Characterization of Electron Number Density of Rocket Exhaust Plumes Through Microwave Transmissions

Jorge Torres
Embry-Riddle Aeronautical University - Daytona Beach

Follow this and additional works at: https://commons.erau.edu/edt
Part of the Electrical and Computer Engineering Commons

Scholarly Commons Citation
https://commons.erau.edu/edt/186

This Thesis - Open Access is brought to you for free and open access by Scholarly Commons. It has been accepted for inclusion in Dissertations and Theses by an authorized administrator of Scholarly Commons. For more information, please contact commons@erau.edu, wolfe309@erau.edu.
Characterization of Electron Number Density of Rocket Exhaust Plumes Through Microwave Transmissions

Author: Jorge Torres
Advisor: Dr. William C. Barott

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Electrical and Computer Engineering in the Radar & Microwaves Laboratory Department of Electrical, Computer, Software, and Systems Engineering

December 2014
Committee Approval

“Characterization of Electron Number Density of Rocket Exhaust Plumes Through Microwave Transmissions,"

Jorge Torres

This thesis is prepared under the direction of the candidate's thesis committee chairman, Dr. William C. Barott, Department of Electrical, Computer, Software, and Systems Engineering, and has been approved by members of his thesis committee. It is submitted to the Electrical, Computer, Software, and Systems Engineering Department in partial fulfillment of the requirements for the degree of Master of Science in Electrical and Computer Engineering.

Dated: 12-3-14

Committee Members:

Dr. William C. Barott: [Signature]

Dr. Jianhua Liu: [Signature]

Dr. William Engblom: [Signature]

ECSSE Department Chair:

Dr. Tim Wilson: [Signature]

Dean of the College of Engineering:

Dr. Maj Mirmirani: [Signature]

Associate Vice President for Academics: Dr. Robert Oxley: [Signature]
Embry-Riddle Aeronautical University

Author: Jorge Torres
Title: Characterization of Electron Number Density of Rocket Exhaust Plumes Through Microwave Transmissions
Department: Electrical, Computer, Software, and Systems Engineering
Degree: MSECE
Convocation: December 2014

Permission is herewith granted to Embry-Riddle Aeronautical University to circulate and to have copied for noncommercial purposes, at its discretion, the above title upon the request of individuals or institutions.

Signature of Author: __________________________

Date: __________________________
“If you can’t take a little bloody nose, maybe you ought to go back home and crawl under your bed. It’s not safe out here. It’s wondrous, with treasures to satiate desires both subtle and gross; but it’s not for the timid.”
Abstract

Master of Science in Electrical and Computer Engineering

Characterization of Electron Number Density of Rocket Exhaust Plumes Through Microwave Transmissions

by Jorge Torres

Dr. William C. Barott, Dr. Jianhua Liu, and Dr. William Engblom
Department of Electrical, Computer, Software, and Systems Engineering

Charged rocket plumes generally exceed the length of their source vehicles, and offer lightning a favorable path to ground. Rocket plumes enhance the induced transient currents in flight electronics, and increase the risk of vehicle failure. The affinity of lightning to the plume can be associated with the plume’s electrical properties, which are coupled to plasma characteristics including the electron number density. However, the electron number density of rocket plumes is not well-known. In this study, the electron number density is characterized through data from static rocket firings. A model of the plume in finite difference time domain (FDTD) simulations also supports the results. Radio frequency and radar methodologies are used to characterize the plume as a dynamic component of an electrical system, supported by the construction of an RF apparatus that includes the design and manufacture of ultra-wideband antenna arrays. The research estimates electron number density using methods exploiting signal processing techniques in time and frequency domain, but the data suggests that other dynamic elements influence delay and attenuation of the radio signal.
Acknowledgements

I would like to acknowledge Dr. William Barott for his assistance and persistent encouragement throughout the research. His support and motivation made it possible to carry on even when challenged by different pieces of this research. I would like to thank my thesis committee, Dr. Jianhua Liu and Dr. William Engblom, for support of this research, as well as Dr. Ebenezer Gnanamanickam, the Embry-Riddle Future Space Explorers and Developers Society (ERFSEDS), and the Experimental Rocket Propulsion Lab (ERPL) for their help with organizing and setting up the live tests. Acknowledged also is the Rogers Corporation, who donated milling substrate to this research. I’d also like to thank Jeanette Barott for her help in editing the document, my family for their encouragement, and lastly, my future wife, Kelly Baczuk. She is my inspiration.
# Contents

Declaration of Authorship i  
Distribution Agreement ii  
Abstract iv  
Acknowledgements v  
Contents vi  
List of Figures viii  
List of Tables xii  
Abbreviations xiii  
Symbols xiv  

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1</td>
<td>Summary</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>Motivation</td>
<td>1</td>
</tr>
<tr>
<td>1.3</td>
<td>Previous Works</td>
<td>3</td>
</tr>
<tr>
<td>1.4</td>
<td>Scope of Works</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>Background and Theory</td>
<td>5</td>
</tr>
<tr>
<td>2.1</td>
<td>Plume Plasma Theory</td>
<td>5</td>
</tr>
<tr>
<td>2.2</td>
<td>Time-Delay Measurements</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>Methodology</td>
<td>10</td>
</tr>
<tr>
<td>3.1</td>
<td>Finite-Difference Time Domain Simulations</td>
<td>10</td>
</tr>
<tr>
<td>3.1.1</td>
<td>FDTD Parameters</td>
<td>10</td>
</tr>
<tr>
<td>3.1.2</td>
<td>Model Construction</td>
<td>11</td>
</tr>
<tr>
<td>3.2</td>
<td>Apparatus</td>
<td>14</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Selection of Radio Frequencies</td>
<td>14</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Antenna Design</td>
<td>14</td>
</tr>
</tbody>
</table>
List of Figures

1.1 The AC-67 Atlas I SLV mission carrying FLTSATCOM resulted in failure due to a lightning strike [2] .................................................. 2
1.2 Lightning strikes the Saturn V vehicle during the Apollo 12 mission [3] . 2
1.3 Increased gain found in Nicholas Coutu’s thesis, believed to have been caused by beam focusing .............................................. 3
3.1 Coutu’s simulated values for $N_e$ ............................................. 11
3.2 Coutu’s simulated values for $v_e$ ............................................. 12
3.3 Composite image showing curves created by contours of $N_e$ and $v_e$ produced in previous CFD plots ........................................... 12
3.4 Solidworks model of the different plume regions ............................ 13
3.5 Model of the apparatus in Agilent EMPro .................................. 13
3.6 Traditional tapered-slot Vivaldi design ...................................... 15
3.7 Antipodal Vivaldi Design from Yang et al. ................................... 15
3.8 Modeled Vivaldi antenna describing parameters adjusted during simulation runs for optimization of gain patterns in an array ............... 16
3.9 Single Vivaldi antenna radiation pattern at 9 GHz .......................... 18
3.10 Single Vivaldi antenna radiation pattern at 13 GHz ....................... 18
3.11 Single Vivaldi antenna radiation pattern at 15 GHz ....................... 19
3.12 Ideal Vivaldi array radiation pattern at 9 GHz ............................ 19
3.13 Ideal Vivaldi array radiation pattern at 13 GHz .......................... 20
3.14 Ideal Vivaldi array radiation pattern at 15 GHz .......................... 20
3.15 S-parameters of 5-stage Wilkinson for 6-16 GHz range .................. 21
3.16 5-stage Wilkinson design with labeled sections .......................... 22
3.17 Complete 8-element Vivaldi array ........................................... 22
3.18 Simulated Vivaldi array radiation pattern at 9 GHz ....................... 23
3.19 Simulated Vivaldi array radiation pattern at 13 GHz ....................... 23
3.20 Simulated Vivaldi array radiation pattern at 15 GHz ....................... 23
3.21 Fabricated Vivaldi array ....................................................... 24
3.22 Block diagram of RF hardware operating in apparatus ................... 25
3.23 Amplifier-mixer board ......................................................... 25
3.24 One of the set of three DC power supplies ................................. 26
3.25 Local oscillator unit ......................................................... 26
3.26 Readying the test equipment for a firing ................................... 27
3.27 Test experiment setup behind the Lehman Science and Technology Building .................................................. 28
3.28 Live testing under canopy ................................................... 28
3.29 Closeup of rocket firing ..................................................... 29
3.30 Frequency response of apparatus, $S_{21}$ measured at VNA ............... 29
3.31 Time-domain response of apparatus, $S_{21}$ measured at VNA .......... 30
3.32 Measuring relative strength of the response of the entire apparatus with the help of a laboratory assistant ................................................................. 31
3.33 Relative strength of the response of the entire apparatus at 9.5 GHz .... 32
3.34 Relative strength of the response of the entire apparatus at 11.5 GHz .... 32
3.35 Relative strength of the response of the entire apparatus 16 GHz ....... 33
3.36 Elliptical RF reflector positioning .............................................................. 34
3.37 Elliptical RF reflector during testing ............................................................ 34
3.38 System time response after adding elliptical section ......................... 35
3.39 System frequency response after the elliptical is added shows multipath interference effects, including a null at approximately 1.9 GHz on the graph 36
3.40 Instrumentation added to the second set of tests .......................... 37
4.1 FDTD simulation of RF beam over the plume ............................................ 38
4.2 FDTD simulation showing some focusing effects ............................... 40
4.3 Skidmark burn direct-path strength over the burn .............................. 41
4.4 Classic burn direct-path strength over the burn ..................................... 41
4.5 Van der Beek study results showing signal gain attributed to air temperature increase .......................................................... 42
4.6 Skidmark burn direct-path strength over the burn, detail ................. 42
4.7 Classic burn direct-path strength over the burn, detail ..................... 43
4.8 Time-delay required to reach peak value of time-domain response over time in Skidmark burn .............................................................. 43
4.9 Time-delay required to reach peak value of time-domain response over time in Classic burn .............................................................. 44
4.10 Time-delay response for Skidmark burn, detail ............................. 44
4.11 Time-delay response for Classic burn, detail ......................................... 45
4.12 Solution curves for Skidmark burn ....................................................... 46
4.13 Solution curves for Classic burn ............................................................. 47
4.14 Collected instrumentation data - IR sensor distance, 3-axis accelerometer G-force for VMax and Longburn firings ............................................. 48
4.15 Collected instrumentation data - IR sensor distance, 3-axis accelerometer G-force for White firing .......................................................... 49
4.16 Collected instrumentation data - ambient temperature and pressure data for VMax and Longburn firings .................................................. 50
4.17 Collected instrumentation data - ambient temperature and pressure data for White firing .............................................................. 50
4.18 Collected instrumentation data - RF amplifier temperature for VMax, Longburn, and White firings. .............................................................. 51
4.19 VMax direct-path strength over the burn ........................................... 52
4.20 VMax direct-path strength over the burn, detail ............................... 52
4.21 VMax direct-path delay over the burn ................................................... 53
4.22 VMax direct-path delay over the burn, detail ...................................... 53
4.23 Longburn direct-path strength over the burn ..................................... 54
4.24 Longburn direct-path strength over the burn, detail ........................................... 54
4.25 Longburn direct-path delay over the burn ........................................... 55
4.26 Longburn direct-path delay over the burn, detail ...................................... 55
4.27 White direct-path strength over the burn .......................................... 56
4.28 White direct-path strength over the burn, detail ................................. 56
4.75 Whitened Skidmark direct-path strength over the burn, detail ............... 82
4.76 Whitened Skidmark direct-path delay over the burn ....................... 83
4.77 Whitened Skidmark direct-path delay over the burn, detail ............... 83
4.78 Whitened Classic direct-path strength over the burn ..................... 84
4.79 Whitened Classic direct-path strength over the burn, detail ............... 84
4.80 Whitened Classic direct-path delay over the burn ......................... 85
4.81 Whitened Classic direct-path delay over the burn, detail ............... 85
4.82 Solution curves for Skidmark firing (whitened) ............................ 86
4.83 Solution curves for Classic firing (whitened) ............................... 86
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Parameters chosen after empirical testing of antenna performance</td>
<td>17</td>
</tr>
<tr>
<td>3.2</td>
<td>Wilkinson section impedances and resistor value, and line width corresponding to the line impedance in 0.508 mm RO4003C (Refer to figure 3.16)</td>
<td>21</td>
</tr>
<tr>
<td>4.1</td>
<td>$S_{21}$ values in decibels (dB) as predicted by the FDTD model, derived from CFD values</td>
<td>39</td>
</tr>
<tr>
<td>4.2</td>
<td>$S_{21}$ values in decibels (dB) as predicted by FDTD model, derived from experimental values</td>
<td>39</td>
</tr>
<tr>
<td>4.3</td>
<td>Experimental results for motor burns 1 and 2</td>
<td>45</td>
</tr>
<tr>
<td>4.4</td>
<td>$N_e$ solutions for both of the motor burns in the first experiment</td>
<td>47</td>
</tr>
<tr>
<td>4.5</td>
<td>Experimental results for whitened VMax, Longburn, and White direct-path beam measurements</td>
<td>71</td>
</tr>
<tr>
<td>4.6</td>
<td>$N_e$ solutions for whitened VMax, Longburn, and White direct-path measurements</td>
<td>72</td>
</tr>
<tr>
<td>4.7</td>
<td>Experimental results for Whitened VMax, Longburn, and White reflected beam measurements</td>
<td>80</td>
</tr>
<tr>
<td>4.8</td>
<td>$N_e$ solutions for whitened reflected-beam measurements</td>
<td>81</td>
</tr>
<tr>
<td>4.9</td>
<td>Experimental results for whitened Classic and Skidmark measurements</td>
<td>85</td>
</tr>
<tr>
<td>4.10</td>
<td>$N_e$ solutions for whitened Skidmark and Classic data</td>
<td>87</td>
</tr>
</tbody>
</table>
# Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAD</td>
<td>Computer-Aided Design</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>ECSSE</td>
<td>Electrical, Computer, Software, and Systems Engineering</td>
</tr>
<tr>
<td>ERAU</td>
<td>Embry-Riddle Aeronautical University</td>
</tr>
<tr>
<td>ERFSEDS</td>
<td>Embry-Riddle Future Space Explorers and Developers Society</td>
</tr>
<tr>
<td>ERPL</td>
<td>Experimental Rocket Propulsion Labs</td>
</tr>
<tr>
<td>FDTD</td>
<td>Finite Difference Time Domain</td>
</tr>
<tr>
<td>GIMP</td>
<td>GNU Image Manipulation Program</td>
</tr>
<tr>
<td>IR</td>
<td>Infra-Red</td>
</tr>
<tr>
<td>LPKF</td>
<td>LeiterPlatten Koppier Frasen</td>
</tr>
<tr>
<td>MSECE</td>
<td>Master of Science in Electrical and Computer Engineering</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>Rx</td>
<td>Receiver</td>
</tr>
<tr>
<td>SMA</td>
<td>SubMiniature, Version A</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SLV</td>
<td>Space Launch Vehicle</td>
</tr>
<tr>
<td>Tx</td>
<td>Transmitter</td>
</tr>
<tr>
<td>VNA</td>
<td>Vector Network Analyzer</td>
</tr>
</tbody>
</table>
## Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>Bandwidth</td>
<td>Hz</td>
</tr>
<tr>
<td>$d$</td>
<td>Vivaldi antenna ground plane length</td>
<td>cm</td>
</tr>
<tr>
<td>$d_p$</td>
<td>Diameter of a slab of the plume</td>
<td>m</td>
</tr>
<tr>
<td>$h$</td>
<td>Vivaldi antenna width</td>
<td>cm</td>
</tr>
<tr>
<td>$l$</td>
<td>Vivaldi antenna length</td>
<td>cm</td>
</tr>
<tr>
<td>$L_{Np}$</td>
<td>Attenuation through a slab of the plume</td>
<td>Np</td>
</tr>
<tr>
<td>$L_{dB}$</td>
<td>Attenuation through a slab of the plume</td>
<td>dB</td>
</tr>
<tr>
<td>$N_e$</td>
<td>Electron number density</td>
<td>Number / cm$^3$</td>
</tr>
<tr>
<td>$r$</td>
<td>Vivaldi antenna feedline cutout radius</td>
<td>cm</td>
</tr>
<tr>
<td>$R$</td>
<td>Range resolution</td>
<td>m</td>
</tr>
<tr>
<td>$t_d$</td>
<td>Time offset</td>
<td>s</td>
</tr>
<tr>
<td>$t_0$</td>
<td>Plasma Relaxation time</td>
<td>s</td>
</tr>
<tr>
<td>$u_p$</td>
<td>Phase velocity</td>
<td>m / s</td>
</tr>
<tr>
<td>$v_e$</td>
<td>Electron-neutral collision frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>$y$</td>
<td>Path length</td>
<td>m</td>
</tr>
<tr>
<td>$Y$</td>
<td>Vivaldi antenna opening function</td>
<td>cm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Attenuation constant</td>
<td>Np / m</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Phase constant</td>
<td>rad/m</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Propagation Constant</td>
<td>m$^{-1}$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Permittivity</td>
<td>F / m</td>
</tr>
<tr>
<td>$\epsilon_r$</td>
<td>Relative permittivity</td>
<td>unitless</td>
</tr>
<tr>
<td>$\epsilon_s$</td>
<td>Static-frequency relative permittivity</td>
<td>unitless</td>
</tr>
<tr>
<td>$\epsilon_\infty$</td>
<td>Infinite-frequency relative permittivity</td>
<td>unitless</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Refractive index</td>
<td>unitless</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
<td>m</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>Relative permeability</td>
<td>unitless</td>
</tr>
<tr>
<td>$\mu_s$</td>
<td>Static-frequency relative permeability</td>
<td>unitless</td>
</tr>
<tr>
<td>$\mu_\infty$</td>
<td>Infinite-frequency relative permeability</td>
<td>unitless</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Conductivity</td>
<td>S/m</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>Magnetic conductivity</td>
<td>S/m</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Time resolution</td>
<td>s</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Radio wave angular frequency</td>
<td>radians/s</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>Plasma frequency</td>
<td>radians/s</td>
</tr>
</tbody>
</table>
I dedicate this study to the scientists, researchers, and engineers whose works were essential to my research. If I have seen further, it is by standing on the shoulders of giants.
Chapter 1

Introduction

1.1 Summary

This thesis is a continuation of research into a precise measurement of the electron number density, $N_e$, in rocket exhaust plumes. $N_e$ is quantified in this study by using radio frequency and radar techniques to measure small changes in radio waves propagating through the plumes.

1.2 Motivation

The combination of pressures, temperatures, and chemical interactions in rocket exhaust plumes creates a dusty, dynamic plasma. Plumes generally exceed the length of their source vehicles, and offer lightning a favorable path to ground. Although aircraft often endure lightning strikes, rocket plumes enhance the induced transient currents in flight electronics, and increase the risk of vehicle failure [1].

Historically, lightning strike events in rocketry include the loss of the U.S. Navy's FLT-SATCOM Satellite in 1987 (Figure 1.1) [2], and the ascent of the Apollo 12 mission in which a lightning strike took the Saturn Command Service Module instrumentation offline [3] (Figure 1.2).
Chapter 1. Introduction

Figure 1.1: The AC-67 Atlas I SLV mission carrying FLTSATCOM resulted in failure due to a lightning strike [2]

Figure 1.2: Lightning strikes the Saturn V vehicle during the Apollo 12 mission [3]

Lightning’s affinity to the plume can be associated with the plume’s electrical properties, which are coupled to plasma characteristics, including the electron number density. These properties can be used to model the plume’s observed effects on radio waves, such
as communications blackouts caused by attenuation and multipath interference [4]. The electron number density can be estimated by measurements of these effects [5].

Rigorous electromagnetic calculations of the plume’s effects on radio waves and its conductivity require knowledge of the plasma frequency $\omega_p$, collision frequency $v_e$, electron number density $N_e$, and radio wave frequency, $\omega$.

### 1.3 Previous Works

Previous work was done by Nicholas Coutu in his thesis, *Implementation of Microwave Transmissions for Rocket Exhaust Plume Diagnostics* [5], in which he constructed an apparatus to measure $N_e$ in J-class solid rocket motor plumes using C-band transmissions. Coutu’s data saw distortion effects believed to have been caused by focusing of antenna beams by the plume. This was manifested as a signal enhancement during motor firing instead of an attenuation, which would have been used to determine the density of the plasma. Due to this, a precise value for $N_e$ was not found in his thesis.

![Figure 1.3: Increased gain found in Nicholas Coutu’s thesis, believed to have been caused by beam focusing (Firing Occurs at 10 s) [5]](image_url)
1.4 Scope of Works

This research extends Coutu’s apparatus to work in higher frequency bands, as was recommended at the end of his thesis. This study attempts to explain whether the distortion effects were caused by focusing through electromagnetics simulations, and experimentally by observing the plume effects on a narrow (as opposed to Coutu’s wide) beamwidth. The apparatus in this research has a higher sampling rate than Coutu’s, allowing deeper analysis of slow-time events.

Additionally, this work introduces a new method of measuring plume properties through time-domain analysis, in terms of the phase velocity $u_p$ of a radio wave propagating through the plume, and the plume’s refraction index, $\eta$. Because of the ability to finely observe small differences in time-domain, multiple beams caused by intentional multipath are also used as a source of data.

After analysis in Chapter 4, a value for $N_e$ is determined through analysis of live rocket fire experiments.
Chapter 2

Background and Theory

The electrical properties of rocket exhaust are dominated by its plasma properties, which are largely determined by $N_e$, and the ion collision frequency, $v_e$ [6]. The amount of electrons in a rocket motor plume can be influenced by many factors, including the propellant formulation, alkali impurities, exhaust temperature, motor size, chamber pressure, flight speed and altitude, and the distance down-stream from the nozzle exit [7]. Solid rocket plumes particularly contain a high amount of alkali impurities, which have low ionization potentials, leading to a higher number of free electrons [8]. Fortunately, characterization of rocket plasma has been the subject of many studies, for both academic [4, 9–11] and commercial interests [6, 8, 12, 13]. However, available estimates for attenuation or conductivity are often an order of magnitude between prediction and tests [14, 15], meaning that additional ways to estimate plume characteristics are needed in order to estimate their electromagnetic effects.

2.1 Plume Plasma Theory

The plume is a dynamic medium, whose interaction with electromagnetic waves is largely frequency dependent. Lightning attractiveness is mostly related to a plume’s conductivity, $\sigma$, while radio wave attenuation is related more with its permittivity, $\epsilon$. The conductivity of a plasma given by

$$\sigma = \frac{q^2 N_e}{m_e (v_e - j\omega)},$$

where $N_e$ is the electron number density, $q_e$ is the elementary charge of an electron, $\omega$ is the frequency of an electrical signal, $m_e$ is the mass of an electron, and $v_e$ is the electron (ion) collision frequency [9].
The relative permittivity of plasmas is given by

$$\epsilon_r = 1 - \frac{w_p^2}{\omega (\omega - jv_e)},$$

(2.2)
in which $\omega_p$ is the plasma frequency \cite{15}, calculated from

$$\omega_p = \sqrt{\frac{N_e q_e^2}{\epsilon_0 m_e}},$$

(2.3)

where $\epsilon_0$ is the permittivity of free space \cite{16}. The electrical permittivity of a material is given by combining the permittivity of free space and its relative permittivity, by

$$\epsilon = \epsilon_r \epsilon_0.$$

(2.4)

Calculating $v_e$ requires situational knowledge of the plasma’s composition. One of Smoot’s works \cite{17} provides the approximation for collision frequency at a single point in a rocket plume, described as

$$v_e = v_{et} \sum_i n_i Q_i,$$

(2.5)

where $v_{et}$ is the mean electron thermal velocity defined by

$$v_{et} = \sqrt{\frac{8kT_e}{\pi m_e}}.$$

(2.6)

In equations 2.5 and 2.6, $k$ is Boltzmann’s constant, $T_e$ is the temperature of the plume at the estimating point, $n_i$ is the number density of species $i$, and $Q_i$ is the electron collision cross-section of species $i$. However, $n_i$ and $Q_i$ are dependent on the chemical interactions in the plume and must be obtained empirically by measurement or by simulation. Many other researchers in the subject \cite{4, 9, 15}, including Coutu \cite{5}, choose to derive both $v_e$ and $N_e$ through the rocket firings by calculating the propagation constant of the waves traveling through a rocket plume. The propagation constant can be calculated using different methods, or measured directly, but in definition is given by

$$\gamma = \alpha + j\beta,$$

(2.7)
where \( \alpha \) and \( \beta \) are the attenuation and phase constants of the propagating wave, respectively. The attenuation constant is given by

\[
\alpha = \frac{L_{Np}}{y},
\]  

(2.8)

where \( L_{Np} \) is the loss (or gain) through a medium in Nepers, and \( y \) is the path length through the medium in meters. Nepers are a logarithmic scale of power, and are converted to decibels using a factor of 10 \( \log e^2 \) [18]. The phase constant is frequency dependent, and is defined by

\[
\beta = \frac{\omega}{u_p},
\]  

(2.9)

where \( u_p \) is the phase velocity of the wave traveling through the medium.

Van der Beek et al. give an analytic calculation for \( \alpha \) and \( \beta \) based on plasma properties [9]:

\[
\alpha = \frac{\omega}{c} \sqrt{-\frac{1-A^2}{2} + \frac{1}{2} (1-A)^2 + \frac{v_e^2}{\omega^2} A^2},
\]  

(2.10)

and

\[
\beta = -\frac{\omega}{c} \sqrt{-\frac{1-A^2}{2} + \frac{1}{2} (1-A)^2 + \frac{v_e^2}{\omega^2} A^2},
\]  

(2.11)

where \( A \) is as described by

\[
A = \frac{\omega_p^2}{v_e^2 + \omega^2}.
\]  

(2.12)

By equating the two relations for \( \alpha \) in equations 2.8 and 2.10, and the two relations for \( \beta \) in equations 2.9 and 2.11, it is possible to characterize both the collision frequency and the electron number density by knowing \( \omega \), \( v_e \), and \( L_{Np} \).

Previous work by Smoot [14] also provides an approximation for the attenuation through a slab of plasma using only \( N_e \), \( v_e \), and \( \omega \), and path length \( y \) as

\[
L_{dB} = 0.46 \left( \frac{y v_e N_e}{v_e^2 + \omega^2} \right),
\]  

(2.13)

which can be used as a secondary form of estimation for \( N_e \) if the collision frequency is known.
2.2 Time-Delay Measurements

In radar theory, the range resolution is determined by the signal bandwidth, $B$ [19]. The smallest time interval that the radar can resolve is the inverse of bandwidth, as shown by

$$\tau = \frac{1}{B}.$$  \hspace{1cm} (2.14)

In order to find the phase velocity $u_p$ of a radio wave traveling through the plume, the delay of the wave in the plume must be characterized by comparing measurements in time-domain to those before and during a rocket motor fire. To the author’s knowledge, characterization of $u_p$ in rocket plumes has not previously been examined for plasma attenuation or electron density studies.

This inherently requires a wide bandwidth system to extract small delays in time-domain, which spreads the value for $N_e$ and $v_e$ in section 2.1, due to varying values of $\omega$. Also, the smaller the rocket motor, the more precise the ranging information must be, since light travels at approximately one foot per nanosecond, but is slowed only slightly by most materials. Sufficient accuracy should require ranging information to be at the picosecond level for this work.

Because resolution is the inverse of system bandwidth, a 1 ps resolution requires an unrealistically high bandwidth of 1 THz. To reduce this requirement, signals in the frequency-domain may be multiplied by a phasor, $e^{-j\omega t_d}$, to shift in the time-domain by $t_d$ seconds [20] as shown by

$$X(j\omega)e^{-j\omega t_d} \Rightarrow x(t - t_d).$$ \hspace{1cm} (2.15)

While the resolution is proportional to the inverse of bandwidth, the ability to detect an impulse in the absence of other signals scales as

$$\tau \propto \frac{1}{B\sqrt{\text{SNR}}},$$ \hspace{1cm} (2.16)

where SNR is signal-to-noise ratio. e.g. A 1 ps accuracy with 3 GHz of bandwidth requires 50 dB of SNR. Phasors of varying time $t_d$ can be multiplied with frequency domain data, and the peak of the resulting time-domain signal would indicate the true direct path maximum. If this is done before and during a rocket motor fire, the change in the $\Delta t_d$ required to find the signal maximum gives a delay in the electromagnetic
wave. This greatly reduces the needed bandwidth of the system, since if the plume is electrically large compared to the wavelengths of the incoming wave, its delay can be measured. This is true so long as the range resolution of the radar-like signal is able to separate the plume from multipath, where range resolution [19] is given by

\[ \Delta R = c\tau. \]  

(2.17)

It is also possible to calculate the electron density directly from the time-delay, since if the propagation velocity \( u_p \) of a wave in the plume is known, the index of refraction can be calculated as a ratio of the velocity of light in air and the velocity of light in the plume as shown by

\[ \eta = \frac{c}{u_p} = \sqrt{\epsilon_r}. \]  

(2.18)
Chapter 3

Methodology

3.1 Finite-Difference Time Domain Simulations

As a component of this study, preliminary simulation work was performed to determine whether focusing effects hypothesized in Coutu’s thesis could be observed on a model of the plume using Finite-Difference Time-Domain (FDTD) software.

A model of Coutu’s experiment was created in Dassault Systèmes’ Solidworks and simulated in Agilent EMPro.

3.1.1 FDTD Parameters

In FDTD simulations, plasmas are typically modeled using the following parameters:

1. Infinite frequency relative permittivity, $\epsilon_\infty$
2. Infinite frequency relative permeability, $\mu_\infty$
3. Static relative permittivity, $\epsilon_r$
4. Static relative permeability, $\mu_r$
5. Electrical conductivity, $\sigma$
6. Magnetic conductivity, $\sigma_m$, and
7. Plasma Relaxation time, $t_0$. 
Zhu, in his master’s thesis, describes how these parameters can be derived from the plasma frequency (which is dependent on $N_e$) and is described in equation 2.3, and the damping (collision) frequency, $v_e$ [21]. The parameters are defined as

$$\sigma = \varepsilon_0 \frac{\omega_p^2}{v_e}, \quad (3.1)$$

$$\sigma_m = \mu_0 \frac{\omega_p^2}{v_e}, \quad (3.2)$$

$$\epsilon_s = - \left( \frac{\omega_p}{v_e} \right)^2 + \epsilon_\infty, \quad (3.3)$$

$$\mu_s = - \left( \frac{\omega_p}{v_e} \right)^2 + \mu_\infty, \quad (3.4)$$

and

$$t_0 = \frac{1}{v_e}. \quad (3.5)$$

In order to simplify the model, Zhu assumed that the values of $\epsilon_\infty$ and $\mu_\infty$ were equal to 1.

### 3.1.2 Model Construction

Since $v_e$ and $N_e$ are only known experimentally in this study, the FDTD plume model was initially constructed with values estimated using the Computational Fluid Dynamics (CFD) analysis in Coutu’s thesis.

![Figure 3.1: Coutu’s simulated values for $N_e$ [5]](image-url)
Using the GNU Image Manipulation Program (GIMP), the two contours were combined to produce a set of regions with different $N_e$ and $v_e$ values as shown in Figure 3.3.

This was then revolved in Solidworks to create a three-dimensional CAD model that could be imported into EMPro.
The rest of the apparatus was also modeled in Solidworks and imported into EMPro.

Figure 3.4: Solidworks model of the different plume regions

Figure 3.5: Model of the apparatus in Agilent EMPro
3.2 Apparatus

The apparatus was built as an extension of Coutu’s apparatus [5] to enable higher frequencies and a vertically compressed beamwidth. In designing the apparatus, the plume is considered as a dynamic component of the RF system, using radio frequency and radar techniques to analyze how it attenuates and delays a radio wave. This is similar to principles established by Baghdady on characterizing radio properties through rocket plumes [10].

3.2.1 Selection of Radio Frequencies

Coutu’s thesis suggested that, for the scale of rocket engines being tested, higher frequencies exhibited stronger results when passing through the plume [5]. This is logical, as a J-class rocket motor plume is not large compared to the size of a wavelength in the C-band range. The region of highest electron densities, which Coutu estimated to be 10 cm wide, is the focus of the experiment. In addition, at the lower edge of the C-band frequencies, the experiment apparatus placed the antennas before the far-field [22], and was therefore susceptible to possible near-field distortions. Being limited by equipment availability and cost, the focus of this research moved the C-band measurements in Coutu’s experiment into the X and Ku-bands. Since this new range of frequencies is outside the range of the original apparatus, including its Vector Network Analyzer (VNA), the apparatus must now incorporate new hardware, including a local oscillator, mixers, filters, and antennas, to support the higher frequency range.

3.2.2 Antenna Design

The antennas are the most crucial part of the apparatus in expanding this study. The antennas for the apparatus must overcome the difficulties faced by the previous experiments in two ways. First, the vertical beamwidth of the antennas must be reduced to reduce the possibility of bending and focusing of the radio waves propagating through the plume, and minimize multipath interference from the ground in order to space the antennas at far-field distances. Second, the antennas must support as narrow a beamwidth as possible at a range of frequencies higher than the original experiment’s, thereby making the wavelength smaller in relation to the size of the plume, and maintain sufficient time resolution to separate the direct path signal from reflections in order to observe time-delay effects caused by the plume. A higher gain also helps mitigate the increased path loss due to higher frequencies.
As in Coutu’s thesis, horn antennas were determined to be prohibitively expensive despite the advantages provided by their wide bandwidth. Additionally, the capability to mill printed circuit boards with high precision at the laboratory enabled fabrication of antennas from RF-specialty laminates. Therefore, an array of Vivaldi antennas were implemented as both the receive and transmit elements to fulfill the specified criteria.

Yang, Wang, and Fathy describe the methodology of designing a Vivaldi antenna array. The paper details the design of both a “traditional” tapered-slot antenna and an antipodal Vivaldi antenna [23].

![Figure 3.6: Traditional tapered-slot Vivaldi design [24]](image1)

![Figure 3.7: Antipodal Vivaldi Design from Yang et al. [23]](image2)

The antipodal design (figure 3.7) is comparable in characteristics to the traditional tapered slot (figure 3.6), but has fewer parameters to adjust, and is therefore easier to design. The antennas were modeled and tested for performance in gain across a range of 8 to 16 GHz using Solidworks and EMPro. To enable rapid prototyping of the antennas, the parameters for antipodal Vivaldis were simplified optimized for an array configuration.
Since the total opening of the Vivaldi antenna affects its bandwidth (i.e., the larger the opening, the lower the minimum frequency of the antenna becomes), the \( w \) value in Yang et al.’s antenna model (figure 3.7) is reduced to zero, where the entire width of one antenna is now \( h \), as shown in figure 3.8.

\[ \text{Figure 3.8: Modeled Vivaldi antenna describing parameters adjusted during simulation runs for optimization of gain patterns in an array} \]

Since it is known that the antenna will be used in an array, it can also be assumed that \( h \) will be locked to

\[ h = \frac{\lambda_{\text{max freq}}}{2}, \]  

(3.6)

in order to maximize bandwidth and without introducing grating lobes into the overall array gain pattern [25].

The parameter \( R \) in Figure 3.8 represents the opening rate of the Vivaldi antennas, as described by Yang et al. [23] in the equation describing the antenna opening rate,

\[ Y = c_1 e^{Rx} + c_2 \]  

(3.7)

where \( c_1 \) and \( c_2 \) are described by
Chapter 3. Methodology

\[ c_1 = \frac{y_2 - y_1}{e^{Rx_2} - e^{Rx_1}} \]  \hspace{1cm} (3.8) \\

and

\[ c_2 = \frac{y_1e^{Rx_2} - y_2e^{Rx_1}}{e^{Rx_2} - e^{Rx_1}} \]  \hspace{1cm} (3.9) \\

respectively. However, assuming that \( y_1 \) and \( x_1 \) are the origin, \([0,0]\), the equation can be reduced into

\[ y = \frac{h}{e^{R(l+d)} - 1} (e^{Rx} - 1), \]  \hspace{1cm} (3.10) \\

the parameters for which are as labeled in figure 3.8. The value of \( R \) affects the gain of the antennas at an inverse of bandwidth. For the apparatus antenna, the \( R \) value of -0.12 was chosen as an optimal compromise between bandwidth and gain, as described in Abbosh’s paper [26].

The values of \( r \) and \( d \) in figure 3.8 are adjusted only for impedance matching the antennas to the rest of the feed. In this case, \( r \) is adjusted to change the impedance of the feed strip into the radiating element. The parameter \( d \) needs to be large enough to make a proper match, and can extend from then on until the end of the antenna printed circuit without affecting performance. For an antenna array, \( d \) encompasses the entire backplane of the antenna leading up to the half of the radiating element. The overall length of the antenna is adjusted to the value at which the \( y \) function in equation 3.10 intersects the corner of the board at a 45-degree angle, past which the antenna sees decremental gains in bandwidth.

Using, Rogers 4003C Laminate with a thickness of 0.508 mm, the selected antenna parameters were as described in table 3.1.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&gt;5 mm</td>
</tr>
<tr>
<td>( l )</td>
<td>43 mm</td>
</tr>
<tr>
<td>( r )</td>
<td>12 mm</td>
</tr>
<tr>
<td>( h )</td>
<td>19.5 mm</td>
</tr>
<tr>
<td>( R )</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

Table 3.1: Parameters chosen after empirical testing of antenna performance
producing an antenna pattern as illustrated in figures 3.9 - 3.11 indicating the principal polarization strength in dBi.

(a) E-plane of 9 GHz Radiation Pattern  
(b) H-plane of 9 GHz Radiation Pattern

Figure 3.9: Single Vivaldi antenna radiation pattern at 9 GHz

(a) E-plane of 13 GHz Radiation Pattern  
(b) H-plane of 13 GHz Radiation Pattern

Figure 3.10: Single Vivaldi antenna radiation pattern at 13 GHz
Chapter 3. Methodology

The RO4003C material available in the laboratory was 12” x 18”, which limited the array size to eight elements if exotic wideband power splitter configurations are avoided. An antenna pattern produced by eight elements of this antenna with power perfectly split would result in pattern described in figures 3.12, 3.13, and 3.14.

Figure 3.11: Single Vivaldi antenna radiation pattern at 15 GHz

(a) E-plane of 15 GHz Radiation Pattern  (b) H-plane of 15 GHz Radiation Pattern

Figure 3.12: Ideal Vivaldi array radiation pattern at 9 GHz

(a) E-plane of 9 GHz Radiation Pattern  (b) H-plane of 9 GHz Radiation Pattern
In order to split power equally among the eight array elements while maintaining an optimal response across the antennas’ bandwidth, a five-stage Wilkinson transformer was chosen as a splitter for the array. The transformer’s impedances were determined with microwaves101’s spreadsheet tool [27], which produced resistor values and line impedances as listed in table 3.2.
Chapter 3. *Methodology*

Table 3.2: Wilkinson section impedances and resistor value, and line width corresponding to the line impedance in 0.508 mm RO4003C (Refer to figure 3.16)

<table>
<thead>
<tr>
<th>Section</th>
<th>Resistor value</th>
<th>Line impedance</th>
<th>Line width for 0.508 mm RO4003C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>52.3 Ω</td>
<td>1.22 kΩ</td>
<td>0.33 mm</td>
</tr>
<tr>
<td>2</td>
<td>59 Ω</td>
<td>206 Ω</td>
<td>0.44 mm</td>
</tr>
<tr>
<td>3</td>
<td>71.5 Ω</td>
<td>210 Ω</td>
<td>0.65 mm</td>
</tr>
<tr>
<td>4</td>
<td>84.5 Ω</td>
<td>122 Ω</td>
<td>0.91 mm</td>
</tr>
<tr>
<td>5</td>
<td>95.3 Ω</td>
<td>65.8 Ω</td>
<td>1.11 mm</td>
</tr>
</tbody>
</table>

The response of the five-stage section calculated by the spreadsheet tool is given by the graph in figure 3.15.

![Figure 3.15: S-parameters of 5-stage Wilkinson for 6-16 GHz range](image)

Utilizing 0402 resistor dimensions, the section was synthesized in Solidworks, producing a section illustrated in figure 3.16.
Assembling all of the previous elements together yields a complete array as depicted in figure 3.17.
Finally, the performance of the entire array is measured in terms of the gain and beam pattern by simulating the array in EMPro, with its patterns described at 9 GHz, 13 GHz, and 15 GHz, in figures 3.18, 3.19, and 3.20 respectively.

**Figure 3.18:** Simulated Vivaldi array radiation pattern at 9 GHz

**Figure 3.19:** Simulated Vivaldi array radiation pattern at 13 GHz

**Figure 3.20:** Simulated Vivaldi array radiation pattern at 15 GHz
As expected, the radiation pattern of the full array with the power dividers implemented is measurably weaker than the ideal eight-way split seen earlier in figures 3.12, 3.13, and 3.14. The five-stage Wilkinson transforms have a flat response across their design bandwidth, at the cost of a higher insertion loss over the 8 GHz frequency range. The realistic case loses about 5 dB in the radiation pattern, due to inefficiencies in the system.

The Vivaldi array was fabricated using an LPKF S103 mill in the Radar & Microwaves Lab, and is shown in figure 3.21.

![Fabricated Vivaldi array](image_url)

**Figure 3.21:** Fabricated Vivaldi array

### 3.2.3 RF Hardware

The apparatus uses an Agilent E5071B Vector Network Analyzer (VNA) provided by the Radar & Microwaves Laboratory as the radio source and data acquisition device. A step-up/step-down heterodyne at 8.5 GHz interfaces the VNA with the antennas, which operate over the measurement band of 8.53 to 13 GHz. Attenuation is measured with a repeatability of 0.01 dB, and the 4.5 GHz bandwidth supports a time resolution of roughly 250 ps, as calculated using equation 2.14, and a range resolution of 3.75 centimeters as calculated using equation 2.17. The VNA captures approximately eight frames per second, with 400 points in a frequency sweep, at an intermediate frequency (IF) bandwidth of 100 KHz.
Most of the RF hardware shown on the block diagram in figure 3.22 is composed of small components from Mini-Circuits with SMA connectors, fastened to a wooden board as shown in figure 3.23. A full list of components in the RF system can be found in Appendix A.

The VNA and the Tx amplifiers are separated from the rest of the test apparatus by a long RG316 SMA-terminated cable, in order to enable the VNA operation at a safe distance away from the firing.

Other equipment used in the study is shown in figure 3.24 and 3.25.
Chapter 3. Methodology

Figure 3.24: One of the set of three DC power supplies

Figure 3.25: Local oscillator unit

The system is shown as deployed in figure 3.26, in the garage area of the Lehman Science and Technology Building.
3.2.4 Experimental Setup

Two sets of live fire experiments were conducted behind the Lehman Science and Technology Building by mounting the J and K-class motors on a large steel frame braced with a waste receptacle behind it. Access to the area during the firings was controlled by volunteers from the campus’s local rocket societies: the Embry-Riddle Future Space Developers and Explorers Society (ERFSEDS), and the Experimental Rocket Propulsion Labs (ERPL).

A rough map of the area is provided in figure 3.27. The image marks a safe viewing distance, corresponding to the Tripoli Rocket Association Safe Launch Practices Manual’s minimum safe distance for a non-complex research launch [28]. The VNA and its operator were placed under a roll-down blast door in the nearby Wind-Tunnel Laboratory.
Because of rainy weather during testing, the two sets of tests were conducted under a canopy as shown in figure 3.28 to protect exposed electronics.

A close-up image of one of the rocket firings is shown in figure 3.29.
3.2.5 System Response

The frequency-domain response of the entire system in the experiment setup can be seen in figure 3.30. Note that, because frequencies are heterodyned, the VNA is offset by 8.5 GHz from the transmitted signal.
The frequency response shows some aliasing due to the reflections in the environment. This seems to be confirmed after the elliptical section described in section 3.2.7 is added, since the frequency response at that time does not exhibit the same rippling behavior. While the antennas and power dividers obtaining other RF components with as wide a frequency range was cost-prohibitive. The high-pass filters introduce the most significant cutoff, and produce a favorable response from only 8-13 GHz.

The system time-domain response obtained after performing an inverse Fourier transform on the frequency-response data is shown in figure 3.31. The time-domain signal exhibits a strong main beam peak, as well as some reflections from the environment.

![Time-domain response of apparatus, S21 measured at VNA](image)

**Figure 3.31:** Time-domain response of apparatus, $S_{21}$ measured at VNA

### 3.2.6 Antenna Testing

Using the thesis apparatus, a same-scale experiment of relative signal strength across a set of measured points was performed using a flat surface and large sheets of paper, as shown in figure 3.32.
The scale of the distance in the measurements reflects a larger distance than in actual field tests. Despite the other components in the apparatus not being able to achieve as wide a bandwidth as the antennas and power splitters, the radiation pattern of the antennas is captured by mapping the strengths of the measurements on paper to a color by using image manipulation software. These colors are "blended" together using the Gaussian blur and smudge tools in the GNU Image Manipulation Program (GIMP). The results are in figures 3.33, 3.34, and 3.35.
Figure 3.33: Relative strength of the response of the entire apparatus at 9.5 GHz

Figure 3.34: Relative strength of the response of the entire apparatus at 11.5 GHz
3.2.7 Elliptical Reflector

The results shown in the first two motor firings (see section 4.2.1), demonstrated that the time domain response of the apparatus exhibited some reflections as shown in figure 3.31. These reflections were strong enough, and the apparatus’s time resolution good enough, that it would be possible to measure their peaks over time if their path through the plume was known. To that end, a reflector was added to the apparatus, in order to introduce another set of data corresponding to an RF signal following a different path through the plasma. Since the apparatus can ideally sample 222 ps, the reflector is placed such that it is on a path approximately 1,110 ps away, assuming the speed of light $c$ is approximately $3 \times 10^8$ m/s. The reflector consists of a 1 ft x 1 ft square metal sheet, mounted on a wooden frame forming a section of a ellipse with the two experiment antennas at the foci and the reflector at an approximately 45-degree angle to the receive antenna. A diagram showing approximate positioning can be seen in figure 3.36.
Figure 3.36: Elliptical RF reflector positioning. Note that this image shows half of the apparatus plane from above where 0,0 corresponds to halfway between the plume, in the antennas’ direct path.

The reflector itself can be seen in testing in figure 3.37.

Figure 3.37: Elliptical RF reflector during testing

The presence of the reflector in the apparatus adds a strong secondary main beam, as shown in the time response graph in figure 3.38.
Figure 3.38: System time response after adding elliptical section. Note the two large peaks a few nanoseconds apart, corresponding to the main beam and the elliptical reflected beam.

As a result, a noticeable null is introduced in the frequency response at about 1.9 GHz (which corresponds to 10.4 GHz actually transmitted) due to the intentional multipath interference.
Figure 3.39: System frequency response after the elliptical is added shows multipath interference effects, including a null at approximately 1.9 GHz on the graph.

3.2.8 Additional Test Instrumentation

In order to observe the effects of heat and antenna movement postulated in section 4.2.1, a microcontroller and a set of sensors was added to the third, fourth, and fifth firing that include a three-axis accelerometer, ambient temperature sensor, barometric pressure sensor, IR spot thermometer, and a distance sensor. The sensors and microcontroller were mounted to the same material the antenna sits on, and the distance sensor was aimed directly at the antenna to quantify how much it moved during a firing. A picture of the sensor instruments is shown in figure 3.40.
An additional microcontroller was added near the RF amplifier-mixer board, shown in figure 3.23, to measure the temperature of components on the board using Type-K thermocouples while testing.

### 3.3 Software

The data generated by the VNA is saved in S2P (touchstone) files, which contain complex frequency-domain data in decibels. Data analysis is performed through Python scripts, which use the scikit-rf library for extracting information from the touchstone files. Time-delay analysis is an exception, being produced in MATLAB by thesis committee chair, Dr. William C. Barott.

The VNA is controlled via telnet by a laptop running a distribution of GNU/Linux with a tcl expect script, which is responsible for communicating with the VNA to store information, and ensuring that live data is being saved. Another Python script can display time or frequency information for those stored files.

The source code used in this thesis is found in Appendix B.
Chapter 4

Results

4.1 FDTD Results

4.1.1 CFD Values

Due to the extremely high simulation times on a detailed model, the simulated area on
the FDTD simulation was confined to the area in the direct path of the plume. Using
the CFD values from Coutu’s thesis [5], simulations performed in Agilent EMPro were
not able to duplicate focusing effects postulated by Coutu.

Figure 4.1: FDTD simulation of RF beam over the plume

Additionally, the attenuation prediction from the FDTD software did not match ex-
pected real-life results provided in the literature. This implies that either the model
generating values for $N_e$ and $v_e$ in CFD was inaccurate, or the FDTD model was inaccurate.

The simulated $S_{21}$ model results are as shown on table 4.1.

<table>
<thead>
<tr>
<th></th>
<th>3 GHz</th>
<th>4 GHz</th>
<th>5 GHz</th>
<th>6 GHz</th>
<th>7 GHz</th>
<th>8 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Plume</td>
<td>-163.7</td>
<td>-150.033</td>
<td>-139.899</td>
<td>-131.271</td>
<td>-132.861</td>
<td>-129.308 dB</td>
</tr>
<tr>
<td>With Plume</td>
<td>-176.577</td>
<td>-160.751</td>
<td>-150.634</td>
<td>-141.865</td>
<td>-142.342</td>
<td>-140.176 dB</td>
</tr>
</tbody>
</table>

**Table 4.1: $S_{21}$ values in decibels (dB) as predicted by the FDTD model, derived from CFD values**

### 4.1.2 Experimental Values

After the results analysis from section 4.2.1, the model was re-made with the experimentally-derived values for $N_e$ and $v_e$, producing the attenuation as shown in table 4.2.

<table>
<thead>
<tr>
<th></th>
<th>3 GHz</th>
<th>4 GHz</th>
<th>5 GHz</th>
<th>6 GHz</th>
<th>7 GHz</th>
<th>8 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Plume</td>
<td>-163.7</td>
<td>-150.033</td>
<td>-139.899</td>
<td>-131.271</td>
<td>-132.861</td>
<td>-129.308 dB</td>
</tr>
<tr>
<td>With Plume</td>
<td>-172.560</td>
<td>-155.323</td>
<td>-144.286</td>
<td>-135.670</td>
<td>-138.146</td>
<td>-132.061 dB</td>
</tr>
<tr>
<td>Difference</td>
<td>-5.86</td>
<td>-5.29</td>
<td>-4.387</td>
<td>-4.399</td>
<td>-5.285</td>
<td>-3.753 dB</td>
</tr>
</tbody>
</table>

**Table 4.2: $S_{21}$ values in decibels (dB) as predicted by FDTD model, derived from experimental values**

In this model, the values for attenuation are still too high. While the model does not show any signal gain, it is closer to the real-life case than when the CFD-derived values were used, suggesting the experimental values are more accurate.

Additionally, the plume in the model seemed to be bending in the RF waves passing around it, which could imply a focusing effect. A still frame of the effect is shown in figure 4.2.
It is possible that the assumptions in the model are not entirely correct at lower frequencies, but using the experimental values has changed the model significantly in a way more closely resembling the effects of the plume. Additionally, the presence of slight focusing can partly validate some of the conclusions from Coutu’s experiments.

4.2 Test Data

4.2.1 Motor Burns 1 and 2

The first live experiment was conducted on 9 September, 2014, with amateur sport rocket Level 2 High Power Rocket motors:

1. Cesaroni J295 54 mm 3-Grain Classic, and
2. Cesaroni J360 54 mm 3-Grain Skidmark.

The Classic motor has a total impulse of 1195 Ns and a burn duration of 4 s. The Skidmark motor has a total impulse of 1016 Ns and a burn duration of 2.8 s, with titanium particulates added to the grain for a more dramatic effect [29].

The first test performed used the Skidmark motor, and data recording was stopped a few seconds after the motor fire. By recording the peak value of the direct-path beam as described in section 3.2.5, the plot in figure 4.3 was produced.
Chapter 4. Results

Figure 4.3: Skidmark burn direct-path strength over the burn

The response of the system shows a clear drop upon the motor firing. However, there appears to be a decaying increase in signal strength immediately after the motor fire. To analyze this behavior, the next motor fire’s experiment recorded data for an extended period of time.

Figure 4.4: Classic burn direct-path strength over the burn

The same pattern appears in figure 4.4 as with the Skidmark motor, with a large initial drop, and a decaying increase in signal strength.
This pattern is similar to one encountered by Van der Beek [9] in a study of rocket plume attenuation using waveguides. The effect was attributed to heating of the air inside the waveguide, changing the permittivity of the medium.

This effect could be a plausible explanation for the increased signal strength. The test was conducted during a hot day under a canopy because of the heavy rain during the experiment. It is possible that these conditions caused heating of the air under the canopy upon the rocket fire, with the air restoring to a normal temperature over time. Assuming these effects are true, the initial drop of the direct-path strength can be considered to be the attenuation due to the plasma.
Performing delay analysis of the two rocket motor burns produces the plots in figures 4.8 and 4.9.
For the delay calculations, a similar pattern emerges from the data as with the attenuation plots. Following the same assumptions, however, the peak delay can be considered the plasma effect before temperature effects take over.

Figure 4.9: Time-delay required to reach peak value of time-domain response over time in Classic burn

Figure 4.10: Time-delay response for Skidmark burn, detail
Chapter 4. Results

Figure 4.11: Time-delay response for Classic burn, detail

The experimental results are summarized in table 4.3.

<table>
<thead>
<tr>
<th></th>
<th>Attenuation</th>
<th>Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classic</td>
<td>0.4315 dB</td>
<td>16.31 ps</td>
</tr>
<tr>
<td>Skidmark</td>
<td>0.2275 dB</td>
<td>8.24 ps</td>
</tr>
</tbody>
</table>

Table 4.3: Experimental results for motor burns 1 and 2

To find $N_e$ and $v_e$, the attenuation and delay are used as described in Section 2.1.

However, since the radio frequency $\omega$ now spans a wide bandwidth, contending with an additional unknown to $N_e$ and $v_e$ in the propagation constant equations 2.10 and 2.9 resulted in an inability to determine a solution for $N_e$ with those equations. However, using Smoot’s model in equation 2.13 and the refractive-index solution in 2.18, two solution sets can be generated and plotted against each other to find a solution for $N_e$ over a range of frequencies.

The solution for $N_e$ in equation 2.18 in terms of the delay, plume width $y$, and $v_e$ is

$$N_e = \frac{-c^2 (c^2 (delay)^2 \omega + 2c (delay)^2 \omega y - j \epsilon_0 m_e v_e^2 \omega y^2)}{q_0^2 (delay)^2 \epsilon_0 m_e v_e^2 \omega y - \epsilon_0 m_e v_e^2 \omega y^2}. \quad (4.1)$$

The solution for $N_e$ in equation 2.13 in terms of the attenuation, $L_{dB}$, $y$, and $v_e$ is as described in equation 4.2,
Chapter 4. *Results*

\[ N_e = \frac{2.17391 L_{dB} \left( v_e^2 + \omega^2 \right)}{y v_e}, \]  

(4.2)

Plotting both solutions with varying values of \( v_e \) as iterated over \( \omega \) produces the graphs in figures 4.12 and 4.13. In these plots, the solid lines represent solutions using equation 4.1, and the dotted-line solutions represent the results using equation 4.2. Both solution sets seem to be inversely related in terms of \( v_e \), which allows for convergence between them for the right values. It follows that \( N_e \) should lie between two lines of either solution at a convergence value of \( v_e \).

**Figure 4.12:** Solution curves for Skidmark burn
The solution for the Skidmark burn has a convergence between a $v_e$ value of $2.5 \times 10^8$ and $5 \times 10^8$, where the Classic burn seems very close at a $v_e$ of $5 \times 10^8$. The solutions for both of the motor burns are summarized in table 4.4.

<table>
<thead>
<tr>
<th></th>
<th>$N_e$</th>
<th>$v_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skidmark</td>
<td>$2.8843 \times 10^{13} - 5.7032 \times 10^{13}$</td>
<td>$4 \times 10^8$ Hz</td>
</tr>
<tr>
<td>Classic</td>
<td>$5.6225 \times 10^{13} - 7.8552 \times 10^{13}$</td>
<td>$5 \times 10^8$ Hz</td>
</tr>
</tbody>
</table>

Table 4.4: $N_e$ solutions for both of the motor burns in the first experiment
4.2.2 Motor Burns 3, 4, and 5

The next experiment was constructed with the addition of the elliptical reflector and instrumentation to the apparatus as described in sections 3.2.7 and 3.2.8. The primary focus of the second experiment was to experimentally confirm whether the very small delays and attenuation observed in the system were caused by the plume or by the antennas being moved by the rocket motor’s firing. The motors used were the following:

1. Cesaroni J1520 54 mm 3-Grain VMax,
2. Cesaroni J140 54 mm 3-Grain White Longburn, and
3. Cesaroni K360 54 mm 3-Grain White.

The VMax motor has a total impulse of 1093 Ns and a burn duration of 0.7 s. The Longburn motor has a total impulse of 1211 Ns and a burn duration of 8.5 s, and the White motor has an impulse of 1266 Ns with a burn duration of 3.5 s [29].

The weather on the day of the test was colder than the previous test day. The canopy from the first experiment was still used because of a forecasted chance of rain; however, no rain occurred during testing.

The raw data collected by the added instrumentation during the first and second motor burns in the experiment is plotted in figures 4.14 - 4.18.

![Graph of Distance from IR sensor to Antenna, Vmax and Longburn Firings](image1)

![Graph of Accelerometer Data, Vmax and Longburn Firings](image2)

**Figure 4.14:** Collected instrumentation data - IR sensor distance, 3-axis accelerometer G-force for VMax and Longburn firings
The top graph of distance as measured by the IR sensor from itself to the antenna shows clearly when the antenna mount was intentionally moved. This was done before the VMax firing to simulate the effects of the rocket’s pressure on the apparatus, in order to determine the impact on attenuation and delay from antenna movements. The other movements correlate well between antenna movements as recorded by the IR distance sensor and the accelerometer readings. The VMax and Longburn firings occurred at 650 s and 3370 s respectively. Between the Longburn and White firings, the microcontroller recording the data reset and time indexes were reset. The White firing occurred in the next set of data at 825 s.

![Distance from IR sensor to Antenna, White Firing](image1)

![Accelerometer Data, White firing](image2)

**Figure 4.15:** Collected instrumentation data - IR sensor distance, 3-axis accelerometer G-force for White firing
Chapter 4. Results

Figure 4.16: Collected instrumentation data - ambient temperature and pressure data for VMax and Longburn firings

Figure 4.17: Collected instrumentation data - ambient temperature and pressure data for White firing
Chapter 4. Results

When aligned, the instrument data seem to correlate between motor firings. The data for temperature and pressure don’t exhibit strong correlation in the first motor burn, most likely due to its short fire duration. Unfortunately, the IR spot thermometer malfunctioned during the experiment and no data was gathered. While ambient temperature is measured adjacent to the antennas and not between, a small temperature increase can be observed in the data at the time of motor firings.

4.2.2.1 Direct-Path Measurements

The $S_{21}$ direct-path curves for the first firing with the VMax motor are as shown in figure 4.19.
Figure 4.19: VMax direct-path strength over the burn

In the attenuation plot for the VMax fire, a small, momentary fluctuation can be seen at the beginning of recording, believed to be the intentional antenna movement. The rocket fire happens at approximately 300 seconds into recording, and seems to have a much more pronounced drop.

Figure 4.20: VMax direct-path strength over the burn, detail

Curiously, patterns of decaying gain are not evident in the data gathered in the VMax firing, and the post-firing signal levels seem significantly lower than on the initial response. Upon reviewing video recordings of the experiment, the multipath reflector
is seen shifted slightly by the thrust of the VMax motor, which may account for the discrepancy.

The delay analysis plots for the VMax motor are as illustrated in figures 4.21 and 4.22.

![Figure 4.21: VMax direct-path delay over the burn](image1)

![Figure 4.22: VMax direct-path delay over the burn, detail](image2)

The same effect as with the attenuation plots can be observed in the delay - a very pronounced delay in the data, much more so than in previous firings. This is also
believed to be an artifact of the multipath environment. The collected signal data for the Longburn and White motors can be found in figures 4.23 - 4.30.

**Figure 4.23:** Longburn direct-path strength over the burn

**Figure 4.24:** Longburn direct-path strength over the burn, detail
Figure 4.25: Longburn direct-path delay over the burn

Figure 4.26: Longburn direct-path delay over the burn, detail
Chapter 4. \textit{Results}

Figure 4.27: White direct-path strength over the burn

Figure 4.28: White direct-path strength over the burn, detail
Figure 4.29: White direct-path delay over the burn

Figure 4.30: White direct-path delay over the burn, detail

The data for the Longburn and White firings exhibit an unclear source of oscillation due to the multipath distortions. A clear number for both attenuation and delay cannot be extracted out of the collected data for either of the firings in the direct-path. The $S_{21}$ measurements for the White plume exhibit similar behavior to Coutu’s firings, where a small gain was seen during the firings, giving plausibility to the theory of multipath beams affecting the collected data.
4.2.2.2 Reflector Measurements

The elliptical multipath reflector described in section 3.2.7 adds another peak to measure data from on each firing, through an adjacent portion of the plume. The collected attenuation and delay data are as plotted in figures 4.31 - 4.42.

**Figure 4.31:** VMax multipath reflector strength over the burn

**Figure 4.32:** VMax multipath reflector strength over the burn, detail
Figure 4.33: VMax multipath reflector delay over the burn

Figure 4.34: VMax multipath reflector delay over the burn, detail
Figure 4.35: Longburn multipath reflector strength over the burn

Figure 4.36: Longburn multipath reflector strength over the burn, detail
Figure 4.37: Longburn multipath reflector delay over the burn

Figure 4.38: Longburn multipath reflector delay over the burn, detail
Figure 4.39: White multipath reflector strength over the burn

Figure 4.40: White multipath reflector strength over the burn, detail
The measurements for the reflector are very similar to the direct-path, in appearance and in terms of noise. The ambiguity in measurements makes a concrete number for attenuation and delay not possible to extract.
4.3 Data Whitening

Because of the high amount of noise encountered in the second set of firings, and because of the lower spatial resolution imposed by the system’s low-pass response, a linear equalization factor was applied to the frequency-domain data of the burns to flatten the frequency response of (whiten) the data and increase temporal resolution [30]. Whitening the frequency response of the system reduces the overall impact of multipath on the response of the measurements, but it also changes the response of the plume over the firings.

![Whitening applied to frequency-domain data](image)

**Figure 4.43:** Whitening applied to frequency-domain data

A 75 dB linear whitening is applied to the VMax, Longburn, and White motors in section 4.3.1 in order to extract a delay and attenuation for each. To analyze the effects of whitening, a 55 dB whitening factor is also applied to the initial Classic and Skidmark burns in section 4.3.2. The whitening data was created by Dr. William C. Barott.

4.3.1 Motor Burns 3, 4, and 5

The VMax, Longburn, and White burns suffered significant noise in measurements as described in section 4.2.2. Whitening makes a correct reading of attenuation and delay
for these firings much easier to extract.

4.3.1.1 Direct-Path Measurements

The whitened data for the direct-path beam is plotted in figures 4.44 - 4.55.

**Figure 4.44:** Whitened VMax direct-path strength over the burn

**Figure 4.45:** Whitened VMax direct-path strength over the burn, detail
Figure 4.46: Whitened VMax direct-path delay over the burn

Figure 4.47: Whitened VMax direct-path delay over the burn, detail
Figure 4.48: Whitened Longburn direct-path strength over the burn

Figure 4.49: Whitened Longburn direct-path strength over the burn, detail
Figure 4.50: Whitened Longburn direct-path delay over the burn

Figure 4.51: Whitened Longburn direct-path delay over the burn, detail
Chapter 4. Results

Figure 4.52: Whitened White motor direct-path strength over the burn

Figure 4.53: Whitened White motor direct-path strength over the burn, detail
Chapter 4. *Results*

Figure 4.54: Whitened White motor direct-path delay over the burn

![Whitened White motor direct-path delay over the burn](image1)

Figure 4.55: Whitened White motor direct-path delay over the burn, detail

![Whitened White motor direct-path delay over the burn, detail](image2)

Extracted measurements for direct-path beam attenuation and delays are as listed in table 4.5.
### Table 4.5: Experimental results for whitened VMax, Longburn, and White direct-path beam measurements

<table>
<thead>
<tr>
<th>Whitening Method</th>
<th>Attenuation</th>
<th>Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whitened VMax</td>
<td>1.809 dB</td>
<td>11.65 ps</td>
</tr>
<tr>
<td>Whitened Longburn</td>
<td>0.262 dB</td>
<td>1.51 ps</td>
</tr>
<tr>
<td>Whitened White</td>
<td>0.295 dB</td>
<td>7.868 ps</td>
</tr>
</tbody>
</table>

Using these results, a set of solution curves created in the same fashion as in section 4.2.1 are created and plotted in figures 4.56 - 4.59.
Based on the solution curves, the values for $N_e$ and $v_e$ can be extracted as listed in table 4.6.

<table>
<thead>
<tr>
<th></th>
<th>$N_e$</th>
<th>$v_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VMax</td>
<td>$2.27 \times 10^{13} - 4.55 \times 10^{13}$</td>
<td>$3.75 \times 10^8$ Hz</td>
</tr>
<tr>
<td>Longburn</td>
<td>$8.53 \times 10^{12} - 1.25 \times 10^{13}$</td>
<td>$2 \times 10^8$ Hz</td>
</tr>
<tr>
<td>White</td>
<td>$1.32 \times 10^{13} - 1.59 \times 10^{13}$</td>
<td>$2.25 \times 10^8$ Hz</td>
</tr>
</tbody>
</table>

Table 4.6: $N_e$ solutions for whitened VMax, Longburn, and White direct-path measurements
The solutions seem to be correlated with the average thrust of the motors. While all three motors in the last solution set have similar impulses, the VMax has approximately 20 times the average thrust as the Longburn, while the White has about three times as much. This is reflected in the values for $N_e$ and $v_e$, where the VMax motor appears to have significantly higher values than the Longburn, and the White lies slightly higher than the latter.

![Figure 4.59: Plot of motor thrust against the value of $N_e$ extracted from the Whitened data](image)

The solutions are, however, inconsistent with the results from section 4.2.1. The average thrust of the White and Skidmark motors is approximately the same, and so it would be expected that both firings should be similar. Based on the two common average thrusts, it can be observed that the whitening process has approximately halved the measurements for $N_e$ and $v_e$. 

4.3.1.2 Reflector Measurements

The whitened data for the reflected beam is as plotted in figures 4.60 - 4.71.

**Figure 4.60**: Whitened VMax multipath reflector strength over the burn

**Figure 4.61**: Whitened VMax multipath reflector strength over the burn, detail
Figure 4.62: Whitened VMax multipath reflector delay over the burn

Figure 4.63: Whitened VMax multipath reflector delay over the burn, detail
Figure 4.64: Whitened Longburn multipath reflector strength over the burn

Figure 4.65: Whitened Longburn multipath reflector strength over the burn, detail
For the reflected beam, the whitening process does not appear to have removed enough noise from the longburn delay to claim a delay value for that firing.
Figure 4.68: Whitened White motor multipath reflector strength over the burn

Figure 4.69: Whitened White motor multipath reflector strength over the burn, detail
Figure 4.70: Whitened White motor multipath reflector delay over the burn

Figure 4.71: Whitened White motor multipath reflector delay over the burn, detail

Extracted measurements for direct-path beam attenuation and delays are as listed in table 4.7.
Table 4.7: Experimental results for Whitened VMax, Longburn, and White reflected beam measurements

<table>
<thead>
<tr>
<th></th>
<th>Attenuation</th>
<th>Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whitened VMax</td>
<td>2.265 dB</td>
<td>16.85 ps</td>
</tr>
<tr>
<td>Whitened Longburn</td>
<td>0.261 dB</td>
<td>-</td>
</tr>
<tr>
<td>Whitened White</td>
<td>0.688 dB</td>
<td>4.85 ps</td>
</tr>
</tbody>
</table>

With the reflected beam results, another of solution curves are created and plotted in figures 4.82 - 4.83. Since a delay value could not be obtained for the longburn firing, there is no solution possible.

Figure 4.72: Solution curves for VMax firing (reflected beam)
The extracted $N_e$ and $v_e$ values for the reflected beam measurements is as shown on table 4.8

<table>
<thead>
<tr>
<th></th>
<th>$N_e$</th>
<th>$v_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VMax</td>
<td>$3.85 \times 10^{13} - 4.21 \times 10^{13}$</td>
<td>$3.5 \times 10^8$ Hz</td>
</tr>
<tr>
<td>Longburn</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>White</td>
<td>$1.71 \times 10^{13} - 1.90 \times 10^{13}$</td>
<td>$2.3 \times 10^8$ Hz</td>
</tr>
</tbody>
</table>

Table 4.8: $N_e$ solutions for whitened reflected-beam measurements

For the VMax and White burns, there is an increase in $N_e$ in the reflected beam, implying that the region of highest density was further back on the plume from were the antennas were placed.

### 4.3.2 Motor Burns 1 and 2

In order to compare the effects of whitening with a known set of data, the previous Skidmark and Classic burns were also whitened in frequency-domain. The resulting new plots are as shown in figures 4.74 - 4.81.
Figure 4.74: Whitened Skidmark direct-path strength over the burn

Figure 4.75: Whitened Skidmark direct-path strength over the burn, detail
Figure 4.76: Whitened Skidmark direct-path delay over the burn

Figure 4.77: Whitened Skidmark direct-path delay over the burn, detail
Figure 4.78: Whitened Classic direct-path strength over the burn

Figure 4.79: Whitened Classic direct-path strength over the burn, detail
Based on the whitened plots, the new attenuation and delay values for the Classic and Skidmark burns are as listed in table 4.9.

<table>
<thead>
<tr>
<th></th>
<th>Attenuation</th>
<th>Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whitened Skidmark</td>
<td>0.36 dB</td>
<td>3.07 ps</td>
</tr>
<tr>
<td>Whitened Classic</td>
<td>0.76 dB</td>
<td>2.22 ps</td>
</tr>
</tbody>
</table>

Table 4.9: Experimental results for whitened Classic and Skidmark measurements
Chapter 4. Results

Figure 4.82: Solution curves for Skidmark firing (whitened)

Figure 4.83: Solution curves for Classic firing (whitened)
The new solutions for the Whitened Skidmark and Classic values are as listed on table 4.10

<table>
<thead>
<tr>
<th></th>
<th>$N_e$</th>
<th>$v_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whitened Skidmark</td>
<td>$1.11 \times 10^{13} - 1.25 \times 10^{13}$</td>
<td>$2 \times 10^8$</td>
</tr>
<tr>
<td>Whitened Classic</td>
<td>$1.80 \times 10^{13} - 1.96 \times 10^{13}$</td>
<td>$2.5 \times 10^8$</td>
</tr>
</tbody>
</table>

**Table 4.10: $N_e$ solutions for whitened Skidmark and Classic data**

The new solutions are consistent with the other whitened data in that they appear to be approximately halved from the non-whitened prediction. The new Skidmark values show a 110% difference, while the Classic shows a 120% difference in values.
Chapter 5

Research Conclusion

Given the commonality of linear equalization as a radar range-resolution sharpening technique, it is likely that the whitened values are more correct than the non-whitened. However, it would be beneficial to repeat the experiments multiple times without the reflector to show the effects of whitening without an increased multipath environment.

The solutions for $N_e$ and $v_e$ found in this study are similar to measured values in previous studies [11, 14, 17]. It is therefore plausible that $N_e$ and $v_e$ can be estimated accurately by finding the phase velocity $u_p$ of the plume through wide bandwidths and ranging. However, additional solutions using different methods based on the delay and attenuation of a radio wave passing through the plume would help further verify the results.

Given the patterns of non-restoring data, even with the thin horizontal beamwidth of the Vivaldi arrays, it is possible that focusing was not the sole cause of the effects in Coutu’s thesis, although focusing effects could be seen at the measured plasma parameters in simulations. Temperature seems to affect the overall response of the system, and how it changes after the rocket fire isn’t well known. However, some focusing could be observed in the FDTD model after experimental values were used in the model.

The analysis suggests a much lower value for $v_e$ than in the CFD models. This is consistent with Van der Beek’s paper’s conclusions [9] and is in the same order of magnitude as Kinefuchi’s model [11].
Chapter 6

Topics for Further Research

The effect of air and system temperatures should be a focus of future studies of similar scales. A more sophisticated set of instruments could theoretically correlate the increase in gain with an air temperature between the antennas as opposed to adjacent to them.

The addition of other peaks by intentionally introducing multipath has appeared to make the data more noisy, but could be filtered to produce more precise results. This may be an alternative to moving platforms [14] in the future to test multiple parts of the plume, without requiring more antennas and RF equipment. Using ranging presents multiple challenges in analyzing data and adding multiple beams seems to compound the challenges; therefore, whatever properties that can be exploited from radar techniques should be utilized. It could prove helpful to send pseudo-random digital data through the plume in order to utilize correlation and eye-diagram techniques for optimizing filters and reducing Inter-Symbol Interference. Improving the RF section of the apparatus to support the full range of the Wilkinson transformers and antennas would increase the precision of the apparatus.

Finding more solutions for $N_e$ and $v_e$ as related to phase velocity and attenuation seem like the most critical factor in improving the quality of the experimental model. Smoot’s model [17] is known to be a very rough estimate for attenuation, and the method for using the index of refraction in this study has, to the author’s knowledge, not been previously used.
### Bill of Materials

<table>
<thead>
<tr>
<th>Description</th>
<th>Manufacturer 1</th>
<th>Model Number 1</th>
<th>Quantity</th>
<th>Part of Coutu's Thesis?</th>
<th>Replacement Cost</th>
<th>Cost to Thesis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RF Equipment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vivaldi Antenna Arrays</td>
<td>-</td>
<td>-</td>
<td>2</td>
<td>✓</td>
<td>$64.47</td>
<td>-</td>
</tr>
<tr>
<td>SMA to SMA PE-P105 Coax, 600 in.</td>
<td>Pasternack</td>
<td>PE3138</td>
<td>2</td>
<td>✓</td>
<td>$88.64</td>
<td>-</td>
</tr>
<tr>
<td>SMA to SMA RG405 Coax, 24 in</td>
<td>Pasternack</td>
<td>PE3831-24</td>
<td>8</td>
<td>✓</td>
<td>$1,395.00</td>
<td>-</td>
</tr>
<tr>
<td>Super Ultra Wideband Amplifier, 0.1-18 GHz</td>
<td>Mini-Circuits</td>
<td>ZVA-183W+</td>
<td>1</td>
<td>✓</td>
<td>$399.60</td>
<td>-</td>
</tr>
<tr>
<td>Connectorized Amplifier</td>
<td>Mini-Circuits</td>
<td>ZX60-8008E+</td>
<td>8</td>
<td>✓</td>
<td>$109.90</td>
<td>$109.90</td>
</tr>
<tr>
<td>Mixers</td>
<td>Mini-Circuits</td>
<td>ZX05-24MH+</td>
<td>2</td>
<td></td>
<td>$24.95</td>
<td>$24.95</td>
</tr>
<tr>
<td>Low-Pass Filter</td>
<td>Mini-Circuits</td>
<td>VLF-7200+</td>
<td>1</td>
<td></td>
<td>$43.90</td>
<td>$43.90</td>
</tr>
<tr>
<td>High-Pass Filters</td>
<td>Mini-Circuits</td>
<td>VHF-8400+</td>
<td>2</td>
<td></td>
<td>$39.95</td>
<td>$39.95</td>
</tr>
<tr>
<td>Power Divider</td>
<td>Mini-Circuits</td>
<td>ZX10-2-126-S+</td>
<td>1</td>
<td></td>
<td>$24.95</td>
<td>$24.95</td>
</tr>
<tr>
<td>Local Oscillator</td>
<td>Polarad</td>
<td>1108E</td>
<td>1</td>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Programmable DC Power Supplies</td>
<td>Tektronix</td>
<td>PS2510G</td>
<td>3</td>
<td></td>
<td>$699.00</td>
<td>-</td>
</tr>
<tr>
<td>Vector Network Analyzer</td>
<td>Agilent</td>
<td>E5071B</td>
<td>1</td>
<td>✓</td>
<td>$50,000.00</td>
<td>-</td>
</tr>
<tr>
<td><strong>Structure</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slotted-Aluminum Frames</td>
<td>80/20 Inc.</td>
<td>1515</td>
<td>1</td>
<td>✓</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Steel Test Stand</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>✓</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Elliptical Reflector</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td></td>
<td>-</td>
<td>$12.95</td>
</tr>
<tr>
<td><strong>Instrumentation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laptop with GNU/Linux OS</td>
<td>Asus</td>
<td>EEE1000HD</td>
<td>1</td>
<td></td>
<td>$300</td>
<td>-</td>
</tr>
<tr>
<td>Data-collection microcontroller</td>
<td>Arduino</td>
<td>Uno</td>
<td>2</td>
<td></td>
<td>$9.95</td>
<td>$9.95</td>
</tr>
<tr>
<td>3-Axis Accelerometer</td>
<td>Sparkfun</td>
<td>MMA8452</td>
<td>1</td>
<td></td>
<td>$12.95</td>
<td>$12.95</td>
</tr>
<tr>
<td>Ambient temperature and pressure sensor</td>
<td>Sparkfun</td>
<td>MPL115A1</td>
<td>1</td>
<td></td>
<td>$13.95</td>
<td>$13.95</td>
</tr>
<tr>
<td>Infrared Proximity Sensor</td>
<td>SHARP</td>
<td>GP2Y0A21YK</td>
<td>1</td>
<td></td>
<td>$59.80</td>
<td>$59.80</td>
</tr>
<tr>
<td>Type-K glass braid thermocouple</td>
<td>Adafruit</td>
<td>269</td>
<td>4</td>
<td></td>
<td>$39.80</td>
<td>$39.80</td>
</tr>
<tr>
<td><strong>Rocket Motors</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cesaroni Pro-54 3-Grain Casing</td>
<td>Cesaroni</td>
<td>P54-3G</td>
<td>1</td>
<td>✓</td>
<td>$69.39</td>
<td>-</td>
</tr>
<tr>
<td>Cesaroni Pro-54 Delay/Eject Closure Adapter</td>
<td>Aero Pack</td>
<td>MC54</td>
<td>1</td>
<td>✓</td>
<td>$14.00</td>
<td>-</td>
</tr>
<tr>
<td>Cesaroni J295 Classic Reload Kit</td>
<td>Cesaroni</td>
<td>1195J295-15A</td>
<td>1</td>
<td></td>
<td>$91.50</td>
<td>$91.50</td>
</tr>
<tr>
<td>Cesaroni J360 Skidmark Reload Kit</td>
<td>Cesaroni</td>
<td>1016J360-16A</td>
<td>1</td>
<td></td>
<td>$99.46</td>
<td>$99.46</td>
</tr>
<tr>
<td>Cesaroni J360 White Reload Kit</td>
<td>Cesaroni</td>
<td>1281J360-13A</td>
<td>1</td>
<td></td>
<td>$91.50</td>
<td>$91.50</td>
</tr>
<tr>
<td>Cesaroni J1520 VMax Reload Kit</td>
<td>Cesaroni</td>
<td>1093J1520-17A</td>
<td>1</td>
<td></td>
<td>$91.50</td>
<td>$91.50</td>
</tr>
</tbody>
</table>

Replacement Total (excl. VNA) $3790.66  
Cost Total $743.51
Appendix B

Source Code

B.1 Data collection scripts

The data collection scripts are responsible for communicating with the VNA to store information, and ensuring that live data is being saved. A tcl expect script spawns a telnet session, and specifying a folder on the VNA’s D drive and its network address will allow telnet to trigger the machine and save files to that folder. A Python script can display time or frequency information for those stored files, assuming the VNA’s folder is mounted locally on the filesystem.

B.1.1 Expect: vna-trig-save-local

```sh
#!/usr/bin/expect
#
# vna-trig-save-local
#
# This script communicates with the VNA to save S2P files to its local memory.
#
# folders are specified on the VNA’s D drive.
# its proper usage should be:
#
# ./vna-trig-save-local IPADDRESS LOCALFOLDER
#
# Needless to say, you will need to install expect script support to be able to run this script.

set host [lindex $argv 0]
set folder [lindex $argv 1]
set prompt "SCPI>"
```
# open a telnet session to talk with the VNA
spawn telnet $host
expect "\]'."
send "\r\n"
sleep 3
send "\r"

# wait for the VNA to respond
expect "$prompt"
send ":TRIG:SCOP ACT\r"
sleep 1
expect "$prompt"
send ":TRIG:SOUR BUS\r"
sleep 1
expect "$prompt"
send ":DISP:ENAB OFF\r"
sleep 1

while {1 > 0} {

# save each file with the current unix time in microseconds
# as its filename.
set date [clock clicks -microseconds]
expect "$prompt"
send ":TRIG:SING\r"
expect "$prompt"

# ask the VNA to tell us when it's ready to save
send "*OPC?\r"
expect "+1"
expect "$prompt"

# send the save command
send ":MMEM:STOR:SNP "D:\${folder}\${date}.S2P"
expect "$prompt"

# ask the VNA to update the screen
send ":DISP:UPD\r"
}

B.1.2 Python: view-data-rt-network.py

```python
#!/usr/bin/python
#
# view-data-rt-network.py
#
# This script plays back an animated realtime plot of the VNA
# information as S2P files are saved to its local memory.
# For this to work, you must have the folder the VNA is saving to
# mounted locally and specified in the dataFolder string

import numpy as np
import matplotlib.pyplot as plt
import pylab

import skrf as rf

# change this to the mount point on the local file system
# of the VNA save directory
dataFolder='../mountVNA'

# this technically reads all the files in a folder, so the
# longer it goes on, the slower it gets.
# works fine for a few thousand files though.
readstuff = rf.read_all(dataFolder, obj_type = 'Network')

networks = rf.NetworkSet(readstuff)

networks.animate('s_db', ylims=(-50,25), xlims=(300000,4500000000),
show=True, savefigs=False, label=None)
```
B.2 Data analysis scripts

The following code was used during the study to analyze certain properties of the gathered S2P (touchstone) files from the VNA. Most of the files are in python using the scikit-rf library for extracting information from the touchstone files, with the exception of the time-delay information, which was produced by Dr. William C. Barott in MATLAB.

B.2.1 Python: analyze-wholefolder1plot.py

```python
#!/usr/bin/python

# This file is a simple, few line script to see all of the S2P frequency information in a folder smashed together in 1 plot

import numpy as np
import matplotlib.pyplot as plt
import pylab
import skrf as rf

# change the path to whatever folder contains the data
dataFolder = '../tests_2014_5_5/test1'

# This puts all of the S2P files in one object
readstuff = rf.read_all(dataFolder, obj_type = 'Network')

networks = rf.NetworkSet(readstuff)

# and this pline creates the plot.
networks.plot_s_mag()

# without the line below, the figure won’t show
pylab.show()
```
B.2.2 Python: analyze-view1FileTimeDomain.py

```python
#!/usr/bin/python

# analyze_view1FileTimeDomain.py
#
# This file is a short script that plots frequency and time
# information for a single S2P file.

import skrf as rf
import matplotlib.pyplot as plt
import pylab

# create a new figure
figure1 = plt.figure()

# open a specified file and create a network object
# - Note - Change the path here to whatever file you’d like
# to see time and frequency information of
network = rf.Network('../tests_2014_9_20/Test_1_Skidmark/1.S2P')

# This plots the S matrix in dB. m=1, n=1 corresponds to S21.
network.plot_s_db(m=1, n=0, show_legend=True)

# open another figure.
figure2 = plt.figure()

# This plots S21’s time data.
network.plot_s_db_time(m=1, n=0, show_legend=True)

# without the line below, the figure won’t show
pylab.show()
```
B.2.3 Python: analyze-savecsv.py

```python
#!/usr/bin/python

#analyze_savecsv.py
#
# This file goes through a folder containing S2P files and
# exports them as CSV’s for that pesky Dr. Barott.
#
import skrf as rf
import os

# This variable points to the path of the folder
# that we want to look at.
directory = '../tests_2014_9_20/test_2_classic/'

#create a list of the files in the directory
fileList = os.listdir(directory)

#store the number of files
numFiles = len(fileList)
count = 0

#Iterate through the files in the directory
for filename in sorted(fileList):
    #only consider touchstone files
    # (exclude desktop.ini and other stuff)
    if filename.endswith('.S2P'):
        count = count + 1

        # this is so we can keep track of how far we are
        print 'processing ' + str(count) + ' out of ' + '\
        str(numFiles) + ', ' + \
        str( round(float(count)/float(numFiles)\
        * 100,2 ) ) + '%'

        #create a scikit-rf network object
        aNetwork = rf.Network(directory + filename)

        #uncomment to write the touchstone again somewhere
        #aNetwork.write_touchstone(\
        #filename=filename,\n        #dir='csv-skd',\n        #write_z0=False,\n        #skrf_comment=False)

        #string with the file’s name.
        filecsv = 'csv-csc/' + filename[:-3] + 'csv'

        #use skrf to write the file.
        aNetwork.write_spreadsheet(\
        file_name=filecsv, file_type='csv',\n        form='ri')
```

B.2.4 Python: analyze-findTDpeak.py

```python
#!/usr/bin/python

#analyze_findTDpeak.py
#
# This file reads in a folder of S2P files, sorts them
# by filename (assumes the filename corresponds to a
# unix timestamp in us), and plots the maximum value
# of the time-domain information of S21 over
# each file (frame).

import skrf as rf
import matplotlib.pyplot as plt
import matplotlib.gridspec as gridspec  # subplots
import pylab
import numpy as np
import plotly
from matplotlib import cm, colors
import os
import plotly.plotly as py
import plotly.tools as tls
from plotly.graph_objs import *

# This is optional, but will export a plot my plotly account,
# which gives me greater flexibility in manipulating plots
py.sign_in("torresja4", "8i1gb0db6t")

# This variable points to the path of the folder we want to
# look at.
directory = '../tests_2014_9_20/Test_1_Skidmark'

#define a frequency range
startFrequency = 300e3
stopFrequency = 4.5e9
numpoints = 400

#calculate BW
bw = abs(stopFrequency - startFrequency)

#and time resolution
t_res = 1/bw

#create a list of the files in the directory
fileList = os.listdir(directory)

#store the number of files
numFiles = len(fileList)

#create some empty lists
peaks_dB = []
fastTime = []
```
slowTime = []
firstTime = 0

# Iterate through the files in the directory
for filename in sorted(fileList):
    # only consider touchstone files
    # (exclude desktop.ini and other stuff)
    if filename.endswith('.S2P '):
        # Since we're using the time difference from the
        # times stamped in the filename, the first file
        # in the folder corresponds to the start time.
        if firstTime == 0:
            firstTime = int(filename[: -4])
        # This corresponds to the difference between the
        # first file and the current.
        timeDiff = (int(filename[: -4]) - firstTime)

        print filename
        # Create a scikit-rf object using the current file
        # in the directory
        aNetwork = rf.Network(directory + filename)
        # copy its s-matrix in dB
        s_time_info_db = aNetwork.s_time_db
        # find the maximum value
        i,j,k = np.unravel_index(s_time_info_db.argmax(),
                                s_time_info_db.shape)
        # append the max to our list of maxes
        peaks_dB.append(s_time_info_db[i,j,k])
        # also append its time index to our list of fast times
        fastTime.append(i/2 * (t_res)/numpoints*100 * 1e9)

        # uncomment this line to see indices
        # instead of time values
        # fastTime.append(i)

        # also append the file's timestamp (in its filename) to
        # our list of slow times
        slowTime.append( timeDiff / 1e6 )

# figure 1 shows a 2x1 subplot, 1 with the index vs dB info of
# all files, and the other with a sample file's time-domain data
figure1 = plt.figure()
plt.subplot(2,1,1)
plt.plot(fastTime,peaks_dB)
plt.autoscale()
plt.xlabel('Time index (ns)')
plt.ylabel('peaks_dB, dB')
plt.title('peak value time-delay in gathered data')

plt.subplot(2,1,2)
aNetwork.plot_s_db_time(m=1,n=0)
plt.title('Sample file time domain information')

# Figure 2 shows the peak-data over time as gathered
# from the filename.
figure2 = plt.figure()
plt.plot(slowTime, peaks_dB)
plt.autoscale()
plt.xlabel('time in seconds')
plt.ylabel('peak value, dB')
plt.title('strength of peak value across gathered data
(skidmark)')

# This is the code that actually uploads figures to plotly
# if you want this to happen, uncomment the lines.
# If they're uploaded, pylab doesn't plot them however.
#py.iplot_mpl(figure1)
#py.iplot_mpl(figure2)
pylab.show()
B.2.5 Python: analyze-findTDpeak-plusminus.py

```python
#!/usr/bin/python
#analyze_findTDpeak_plusminus.py
# This file reads in a folder of S2P files, sorts them by filename (assumes the filename corresponds to a unix timestamp in us), and plots the maximum value AND a set offset index plus and minus from it of the time-domain information of S21 over each file (frame).

import skrf as rf
import matplotlib.pyplot as plt
import pylab
import numpy as np
import os

# This variable points to the path of the folder we want to look at.
directory = '../tests_2014_9_20/Test_1_Skidmark'

# Defines the offsets to look ahead of and before the maximum
# Each offset corresponds to 0.22 nS (our time resolution)
offset = 5

# define a frequency range
startFrequency = 300e3
stopFrequency = 4.5e9
numpoints = 400

# calculate BW
bw = abs(stopFrequency - startFrequency)

# and time resolution
t_res = 1/bw

# create a list of the files in the directory
fileList = os.listdir(directory)

# store the number of files
numFiles = len(fileList)

# create some empty lists
peaks_dB1 = []
fastTime1 = []

peaks_dBplus = []
fastTime2 = []

peaks_dBminus = []
fastTime3 = []
slowTime = []
```
```
firstTime = 0

# Iterate through the files in the directory
for filename in sorted(fileList):
    
    # only consider touchstone files
    #(exclude desktop.ini and other stuff)
    if filename.endswith('.S2P'):
        
        # Since we're using the time difference from the times
        # stamped in the filename, the first file in the folder
        # corresponds to the start time.
        if firstTime == 0:
            firstTime = int(filename[:-4])

        # This corresponds to the difference between the
        # first file and the current.
        timeDiff = (int(filename[:-4]) - firstTime)

        print filename

        # Create a scikit-rf object using the current file in
        # the directory
        aNetwork = rf.Network(directory + filename)

        # copy its s-matrix in dB
        s_time_info_db = aNetwork.s_time_db

        # find the maximum value
        i,j,k = np.unravel_index(s_time_info_db.argmax(),
                                 s_time_info_db.shape)

        # append the max to our list of maxes
        peaks_dB1.append(s_time_info_db[i,j,k])

        # also append its time index to our list of fast times
        fastTime1.append(i/2 * (t_res)/numpoints*100 * 1e9)

        # append max + offset to our list of max + offsets
        peaks_dBplus.append(s_time_info_db[i+offset,j,k])
        # and its corresponding index to index + offsets
        fastTime2.append(((i+offset)/2 * (t_res)/numpoints*100 * 1e9)

        # same as previous, but with a minus offset
        peaks_dBminus.append(s_time_info_db[i-offset,j,k])
        fastTime3.append((i-offset)/2 * (t_res)/numpoints*100 * 1e9)

        # also append the file's timestamp (in its filename) to
        # our list of slow times
        slowTime.append( timeDiff / 1e6 )

# figure 1 shows a 2x2 subplot -
```
Appendix B. Source Code

```python
# 1 has the index vs dB info of all files,
# 2 has a sample file’s time-domain data
# 3 is index vs dB of the offset+ data
# 4 is index vs dB of the offset- data

figure1 = plt.figure()
plt.title('classic burn data uncertainty')
plt.subplot(2,2,1)
plt.plot(fastTime1, peaks_dB1)
plt.autoscale()
plt.xlabel('Time index (ns)')
plt.ylabel('peaks_dB, dB')
plt.title('peak value time-delay in 
gathered data (classic)')

plt.subplot(2,2,2)
aNetwork.plot_s_db_time(m=1,n=0)
plt.title('Sample file time domain information (classic)')

plt.subplot(2,2,3)
plt.plot(fastTime2, peaks_dBplus)
plt.autoscale()
plt.xlabel('Time index (ns)')
plt.ylabel('peaks_dB, dB')
plt.title('+1.1 nS offset of peak value time-delay in 
gathered data (classic)')

plt.subplot(2,2,4)
plt.plot(fastTime3, peaks_dBminus)
plt.autoscale()
plt.xlabel('Time index (ns)')
plt.ylabel('peaks_dB, dB')
plt.title('-1.1 nS offset of peak value time-delay in 
gathered data (classic)')

# figure 2 also shows a 2x2 subplot -
# 1 has the peak value over slow time (as put in the filename)
# 2 has a sample file’s time-domain info
# 3 is slowtime vs dB of the offset+ data
# 4 is slowtime vs dB of the offset- data

figure2 = plt.figure()
plt.title('classic burn data')
plt.subplot(2,2,1)
plt.plot(slowTime, peaks_dB1)
plt.autoscale()
plt.xlabel('time in seconds')
plt.ylabel('peak value, dB')
plt.title('strength of peak value across 
gathered data (classic)')

plt.subplot(2,2,2)
aNetwork.plot_s_db_time(m=1,n=0)
plt.title('Sample file time domain information (classic)')

plt.subplot(2,2,3)
plt.plot(slowTime, peaks_dBplus)
plt.autoscale()
```
plt.xlabel('time in seconds')
plt.ylabel('peak value, dB')
plt.title('+1.1nS offset of peak value across gathered data (classic)')
plt.subplot(2,2,4)
plt.plot(slowTime, peaks_dBminus)
plt.autoscale()
plt.xlabel('time in seconds')
plt.ylabel('peak value, dB')
plt.title('-1.1nS offset of peak value across gathered data (classic)')

#Need this line for the plots to actually appear.
pylab.show()
B.2.6 Python: analyze-datain.py

```python
#!/usr/bin/python

#analyze_datain.py
#
# This file iterates through our range of frequencies, and uses
# the given solutions to plot values of Ne vs Ve and w

import skrf as rf
import matplotlib.pyplot as plt
import pylab
import numpy as np
import math
import os

import plotly.plotly as py
import plotly.tools as tls
from plotly.graph_objs import *

# This is optional, but will export a plot my plotly account,
# which gives me greater flexibility in manipulating plots
py.sign_in("torresja4", "8i1gb0db6t")

# define a frequency range
startFrequency = 300000
stopFrequency = 4500000000
numpoints = 400

fspan = stopFrequency - startFrequency

# note to self, this is why we usually use points ending in xx1,
# i.e. 401 vs 400
fstep = fspan/(numpoints - 1)

# bandwidth
bw = abs(stopFrequency - startFrequency)

t_res = 1/bw

# Enter measured parameters here:

# Experimentally derived - delays for classic
delaymin_csc = 58.92e-12
delaymax_csc = 75.23e-12
delay_csc = delaymax_csc - delaymin_csc

# Experimentally derived - delays for skidmark
delaymin_skd = 55.10e-12
delaymax_skd = 63.34e-12
```
```python
delay_skd = delaymax_skd - delaymin_skd

# Experimentally derived - signal levels at drop for classic
attenb4_csc = 5.3661
attenaft_csc = 4.9346
atten_csc = attenb4_csc - attenaft_csc

# Experimentally derived - signal levels at drop for skidmark
attenb4_skd = 5.3925
attenaft_skd = 5.165
atten_skd = attenb4_skd - attenaft_skd

# y represents the plume diameter
y = 0.10

# physical constants
e0 = 8.85418782e-12
qe = 1.60217657e-19
me = 9.10938291e-31

# speed of light in air has a refractive index
# of 1.0003 approximately

\[ c = \frac{299792458}{1.0003} \]

# generic frequency object to create networks if we need it later
frequency_v = rf.Frequency(startFrequency, stopFrequency, numpoints, 'hz')

# initializing solution vectors
# We analytically came up with four solutions for each variable, and we need one for each burn
Nec = np.zeros((400,16), dtype=np.complex128)
Nes = np.zeros((400,16), dtype=np.complex128)
ves=[5e7,1e8,2.5e8,5e8,7.5e8,1e9,5e9,1e10]

m = 0
n = 0
freqsteps = frequency_v.f

# Iterate through the files in the directory
for freq in freqsteps:
    # These were the obtained solutions and they are not pretty.
w = (freq + 8.5e9) * 2 * math.pi
wf = w # oops
m = 0

    for ve in ves:
```

Appendix B. Source Code

106
# ugly equations corresponding to the solutions for
# finding Ne go here

\[
\text{Nec}[n,m] = \text{np.abs}((1j*(-e0 * me * ve**2 * w - \\
20 * c * delay_csc * e0 * me * ve**2 * w \\
- 100 * c**2 * delay_csc**2 * e0 * me * \\
ve**2 * w + e0**2 * me * ve**2 * w)) \\
/(q*e**2 * (e0 * ve + 1j * w + 20 * 1j \\
* c * delay_csc * w + 100 * 1j * c**2 * \\
delay_csc**2 * w)))
\]

\[
\text{Nec}[n,m+8] = \text{np.abs}((21.7391 * atten_csc * \\
(ve**2 + w**2))/ve)
\]

\[
\text{Nes}[n,m] = \text{np.abs}((1j*(-e0 * me * ve**2 * w - \\
20 * c * delay_skd * e0 * me * ve**2 * w \\
- 100 * c**2 * delay_skd**2 * e0 * me * \\
ve**2 * w + e0**2 * me * ve**2 * w)) \\
/(q*e**2 * (e0 * ve + 1j * w + 20 * 1j \\
* c * delay_skd * w + 100 * 1j * c**2 * \\
delay_skd**2 * w)))
\]

\[
\text{Nes}[n,m+8] = \text{np.abs}((21.7391 * atten_skd * \\
(ve**2 + w**2))/ve)
\]

\[
m = m + 1
\]

\[
n = n + 1
\]

\[
\text{print str(n) + " out of 400"}
\]

\[
\text{print str(wf) + " through " + str(stopFrequency)}
\]

# plot this funky business.

\[
m = 0
\]

\[
\text{classicNeSolutions = plt.figure()}
\]

#plot each line in a single figure

\[
\text{for ve in ves:}
\]

\[
\text{plt.plot(freqsteps,Ne[::,m],label="Ve = " + str(ve))}
\]

\[
\text{plt.plot(freqsteps,Ne[::,m+8],label="Ve = " + str(ve),\}
\]

\[
\text{ls='dashed')}
\]

\[
\text{m = m + 1}
\]

\[
\text{plt.autoscale()}
\]

\[
\text{plt.xlabel('frequency, Hz')}
\]

\[
\text{plt.ylabel('value for Ne, number/cm^3')}\]

\[
\text{plt.title('Ne solutions on iterated w using two models, classic')}
\]

#legend()

\[
m = 0
\]

\[
\text{skidmarkNeSolutions = plt.figure()}
\]

\[
\text{for ve in ves:}
\]

\[
\text{plt.plot(freqsteps,Ne[::,m],label="Ve = " + str(ve))}
\]
Appendix B. Source Code

plt.plot(freqsteps,Nes[:,m+8],label="Ve = " + str(ve),
          ls='dashed')
m = m + 1
plt.autoscale()
plt.xlabel('frequency')
plt.ylabel('value for Ne, number/cm^3')
plt.title('Ne solutions on iterated w using two models, skidmark')

# This is the code that actually uploads figures to plotly
# If you want this to happen, uncomment the lines.
# If they're uploaded, pylab doesn't plot them however.
# py.iplot_mpl(classicNeSolutions)
# py.iplot_mpl(skidmarkNeSolutions)

# plots don't show up if you don't have this line,
# unless you're in interactive mode.
pylab.show()
B.2.7 MATLAB: jorge-reader.m

```matlab
% Matlab code reader for the rocket CSV data from Jorge
% Created WCB 2014/10/23

function jorge_reader

% set the directory for the rocket data
directory = 'E:\users\jorge\Desktop\Google Drive\Thesis work\laptop_sep20\Thesis\code\commasv-csc'

% create a list of all of the file names
fns = dir(directory);

% Loop through each file
for n = 1:length(fns)
    % Call csvread with starting on row 1 and col 0 to kill the header data
dd = csvread(strcat(char(directory), '\', char(fns(3).name)), 1, 0);
    % Call the single reader and return the result in r. The single reader
    % will implement all math.
    if n == 1
        % On the first loop, don’t force a "firstpeak". Let the reader
        % identify the first peak bin itself.
        r(n) = jorge_single_reader(dd);
    else
        % on subsequent loops, force the firstpeak to be the bin identified
        % on the first time through.
        r(n) = jorge_single_reader(dd, r(1).firstPeak);
    end

disp(n)
end

disp('done')

figure
plot(20* log10([r. peakValue]))
xlabel('Frame number')
ylabel('power, dB-Arb')
title('Peak value extracted from the plume after correction')

figure
plot([r. peakDelay])
xlabel('Frame number')
ylabel('Delay, seconds')
title('Delay required for peak value')

function r = jorge_single_reader (dd, starter)

% Parse the complex value
```
Appendix B. Source Code

```matlab
flist = dd(:,1);
dreal = dd(:,6);
dimag = dd(:,7);
d = dreal + j*dimag;

% Create a list of frequencies, a bin separation time, and a list of time
flist(end) - flist(1);
tcoh = 1/(flist(end) - flist(1));
tlist = ((1:size(dd,1)) - (size(dd,1)/2))*tcoh;

% if the starter peak wasn’t specified (1 argument), then find it. If it
% was specified, use it.
if nargin == 1
    tseries = fft(d);
    [a,b] = max(abs(tseries)); % B is the index
else
    b = starter;
end

% Create a list of delays to try. Go +/- 1 sample.
dlist = linspace(-tcoh, tcoh, 10000);
for n = 1:length(dlist)
    cdelay = dlist(n); % Pull the current one
    % Use the current one to make an exponential response, but I screwed up
    % and pulled it again (no worries, just didn’t use “cdelay” it’s late
    dspec = exp(-j*2*pi*flist*dlist(n));
    % store the results in a matrix
    fsmat(:,n) = d.*dspec;
end

% create an FFT of the prev matrix to get slow time vs fast time
tsmat = fft(fsmat);
% convert to magnitude but also extract the row corresponding to the “peak”
% value, so that we’re only looking at that row
pkvdel = abs(tsmat(b,:));
% find the delay and peak value corresponding to the delay that causes the
% peak row (eg starter) to exhibit a maxima over all time delays that we
% tried
[a2,b2] = max(pkvdel);

% store and report
r.peakValue = a2;
r.peakDelay = dlist(b2);
r.firstPeak = b;
```
References


