];

b2 = regress(y,x2)
F.11 MageMLR.m

```matlab
clc;
clear all;
close all;

y = [11809 16584 16847 19654 16994 19189 13573 15833 16084]';
gamma = [-0.5 -0.345 -0.520 -0.450 -0.8 0.12 -0.510 -0.630 -0.12]';
eta = [0.8066 0.5974 0.6026 0.6026 0.4173 0.4288 0.4763 0.5923 0.6650]';
epsilon = [43.66 42.76 42.76 42.76 43.55 43.73 43.60 42.76 42.76]';
lamda = [2.6858 4.4721 4.4721 4.4721 2.8925 2.5443 2.8069 4.4721 4.4721]';

x1 = [ones(size(gamma)) gamma eta epsilon lamda...
gamma.*eta gamma.*epsilon gamma.*lamda...
etas.*eta.*epsilon...
gamma.*epsilon.*lamda...
gamma.*eta.*epsilon.*lamda];
b1 = regress(y,x1)

X = [gamma eta epsilon lamda...
gamma.*eta gamma.*epsilon gamma.*lamda...
etas.*epsilon etas.*lamda...
etas.*epsilon.*lamda];
inmodel = [true true true true false false false false false false];
[b,sel,pval,inmodel,stats] = stepwisefit(X,y,'penter',0.99,'premove',0.99);
mean(stats.PVAL)

x2 = [ones(size(gamma)) gamma eta epsilon lamda...
gamma.*eta gamma.*epsilon eta.*lamda];
b2 = regress(y,x2)
```

212
```matlab
clc;
clear all;
close all;

y = [11809 16584 16847 19654 16994 19189 13573 15833 16084]';
gamma = [-0.3994 0.6296 0.4568 0.9020 0.5529 -0.6854 0.6635 0.6956 1.4459]';
epsilon = [20 22 22 22 22 20 22 22 22]';
lamda = [50 49 49 50 49 49 50 50 50]';

x1 = [ones(size(gamma)) gamma eta epsilon lamda... 
gamma.*eta gamma.*epsilon gamma.*lamda...
eta.*epsilon eta.*lamda...
epsilon.*lamda...
gamma.*eta.*epsilon gamma.*eta.*lamda gamma.*epsilon.*lamda...
eta.*epsilon.*lamda...

b1 = regress(y,x1)

X = [gamma eta epsilon lamda... 
gamma.*eta gamma.*epsilon gamma.*lamda...
eta.*epsilon eta.*lamda...
epsilon.*lamda];
inmodel = [true true true true false false false false false false];
[b,sel,pval,inmodel,stats] = stepwisefit(X,y,'penter',0.99,'premove',0.99);
mean(stats.PVAL)

x2 = [ones(size(gamma)) gamma eta epsilon lamda... 
gamma.*eta gamma.*lamda...
eta.*epsilon];

b2 = regress(y,x2)
```
F.13 WeibullMLR.m

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% MLR Results - Weibull Distribution
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
clc;
clear all;
close all;

y = [11809 16584 16847 19654 16994 19189 13573 15833 16084]';


x1 = [ones(size(alpha)) alpha beta alpha.*beta];
b1 = regress(y,x1)

X = [alpha alpha.*beta];

[b.sel,pval,inmodel,stats] = stepwisefit(X,y,'penter',0.99,'premove',0.99);
mean(stats.PVAL);
Rsquare = stats.SStotal/(stats.SStotal+stats.SStotal)

x2 = [ones(size(alpha)) alpha beta alpha.*beta];
b2 = regress(y,x2)
APPENDIX G: ANOVA RESULTS

G.1 Bounded Johnson Distribution (Slifker and Shapiro) ANOVA Results

Step 1:

Initial Variables = [Gamma Eta Epsilon Lambda]

>> Gamma*Eta

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.1703e+004</td>
<td>9.7809e+003</td>
<td>'In'</td>
<td>0.1132</td>
</tr>
<tr>
<td>6.3956e+004</td>
<td>3.4631e+004</td>
<td>'In'</td>
<td>0.1619</td>
</tr>
<tr>
<td>3.5628e+004</td>
<td>1.7214e+004</td>
<td>'In'</td>
<td>0.1303</td>
</tr>
<tr>
<td>2.3516e+004</td>
<td>1.1271e+004</td>
<td>'In'</td>
<td>0.1282</td>
</tr>
<tr>
<td>3.2601e+004</td>
<td>1.6172e+004</td>
<td>'In'</td>
<td>0.1372 Avg = 0.1342</td>
</tr>
</tbody>
</table>

>> Gamma*Epsilon

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.6428e+005</td>
<td>8.0101e+004</td>
<td>'In'</td>
<td>0.1326</td>
</tr>
<tr>
<td>6.3956e+004</td>
<td>3.4631e+004</td>
<td>'In'</td>
<td>0.1619</td>
</tr>
<tr>
<td>3.5105e+004</td>
<td>1.6956e+004</td>
<td>'In'</td>
<td>0.1302</td>
</tr>
<tr>
<td>2.1580e+004</td>
<td>1.0319e+004</td>
<td>'In'</td>
<td>0.1276</td>
</tr>
<tr>
<td>3.6298e+003</td>
<td>1.8006e+003</td>
<td>'In'</td>
<td>0.1372 Avg = 0.1379</td>
</tr>
</tbody>
</table>

>> Gamma*Lambda

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5.3468e+003</td>
<td>3.2123e+003</td>
<td>'In'</td>
<td>0.1946</td>
</tr>
<tr>
<td>Coef</td>
<td>Std.Err.</td>
<td>Status</td>
<td>P</td>
</tr>
<tr>
<td>---------</td>
<td>----------</td>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>6.3956e+004</td>
<td>3.4631e+004</td>
<td>'In'</td>
<td>0.1619</td>
</tr>
<tr>
<td>3.5599e+004</td>
<td>1.7200e+004</td>
<td>'In'</td>
<td>0.1303</td>
</tr>
<tr>
<td>2.1140e+004</td>
<td>1.0102e+004</td>
<td>'In'</td>
<td>0.1275</td>
</tr>
<tr>
<td>-3.3707e+003</td>
<td>1.6721e+003</td>
<td>'In'</td>
<td>0.1372 Avg = 0.1503</td>
</tr>
</tbody>
</table>

>> Eta*Epsilon

<table>
<thead>
<tr>
<th>Coef</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1254e+003</td>
<td>5.8069e+003</td>
<td>'In'</td>
<td>0.3073</td>
</tr>
<tr>
<td>4.0831e+005</td>
<td>2.0523e+005</td>
<td>'In'</td>
<td>0.1407</td>
</tr>
<tr>
<td>4.2711e+004</td>
<td>2.0711e+004</td>
<td>'In'</td>
<td>0.1312</td>
</tr>
<tr>
<td>2.4091e+004</td>
<td>1.1554e+004</td>
<td>'In'</td>
<td>0.1284</td>
</tr>
<tr>
<td>-7.1383e+003</td>
<td>3.5411e+003</td>
<td>'In'</td>
<td>0.1372 Avg = 0.1690</td>
</tr>
</tbody>
</table>

>> Eta*Lambda

<table>
<thead>
<tr>
<th>Coef</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9174e+003</td>
<td>6.1475e+003</td>
<td>'In'</td>
<td>0.2881</td>
</tr>
<tr>
<td>9.0098e+004</td>
<td>4.7526e+004</td>
<td>'In'</td>
<td>0.1543</td>
</tr>
<tr>
<td>3.4443e+004</td>
<td>1.6630e+004</td>
<td>'In'</td>
<td>0.1301</td>
</tr>
<tr>
<td>1.7656e+004</td>
<td>8.3937e+003</td>
<td>'In'</td>
<td>0.1261</td>
</tr>
<tr>
<td>5.8455e+003</td>
<td>2.8998e+003</td>
<td>'In'</td>
<td>0.1372 Avg = 0.1672</td>
</tr>
</tbody>
</table>

>> Epsilon*Lambda

<table>
<thead>
<tr>
<th>Coef</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.9118e+003</td>
<td>2.9788e+003</td>
<td>'In'</td>
<td>0.4004</td>
</tr>
</tbody>
</table>

216
Step 2:

Initial Variables = [Gamma Eta Epsilon Lambda Gamma*Eta]

Table G.1: ANOVA Results – First Analysis (Slifker and Shapiro)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>5</td>
<td>9.67E+06</td>
<td>3.21</td>
<td>0.1830</td>
</tr>
<tr>
<td>Residual</td>
<td>9.03E+06</td>
<td>3</td>
<td>3.01E+06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5.74E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.8425</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

>> Gamma*Epsilon

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-2.1703e+04]</td>
<td>[9.7809e+03]</td>
<td>'In'</td>
<td>[0.1132]</td>
</tr>
<tr>
<td>[6.3956e+004]</td>
<td>[3.4631e+004]</td>
<td>'In'</td>
<td>[0.1619]</td>
</tr>
<tr>
<td>[3.5628e+004]</td>
<td>[1.7214e+004]</td>
<td>'In'</td>
<td>[0.1303]</td>
</tr>
<tr>
<td>[2.3516e+004]</td>
<td>[1.1271e+004]</td>
<td>'In'</td>
<td>[0.1282]</td>
</tr>
<tr>
<td>[3.2601e+004]</td>
<td>[1.6172e+004]</td>
<td>'In'</td>
<td>[0.1372]</td>
</tr>
<tr>
<td>[0]</td>
<td>[22.9934]</td>
<td>'Out'</td>
<td>[1]</td>
</tr>
</tbody>
</table>

Avg = 0.2785

>> Gamma*Lambda

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

217
<table>
<thead>
<tr>
<th>Coef</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.1703e+004</td>
<td>9.7809e+003</td>
<td>'In'</td>
<td>0.1132</td>
</tr>
<tr>
<td>6.3956e+004</td>
<td>3.4631e+004</td>
<td>'In'</td>
<td>0.1619</td>
</tr>
<tr>
<td>3.5628e+004</td>
<td>1.7214e+004</td>
<td>'In'</td>
<td>0.1303</td>
</tr>
<tr>
<td>2.3516e+004</td>
<td>1.1271e+004</td>
<td>'In'</td>
<td>0.1282</td>
</tr>
<tr>
<td>3.2601e+004</td>
<td>1.6172e+004</td>
<td>'In'</td>
<td>0.1372</td>
</tr>
</tbody>
</table>

Avg = 0.2785

>> Eta*Epsilon

<table>
<thead>
<tr>
<th>Coef</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.1703e+004</td>
<td>9.7809e+003</td>
<td>'In'</td>
<td>0.1132</td>
</tr>
<tr>
<td>6.3956e+004</td>
<td>3.4631e+004</td>
<td>'In'</td>
<td>0.1619</td>
</tr>
<tr>
<td>3.5628e+004</td>
<td>1.7214e+004</td>
<td>'In'</td>
<td>0.1303</td>
</tr>
<tr>
<td>2.3516e+004</td>
<td>1.1271e+004</td>
<td>'In'</td>
<td>0.1282</td>
</tr>
<tr>
<td>3.2601e+004</td>
<td>1.6172e+004</td>
<td>'In'</td>
<td>0.1372</td>
</tr>
</tbody>
</table>

Avg = 0.2785

>> Eta*Lambda

<table>
<thead>
<tr>
<th>Coef</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.1703e+004</td>
<td>9.7809e+003</td>
<td>'In'</td>
<td>0.1132</td>
</tr>
<tr>
<td>6.3956e+004</td>
<td>3.4631e+004</td>
<td>'In'</td>
<td>0.1619</td>
</tr>
<tr>
<td>3.5628e+004</td>
<td>1.7214e+004</td>
<td>'In'</td>
<td>0.1303</td>
</tr>
<tr>
<td>2.3516e+004</td>
<td>1.1271e+004</td>
<td>'In'</td>
<td>0.1282</td>
</tr>
<tr>
<td>3.2601e+004</td>
<td>1.6172e+004</td>
<td>'In'</td>
<td>0.1372</td>
</tr>
</tbody>
</table>

Avg = 0.2785


>> Epsilon*Lambda

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.1703e+004</td>
<td>9.7809e+003</td>
<td>'In'</td>
<td>0.1132</td>
</tr>
<tr>
<td>6.3956e+004</td>
<td>3.4631e+004</td>
<td>'In'</td>
<td>0.1619</td>
</tr>
<tr>
<td>3.5628e+004</td>
<td>1.7214e+004</td>
<td>'In'</td>
<td>0.1303</td>
</tr>
<tr>
<td>2.3516e+004</td>
<td>1.1271e+004</td>
<td>'In'</td>
<td>0.1282</td>
</tr>
<tr>
<td>3.2601e+004</td>
<td>1.6172e+004</td>
<td>'In'</td>
<td>0.1372</td>
</tr>
</tbody>
</table>

Avg = 0.2785

Final Variables = [Gamma Eta Epsilon Lambda Gamma*Eta]

Table G.2: ANOVA Results – Final Analysis (Slifker and Shapiro)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>5</td>
<td>9.67E+06</td>
<td>3.21</td>
<td>0.1830</td>
</tr>
<tr>
<td>Residual</td>
<td>9.03E+06</td>
<td>3</td>
<td>3.01E+06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5.74E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.8425</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

219
G.2 Bounded Johnson Distribution (Mage) ANOVA Results

Step 1:

Initial Variables = [Gamma Eta Epsilon Lambda]

>> Gamma*Eta

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0829e+004</td>
<td>1.2871e+004</td>
<td>In</td>
<td>0.2040</td>
</tr>
<tr>
<td>-2.3086e+004</td>
<td>1.1725e+004</td>
<td>In</td>
<td>0.1436</td>
</tr>
<tr>
<td>-7.2363e+005</td>
<td>4.2098e+005</td>
<td>In</td>
<td>0.1841</td>
</tr>
<tr>
<td>-3.6239e+005</td>
<td>2.1178e+005</td>
<td>In</td>
<td>0.1856</td>
</tr>
<tr>
<td>-3.2570e+004</td>
<td>2.5165e+004</td>
<td>In</td>
<td>0.2862 Avg = 0.2007</td>
</tr>
</tbody>
</table>

>> Gamma*Epsilon

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.9820e+005</td>
<td>2.6896e+005</td>
<td>In</td>
<td>0.3484</td>
</tr>
<tr>
<td>-7.0206e+003</td>
<td>6.5854e+003</td>
<td>In</td>
<td>0.3646</td>
</tr>
<tr>
<td>-6.3786e+005</td>
<td>4.1986e+005</td>
<td>In</td>
<td>0.2260</td>
</tr>
<tr>
<td>-3.2097e+005</td>
<td>2.1170e+005</td>
<td>In</td>
<td>0.2268</td>
</tr>
<tr>
<td>6.9765e+003</td>
<td>6.1995e+003</td>
<td>In</td>
<td>0.3424 Avg = 0.3016</td>
</tr>
</tbody>
</table>

>> Gamma*Lambda

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5801e+004</td>
<td>1.0382e+004</td>
<td>In</td>
<td>0.2254</td>
</tr>
<tr>
<td>-7.0241e+003</td>
<td>6.5841e+003</td>
<td>In</td>
<td>0.3643</td>
</tr>
<tr>
<td>Coeff</td>
<td>Std.Err.</td>
<td>Status</td>
<td>P</td>
</tr>
<tr>
<td>--------------</td>
<td>------------------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>-6.4264e+05</td>
<td>4.2114e+05</td>
<td>In</td>
<td>0.2244</td>
</tr>
<tr>
<td>-3.2337e+05</td>
<td>2.1236e+05</td>
<td>In</td>
<td>0.2252</td>
</tr>
<tr>
<td>-3.5076e+03</td>
<td>3.1169e+03</td>
<td>In</td>
<td>0.3423</td>
</tr>
</tbody>
</table>

>> Eta*Epsilon

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.7458e+03</td>
<td>4.1168e+03</td>
<td>In</td>
<td>0.1565</td>
</tr>
<tr>
<td>-2.2641e+06</td>
<td>2.2160e+06</td>
<td>In</td>
<td>0.3821</td>
</tr>
<tr>
<td>-6.5597e+05</td>
<td>4.4074e+05</td>
<td>In</td>
<td>0.2334</td>
</tr>
<tr>
<td>-3.1272e+05</td>
<td>2.1678e+05</td>
<td>In</td>
<td>0.2448</td>
</tr>
<tr>
<td>5.1710e+04</td>
<td>5.0834e+04</td>
<td>In</td>
<td>0.3839</td>
</tr>
</tbody>
</table>

>> Eta*Lambda

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.7419e+03</td>
<td>4.1125e+03</td>
<td>In</td>
<td>0.1563</td>
</tr>
<tr>
<td>6.3511e+04</td>
<td>7.2402e+04</td>
<td>In</td>
<td>0.4450</td>
</tr>
<tr>
<td>-6.3381e+05</td>
<td>4.3344e+05</td>
<td>In</td>
<td>0.2398</td>
</tr>
<tr>
<td>-3.0156e+05</td>
<td>2.1382e+05</td>
<td>In</td>
<td>0.2532</td>
</tr>
<tr>
<td>-2.6043e+04</td>
<td>2.5591e+04</td>
<td>In</td>
<td>0.3838</td>
</tr>
</tbody>
</table>

>> Epsilon*Lambda

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>646.2083</td>
<td>4.0960e+03</td>
<td>In</td>
<td>0.8847</td>
</tr>
<tr>
<td>-1.1562e+04</td>
<td>6.1825e+03</td>
<td>In</td>
<td>0.1582</td>
</tr>
</tbody>
</table>
Step 2:

Initial Variables = [Gamma Eta Epsilon Lambda Gamma*Eta]

Table G.3: ANOVA Results – First Analysis (Mage)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>5</td>
<td>9.67E+06</td>
<td>3.52</td>
<td>0.1646</td>
</tr>
<tr>
<td>Residual</td>
<td>8.23E+06</td>
<td>3</td>
<td>2.74E+06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5.66E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td></td>
<td></td>
<td>0.8544</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

>> Gamma*Epsilon

<table>
<thead>
<tr>
<th>'Coeff'</th>
<th>'Std.Err.'</th>
<th>'Status'</th>
<th>'P'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 6.9661e+005]</td>
<td>[1.6527e+006]</td>
<td>'In'</td>
<td>0.7144</td>
</tr>
<tr>
<td>[-5.4307e+004]</td>
<td>[7.7587e+004]</td>
<td>'In'</td>
<td>0.5564</td>
</tr>
<tr>
<td>[-8.5483e+005]</td>
<td>[5.9014e+005]</td>
<td>'In'</td>
<td>0.2845</td>
</tr>
<tr>
<td>[-4.2478e+005]</td>
<td>[2.9218e+005]</td>
<td>'In'</td>
<td>0.2832</td>
</tr>
<tr>
<td>[-9.4540e+004]</td>
<td>[1.5441e+005]</td>
<td>'In'</td>
<td>0.6027</td>
</tr>
<tr>
<td>[-1.4859e+004]</td>
<td>[3.6339e+004]</td>
<td>'In'</td>
<td>0.7222</td>
</tr>
</tbody>
</table>

>> Gamma*Lambda

<table>
<thead>
<tr>
<th>'Coeff'</th>
<th>'Std.Err.'</th>
<th>'Status'</th>
<th>'P'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 2.7819e+004]</td>
<td>[2.2836e+004]</td>
<td>'In'</td>
<td>0.3473</td>
</tr>
</tbody>
</table>
[ -5.4313e+004] [ 7.7588e+004] 'In' [0.5564]
[ -8.4470e+005] [ 5.7702e+005] 'In' [0.2808]
[ -4.1968e+005] [ 2.8585e+005] 'In' [0.2798]
[ -9.4567e+004] [ 1.5445e+005] 'In' [0.6027]
 [ 7.4739e+003] [ 1.8274e+004] 'In' [0.7222] Avg = 0.4649

>> Eta*Epsilon

'Coeff' 'Std.Err.' 'Status' 'P'
[ 1.9585e+004] [ 2.1251e+004] 'In' [0.4540]
[-3.8374e+005] [ 4.1439e+006] 'In' [0.9347]
[-7.2464e+005] [ 5.1475e+005] 'In' [0.2945]
[-3.6037e+005] [ 2.5993e+005] 'In' [0.2999]
[-2.9046e+004] [ 5.0867e+004] 'In' [0.6255]
[ 8.3059e+003] [ 9.5435e+004] 'In' [0.9386] Avg = 0.5912

>> Eta*Lambda

'Coeff' 'Std.Err.' 'Status' 'P'
[ 1.9584e+004] [ 2.1270e+004] 'In' [0.4544]
[-9.8728e+003] [ 1.5264e+005] 'In' [0.9543]
[-7.2106e+005] [ 5.1547e+005] 'In' [0.2968]
[-3.5857e+005] [ 2.6259e+005] 'In' [0.3054]
[-2.9045e+004] [ 5.0895e+004] 'In' [0.6258]
[-4.1814e+003] [ 4.8087e+004] 'In' [0.9386] Avg = 0.5959

223
Epsilon*Lambda

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.7963e+004</td>
<td>1.4105e+005</td>
<td>'In'</td>
<td>0.5592</td>
</tr>
<tr>
<td>-6.5913e+004</td>
<td>7.9028e+004</td>
<td>'In'</td>
<td>0.4920</td>
</tr>
<tr>
<td>-1.5540e+006</td>
<td>1.5848e+006</td>
<td>'In'</td>
<td>0.4302</td>
</tr>
<tr>
<td>-3.4828e+006</td>
<td>5.6799e+006</td>
<td>'In'</td>
<td>0.6022</td>
</tr>
<tr>
<td>-1.5510e+005</td>
<td>2.2468e+005</td>
<td>'In'</td>
<td>0.5613</td>
</tr>
<tr>
<td>6.5307e+004</td>
<td>1.1877e+005</td>
<td>'In'</td>
<td>0.6376</td>
</tr>
</tbody>
</table>

Avg = 0.5471

Step 3:

Initial Variables = [Gamma Eta Epsilon Lambda Gamma*Eta Gamma*Lambda]

**Table G.4: ANOVA Results – SecondAnalysis (Mage)**

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>6</td>
<td>8.05E+06</td>
<td>2.12</td>
<td>0.3547</td>
</tr>
<tr>
<td>Residual</td>
<td>7.59E+06</td>
<td>2</td>
<td>3.80E+06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5.59E+07</td>
<td>8</td>
<td>3.80E+06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.8642</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Gamma*Epsilon

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.7614e+010</td>
<td>2.3459e+010</td>
<td>'In'</td>
<td>0.5900</td>
</tr>
<tr>
<td>-8.1167e+004</td>
<td>9.4754e+004</td>
<td>'In'</td>
<td>0.5491</td>
</tr>
<tr>
<td>2.6638e+008</td>
<td>3.5590e+008</td>
<td>'In'</td>
<td>0.5910</td>
</tr>
<tr>
<td>1.3406e+008</td>
<td>1.7910e+008</td>
<td>'In'</td>
<td>0.5909</td>
</tr>
<tr>
<td>Coeff</td>
<td>Std.Err.</td>
<td>Status</td>
<td>P</td>
</tr>
<tr>
<td>-----------</td>
<td>----------</td>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>[-5.3998e+005]</td>
<td>[6.1840e+005]</td>
<td>'In'</td>
<td>[0.5430]</td>
</tr>
<tr>
<td>[1.9682e+008]</td>
<td>[2.6211e+008]</td>
<td>'In'</td>
<td>[0.5900]</td>
</tr>
<tr>
<td>[3.9135e+008]</td>
<td>[5.2121e+008]</td>
<td>'In'</td>
<td>[0.5900] Avg = 0.5777</td>
</tr>
</tbody>
</table>

>> Eta*Epsilon

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1.0103e+005]</td>
<td>[1.0087e+005]</td>
<td>'In'</td>
<td>[0.4995]</td>
</tr>
<tr>
<td>[9.5719e+006]</td>
<td>[1.2821e+007]</td>
<td>'In'</td>
<td>[0.5917]</td>
</tr>
<tr>
<td>[-1.5001e+006]</td>
<td>[1.0899e+006]</td>
<td>'In'</td>
<td>[0.4000]</td>
</tr>
<tr>
<td>[-7.9769e+005]</td>
<td>[5.9830e+005]</td>
<td>'In'</td>
<td>[0.4097]</td>
</tr>
<tr>
<td>[-5.3998e+005]</td>
<td>[6.1840e+005]</td>
<td>'In'</td>
<td>[0.5430]</td>
</tr>
<tr>
<td>[4.9622e+004]</td>
<td>[5.9818e+004]</td>
<td>'In'</td>
<td>[0.5591]</td>
</tr>
<tr>
<td>[-2.2575e+005]</td>
<td>[3.0066e+005]</td>
<td>'In'</td>
<td>[0.5900] Avg = 0.5133</td>
</tr>
</tbody>
</table>

>> Eta*Lambda

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1.0108e+005]</td>
<td>[1.0093e+005]</td>
<td>'In'</td>
<td>[0.4995]</td>
</tr>
<tr>
<td>[-5.8974e+005]</td>
<td>[7.1848e+005]</td>
<td>'In'</td>
<td>[0.5624]</td>
</tr>
<tr>
<td>[-1.5972e+006]</td>
<td>[1.1960e+006]</td>
<td>'In'</td>
<td>[0.4092]</td>
</tr>
<tr>
<td>[-8.4660e+005]</td>
<td>[6.5406e+005]</td>
<td>'In'</td>
<td>[0.4188]</td>
</tr>
<tr>
<td>[-5.3998e+005]</td>
<td>[6.1840e+005]</td>
<td>'In'</td>
<td>[0.5430]</td>
</tr>
<tr>
<td>[4.9611e+004]</td>
<td>[5.9804e+004]</td>
<td>'In'</td>
<td>[0.5591]</td>
</tr>
<tr>
<td>[1.1372e+005]</td>
<td>[1.5146e+005]</td>
<td>'In'</td>
<td>[0.5900] Avg = 0.5117</td>
</tr>
</tbody>
</table>

225
>> Epsilon*Lambda

<table>
<thead>
<tr>
<th>'Coeff'</th>
<th>'Std.Err.'</th>
<th>'Status'</th>
<th>'P'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[7.4004e+005]</td>
<td>[9.4891e+005]</td>
<td>'In'</td>
<td>[0.5783]</td>
</tr>
<tr>
<td>[-8.1167e+004]</td>
<td>[9.4754e+004]</td>
<td>'In'</td>
<td>[0.5491]</td>
</tr>
<tr>
<td>[-7.8946e+006]</td>
<td>[9.4119e+006]</td>
<td>'In'</td>
<td>[0.5557]</td>
</tr>
<tr>
<td>[-3.2271e+007]</td>
<td>[4.2422e+007]</td>
<td>'In'</td>
<td>[0.5860]</td>
</tr>
<tr>
<td>[-5.3998e+005]</td>
<td>[6.1840e+005]</td>
<td>'In'</td>
<td>[0.5430]</td>
</tr>
<tr>
<td>[-9.3266e+004]</td>
<td>[1.3575e+005]</td>
<td>'In'</td>
<td>[0.6168]</td>
</tr>
<tr>
<td>[6.8278e+005]</td>
<td>[9.0935e+005]</td>
<td>'In'</td>
<td>[0.5900] Avg = 0.5741</td>
</tr>
</tbody>
</table>

Final Variables = [Gamma Eta Epsilon Lambda Gamma*Eta Gamma*Lambda Eta*Lambda]

Table G.5: ANOVA Results – Final Analysis (Mage)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>7</td>
<td>6.90E+06</td>
<td>1.42</td>
<td>0.5709</td>
</tr>
<tr>
<td>Residual</td>
<td>4.86E+06</td>
<td>1</td>
<td>4.86E+06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5.32E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.9086</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### G.3 Bounded Johnson Distribution (Linearization) ANOVA Results

Step 1:

**Initial Variables = [Gamma Eta Epsilon Lambda]**

>> **Gamma*Eta**

<table>
<thead>
<tr>
<th>'Coeff'</th>
<th>'Std.Err.'</th>
<th>'Status'</th>
<th>'P'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[8.6959e+004]</td>
<td>[6.0818e+004]</td>
<td>'In'</td>
<td>[0.2481]</td>
</tr>
<tr>
<td>[-1.7166e+003]</td>
<td>[1.9313e+003]</td>
<td>'In'</td>
<td>[0.4396]</td>
</tr>
<tr>
<td>[-4.4880e+003]</td>
<td>[8.6199e+003]</td>
<td>'In'</td>
<td>[0.6386]</td>
</tr>
<tr>
<td>[-5.1416e+003]</td>
<td>[2.5876e+003]</td>
<td>'In'</td>
<td>[0.1411]</td>
</tr>
<tr>
<td>[-7.3864e+003]</td>
<td>[8.6199e+003]</td>
<td>'In'</td>
<td>[0.2772]</td>
</tr>
</tbody>
</table>

Avg = 0.3207

>> **Gamma*Epsilon**

<table>
<thead>
<tr>
<th>'Coeff'</th>
<th>'Std.Err.'</th>
<th>'Status'</th>
<th>'P'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5.7782e+005]</td>
<td>[6.9565e+005]</td>
<td>'In'</td>
<td>[0.4671]</td>
</tr>
<tr>
<td>[-1.0414e+004]</td>
<td>[8.5380e+003]</td>
<td>'In'</td>
<td>[0.3097]</td>
</tr>
<tr>
<td>[-5.2967e+004]</td>
<td>[4.8450e+004]</td>
<td>'In'</td>
<td>[0.3542]</td>
</tr>
<tr>
<td>[-5.3375e+003]</td>
<td>[3.9888e+003]</td>
<td>'In'</td>
<td>[0.2732]</td>
</tr>
<tr>
<td>[-2.5516e+004]</td>
<td>[3.1069e+004]</td>
<td>'In'</td>
<td>[0.4717]</td>
</tr>
</tbody>
</table>

Avg = 0.3752

>> **Gamma*Lambda**

<table>
<thead>
<tr>
<th>'Coeff'</th>
<th>'Std.Err.'</th>
<th>'Status'</th>
<th>'P'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1.0775e+005]</td>
<td>[4.6678e+005]</td>
<td>'In'</td>
<td>[0.8323]</td>
</tr>
<tr>
<td>[-4.6581e+003]</td>
<td>[5.5209e+003]</td>
<td>'In'</td>
<td>[0.4608]</td>
</tr>
<tr>
<td>Coeff</td>
<td>Std.Err.</td>
<td>Status</td>
<td>P</td>
</tr>
<tr>
<td>---------------</td>
<td>------------------</td>
<td>--------</td>
<td>-----</td>
</tr>
<tr>
<td>1.6472e+004</td>
<td>1.2676e+004</td>
<td>'In'</td>
<td>0.2846</td>
</tr>
<tr>
<td>6.7901e+004</td>
<td>8.6978e+004</td>
<td>'In'</td>
<td>0.4919</td>
</tr>
<tr>
<td>1.4252e+004</td>
<td>3.4310e+004</td>
<td>'In'</td>
<td>0.7058</td>
</tr>
<tr>
<td>-5.3375e+003</td>
<td>3.9888e+003</td>
<td>'In'</td>
<td>0.2732</td>
</tr>
<tr>
<td>-3.5598e+003</td>
<td>4.3345e+003</td>
<td>'In'</td>
<td>0.4717 Avg = 0.4454</td>
</tr>
</tbody>
</table>

$$\text{Epsilon} \times \Lambda$$

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6472e+004</td>
<td>1.2676e+004</td>
<td>'In'</td>
<td>0.2846</td>
</tr>
<tr>
<td>-1.0414e+004</td>
<td>8.5380e+003</td>
<td>'In'</td>
<td>0.3097</td>
</tr>
</tbody>
</table>
Step 2:

Initial Variables = [Gamma Eta Epsilon Lambda Gamma*Eta]

Table G.6: ANOVA Results – First Analysis (Linearization)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>5</td>
<td>9.67E+06</td>
<td>3.10</td>
<td>0.1903</td>
</tr>
<tr>
<td>Residual</td>
<td>9.36E+06</td>
<td>3</td>
<td>3.12E+06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5.77E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R$^2$</td>
<td>0.8377</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

>> Gamma*Epsilon

<table>
<thead>
<tr>
<th>'Coeff'</th>
<th>'Std.Err.'</th>
<th>'Status'</th>
<th>'P'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-1.4717e+06]</td>
<td>[1.7070e+06]</td>
<td>'In'</td>
<td>[0.4795]</td>
</tr>
<tr>
<td>[ 2.2642e+04]</td>
<td>[2.6733e+04]</td>
<td>'In'</td>
<td>[0.4862]</td>
</tr>
<tr>
<td>[ 1.3235e+05]</td>
<td>[1.5002e+05]</td>
<td>'In'</td>
<td>[0.4707]</td>
</tr>
<tr>
<td>[-1.7858e+03]</td>
<td>[4.5359e+03]</td>
<td>'In'</td>
<td>[0.7318]</td>
</tr>
<tr>
<td>[-2.2165e+04]</td>
<td>[1.7161e+04]</td>
<td>'In'</td>
<td>[0.3256]</td>
</tr>
<tr>
<td>[ 7.6801e+04]</td>
<td>[8.4054e+04]</td>
<td>'In'</td>
<td>[0.4573] Avg = 0.4919</td>
</tr>
</tbody>
</table>

>> Gamma*Lambda

<table>
<thead>
<tr>
<th>'Coeff'</th>
<th>'Std.Err.'</th>
<th>'Status'</th>
<th>'P'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-9.1482e+05]</td>
<td>[2.5215e+05]</td>
<td>'In'</td>
<td>[0.0683]</td>
</tr>
<tr>
<td>Coeff</td>
<td>Std.Err.</td>
<td>Status</td>
<td>P</td>
</tr>
<tr>
<td>--------</td>
<td>----------</td>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>2.1793e+005</td>
<td>1.5640e+005</td>
<td>'In'</td>
<td>0.2982</td>
</tr>
<tr>
<td>-2.1308e+005</td>
<td>2.3133e+005</td>
<td>'In'</td>
<td>0.4542</td>
</tr>
<tr>
<td>-6.9975e+004</td>
<td>7.2217e+004</td>
<td>'In'</td>
<td>0.4348</td>
</tr>
<tr>
<td>-1.7858e+003</td>
<td>4.5359e+003</td>
<td>'In'</td>
<td>0.7318</td>
</tr>
<tr>
<td>-2.2165e+004</td>
<td>1.7161e+004</td>
<td>'In'</td>
<td>0.3256</td>
</tr>
<tr>
<td>1.0715e+004</td>
<td>1.1727e+004</td>
<td>'In'</td>
<td>0.4573 Avg = 0.4530</td>
</tr>
</tbody>
</table>

>> Eta*Lambda

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0850e+005</td>
<td>1.5687e+005</td>
<td>'In'</td>
<td>0.3152</td>
</tr>
<tr>
<td>4.6598e+005</td>
<td>5.5133e+005</td>
<td>'In'</td>
<td>0.4870</td>
</tr>
<tr>
<td>9.5074e+004</td>
<td>1.1771e+005</td>
<td>'In'</td>
<td>0.5041</td>
</tr>
<tr>
<td>8.1103e+004</td>
<td>1.0170e+005</td>
<td>'In'</td>
<td>0.5088</td>
</tr>
<tr>
<td>-2.1133e+004</td>
<td>1.7231e+004</td>
<td>'In'</td>
<td>0.3448</td>
</tr>
<tr>
<td>-8.8969e+003</td>
<td>1.0488e+004</td>
<td>'In'</td>
<td>0.4856 Avg = 0.5291</td>
</tr>
</tbody>
</table>

Avg = 0.4530

Avg = 0.5291
>> Epsilon*Lambda

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Coef</td>
<td>Std.Err</td>
<td>Status</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>2.1793e+005</td>
<td>1.5640e+005</td>
<td>'In'</td>
<td>0.2982</td>
<td></td>
</tr>
<tr>
<td>2.2642e+004</td>
<td>2.6733e+004</td>
<td>'In'</td>
<td>0.4862</td>
<td></td>
</tr>
<tr>
<td>-9.9658e+005</td>
<td>1.0858e+006</td>
<td>'In'</td>
<td>0.4556</td>
<td></td>
</tr>
<tr>
<td>-4.8502e+005</td>
<td>5.2520e+005</td>
<td>'In'</td>
<td>0.4532</td>
<td></td>
</tr>
<tr>
<td>-2.2165e+004</td>
<td>1.7161e+004</td>
<td>'In'</td>
<td>0.3256</td>
<td></td>
</tr>
<tr>
<td>2.1965e+004</td>
<td>2.4039e+004</td>
<td>'In'</td>
<td>0.4573</td>
<td></td>
</tr>
</tbody>
</table>

Avg = 0.4127

Step 3:

Initial Variables = [Gamma Eta Epsilon Lambda Gamma*Eta Gamma*Lambda]

Table G.7: ANOVA Results – Second Analysis (Linearization)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>6</td>
<td>8.05E+06</td>
<td>15.43</td>
<td>0.0621</td>
</tr>
<tr>
<td>Residual</td>
<td>1.04E+06</td>
<td>2</td>
<td>5.22E+05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>4.94E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R^2</td>
<td>0.9789</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

>> Gamma*Epsilon

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Coef</td>
<td>Std.Err</td>
<td>Status</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>8.0375e+004</td>
<td>4.6366e+005</td>
<td>'In'</td>
<td>0.8907</td>
<td></td>
</tr>
<tr>
<td>-177.6097</td>
<td>7.1318e+003</td>
<td>'In'</td>
<td>0.9841</td>
<td></td>
</tr>
<tr>
<td>-2.4647e+004</td>
<td>4.2675e+004</td>
<td>'In'</td>
<td>0.6666</td>
<td></td>
</tr>
<tr>
<td>-2.5799e+004</td>
<td>4.0494e+003</td>
<td>'In'</td>
<td>0.0991</td>
<td></td>
</tr>
<tr>
<td>Coeff</td>
<td>Std.Err.</td>
<td>Status</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>----------</td>
<td>--------</td>
<td>--------</td>
<td></td>
</tr>
<tr>
<td>-1.4353e+006</td>
<td>2.7191e+005</td>
<td>'In'</td>
<td>0.1192</td>
<td></td>
</tr>
<tr>
<td>2.1127e+005</td>
<td>8.6947e+004</td>
<td>'In'</td>
<td>0.2485</td>
<td></td>
</tr>
<tr>
<td>1.5684e+005</td>
<td>4.0472e+004</td>
<td>'In'</td>
<td>0.1608</td>
<td></td>
</tr>
<tr>
<td>-2.5799e+004</td>
<td>4.0494e+003</td>
<td>'In'</td>
<td>0.0991</td>
<td></td>
</tr>
<tr>
<td>-1.4841e+004</td>
<td>4.0842e+003</td>
<td>'In'</td>
<td>0.1710</td>
<td></td>
</tr>
<tr>
<td>3.2098e+004</td>
<td>5.2339e+003</td>
<td>'In'</td>
<td>0.1029</td>
<td></td>
</tr>
<tr>
<td>-9.6113e+003</td>
<td>4.2553e+003</td>
<td>'In'</td>
<td>0.2653</td>
<td></td>
</tr>
</tbody>
</table>

Avg = 0.1667

>> Eta*Lambda

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Std.Err.</th>
<th>Status</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.3436e+006</td>
<td>3.4263e+005</td>
<td>'In'</td>
<td>0.1590</td>
</tr>
<tr>
<td>-3.5924e+005</td>
<td>2.4553e+005</td>
<td>'In'</td>
<td>0.3817</td>
</tr>
<tr>
<td>1.1948e+004</td>
<td>3.9932e+004</td>
<td>'In'</td>
<td>0.8149</td>
</tr>
<tr>
<td>-9.1892e+004</td>
<td>4.8726e+004</td>
<td>'In'</td>
<td>0.3104</td>
</tr>
<tr>
<td>-1.5863e+004</td>
<td>5.3308e+003</td>
<td>'In'</td>
<td>0.2064</td>
</tr>
<tr>
<td>3.0433e+004</td>
<td>6.6537e+003</td>
<td>'In'</td>
<td>0.1370</td>
</tr>
<tr>
<td>7.2230e+003</td>
<td>4.7388e+003</td>
<td>'In'</td>
<td>0.3696</td>
</tr>
</tbody>
</table>

Avg = 0.3399
>> Epsilon*Lambda

<table>
<thead>
<tr>
<th>'Coeff'</th>
<th>'Std.Err.'</th>
<th>'Status'</th>
<th>'P'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-1.4353e+006]</td>
<td>[2.7191e+005]</td>
<td>'In'</td>
<td>[0.1192]</td>
</tr>
<tr>
<td>[-177.6097]</td>
<td>[7.1318e+003]</td>
<td>'In'</td>
<td>[0.9841]</td>
</tr>
<tr>
<td>[9.8803e+005]</td>
<td>[4.0718e+005]</td>
<td>'In'</td>
<td>[0.2489]</td>
</tr>
<tr>
<td>[4.0767e+005]</td>
<td>[1.8835e+005]</td>
<td>'In'</td>
<td>[0.2755]</td>
</tr>
<tr>
<td>[-1.4841e+004]</td>
<td>[4.0842e+003]</td>
<td>'In'</td>
<td>[0.1710]</td>
</tr>
<tr>
<td>[3.2098e+004]</td>
<td>[5.2339e+003]</td>
<td>'In'</td>
<td>[0.1029]</td>
</tr>
<tr>
<td>[-1.9703e+004]</td>
<td>[8.7234e+003]</td>
<td>'In'</td>
<td>[0.2653] Avg = 0.3096</td>
</tr>
</tbody>
</table>

Final Variables = [Gamma Eta Epsilon Lambda Gamma*Eta Gamma*Lambda Eta*Epsilon]

**Table G.8: ANOVA Results – Final Analysis (Linearization)**

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>7</td>
<td>6.90E+06</td>
<td>40.35</td>
<td>0.1206</td>
</tr>
<tr>
<td>Residual</td>
<td>1.71E+05</td>
<td>1</td>
<td>1.71E+05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>4.85E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R^2</td>
<td>0.9965</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
G.4 Weibull Distribution ANOVA Results

Step 1:

Initial variables = []

>> Alpha

<table>
<thead>
<tr>
<th>'Coeff'</th>
<th>'Std.Err.'</th>
<th>'Status'</th>
<th>'P'</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.9645e+003</td>
<td>2.2106e+003</td>
<td>'In'</td>
<td>0.0596</td>
</tr>
</tbody>
</table>

Avg = 0.0596

Table G.9: ANOVA Results – First Analysis (Weibull)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>1</td>
<td>4.83E+07</td>
<td>12.04</td>
<td>0.0596</td>
</tr>
<tr>
<td>Residual</td>
<td>2.81E+07</td>
<td>7</td>
<td>4.01E+06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>7.64E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td></td>
<td></td>
<td>0.6324</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

>> Beta

<table>
<thead>
<tr>
<th>'Coeff'</th>
<th>'Std.Err.'</th>
<th>'Status'</th>
<th>'P'</th>
</tr>
</thead>
<tbody>
<tr>
<td>-418.7858</td>
<td>2.3399e+003</td>
<td>'In'</td>
<td>0.863</td>
</tr>
</tbody>
</table>

Avg = 0.8630

Table G.10: ANOVA Results – Second Analysis (Weibull)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>1</td>
<td>4.83E+07</td>
<td>7.03</td>
<td>0.863</td>
</tr>
<tr>
<td>Residual</td>
<td>4.81E+07</td>
<td>7</td>
<td>6.87E+06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>9.64E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td></td>
<td></td>
<td>0.5011</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

>> Alpha*Beta

<table>
<thead>
<tr>
<th>'Coeff'</th>
<th>'Std.Err.'</th>
<th>'Status'</th>
<th>'P'</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table G.11: ANOVA Results – Third Analysis (Weibull)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>1</td>
<td>4.83E+07</td>
<td>10.02</td>
<td>0.1256</td>
</tr>
<tr>
<td>Residual</td>
<td>3.37E+07</td>
<td>7</td>
<td>4.82E+06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>8.21E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.5888</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 2:

Initial Variables = [Alpha]

>> Alpha*Beta

<table>
<thead>
<tr>
<th>'Coeff'</th>
<th>'Std.Err.'</th>
<th>'Status'</th>
<th>'P'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-3.8023e+003]</td>
<td>[2.8493e+003]</td>
<td>'In'</td>
<td>[0.2305]</td>
</tr>
<tr>
<td>[-392.6831]</td>
<td>[569.0545]</td>
<td>'In'</td>
<td>[0.5159] Avg = 0.3732</td>
</tr>
</tbody>
</table>

Table G.12: ANOVA Results – Final Analysis (Weibull)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td>4.83E+07</td>
<td>2</td>
<td>2.42E+07</td>
<td>5.57</td>
<td>0.1561</td>
</tr>
<tr>
<td>Residual</td>
<td>2.60E+07</td>
<td>6</td>
<td>4.34E+06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>7.43E+07</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.6500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Final Variables = [Alpha Alpha*Beta]