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Jaime Aguilar Guerrero

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INVESTIGATION OF MOUNTAIN WAVES IN THE MESOSPHERE OVER THE ANDES MOUNTAINS

by

JAIME AGUILAR GUERRERO

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INVESTIGATION OF MOUNTAIN WAVES IN THE
MESOSPHERE OVER THE ANDES MOUNTAINS

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This thesis was prepared under the direction of the candidate’s Thesis Committee Chair, Dr. Alan Z. Liu, and has been approved by the Thesis Committee. It was submitted to the College of Arts and Sciences in partial fulfillment of the requirements of the degree of

MASTER OF SCIENCE IN ENGINEERING PHYSICS

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Abstract

An image processing algorithm has been developed to analyze data from a NIR All-Sky Imager of OH airglow emission (from about 87 km altitude), located in the Andes, with the purpose of investigating the atmospheric gravity waves generated when low level wind blows over the high mountains (referred to as Mountain Waves). These types of waves are a special case of atmospheric gravity waves, which carry significant momentum and exert strong forcing to the background upper atmosphere. The imager is located at the Andes Lidar Observatory (ALO) at Cerro Pachón, Chile (30°S, 71°W), which also houses a Na Doppler Lidar and other passive optical instruments. This thesis work reports the successful identification of numerous strong mountain wave events during the austral winters (2010 – 2014) when low level horizontal winds are eastward and exceed 40 m/s. These events span hundreds of kilometers in the night sky with average wavelengths of 20 to 40 km. A database of high resolution videos and statistics on the wave parameters has been produced. Additionally, an airglow imager simulation model has been built for testing the image processing algorithm.
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List of Abbreviations and symbols

AGW - Atmospheric Gravity Wave
ASI - All-Sky Imager
FOV - Field of View
IPA - Imager Processing Algorithm
\( \varepsilon \) - Volume Emission Rate
IVER - Integrated Volume Emission Rate
FT - Fourier Transform
FFT - Fast Fourier Transform algorithm
GCM - General Circulation Model
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Chapter 1

Introduction: Gravity Waves, Mountain Waves and their Importance

Atmospheric gravity waves, sometimes referred to as internal gravity waves, are a dynamic phenomena that constitutes an essential part of Earth’s atmosphere. The density of air decreases smoothly with altitude and our atmosphere is \textit{stably stratified}. Such fluids support wave motions. In fact, these waves are present all the time, and when they interact with the atmosphere we are able to see them. They are in essence a trait of the atmosphere and they come in many varieties. Some times they are visible when the perturbation is enough to create clouds in forms of waves, such as the ones shown in Figure 1.1. Other times they manifest in strong temperature variations that can be measured in situ or by remote sensing. At higher altitudes, the waves are strong enough to perturb the chemical compositions of Earth’s atmosphere and create an enhancement to the natural occurring airglow. Sometimes the observed waves are linear and monochromatic, but oftentimes nonlinearity, turbulence and instabilities contribute to the global circulation of energy and constituents. All of these interactions provide capabilities for probing into different layers of the atmosphere in an effort to better understand the constitution and dynamics of it.
While the existence of these waves has been known for a long time, it wasn’t until the work of (Hines 1960) that we started gaining a deeper understanding of the physics and theory that describes these phenomena. A recollection of the early days of gravity waves was written by (Hines 1989) and provides a great historical insight on the matter. He was also responsible for laying the groundwork of what is known now as the *linear wave theory*, that has been very successful in describing many of the waves we can observe and measure.

**Figure 1.1.** Wave structures over the College of Arts & Sciences, Embry-Riddle University, Daytona Beach, FL.

Waves that are supported by a stably stratified atmosphere are called gravity waves. However, this name fails to describe that the restoring buoyancy force and gravity work in combination to enable these waves. These can be small-scale waves with periods of several minutes and short horizontal wavelengths of several tens of kilometers. They can also be very large with periods of many hours and horizontal wavelengths of thousands of kilometers. If the atmosphere is compressible, acoustic waves can also appear. These are very fast propagating waves that can travel all the way from the
troposphere up to the thermosphere and interact with the ionosphere (Zettergren and Snively 2013). We can also find propagating gravity waves, evanescent waves or stationary ones, all of which carry momentum and energy. The understanding of all the processes that make this energy transfer has been a major scientific pursuit ever since the groundbreaking work by Hines. Here is an outline of some of the major features of atmospheric gravity waves (AGWs) that make them so important for scientific pursuit:

1. AGWs are ubiquitous and are one of the major players in the dynamics of atmosphere. They affect circulation, overall structure (thermal, densities etc.) and seasonal variability (Nappo 2002, Ern, Preusse et al. 2011).
2. Large-scale models greatly benefit from better understanding of AGWs, even if they are small-scale phenomena. Nowadays AGWs in GCMs are generally described by parameterization based on our current understandings (McLandress 2014).
3. AGWs influence extends from the lower atmosphere to the middle and upper atmosphere.
4. Since AGWs carry energy and momentum they exert a force on the atmosphere when they dissipated. This drives a seasonal circulation at the mesopause which changes the overall temperature of the poles and is one of the reasons of why the summer polar mesopause is the coldest place on Earth. This is where noctilucent clouds can be observed.

These four examples provide insight on possible applications of enhanced AGWs models, such as developing better space weather models to improve satellite performance or predicting long-term weather effects; for example, gravity waves can easily interact with storms and may even cause tornados by compressing the storm and increasing its spin (Coulter 2008).
1.1. Mountain Waves

This research focuses on the study of Mountain Waves, a special type of gravity waves that occur when winds flow over topography. They can be responsible for transporting significant amounts of momentum and energy to the middle and upper atmosphere (Eliassen and Palm 1961). In mountainous regions, the effect becomes quite significant for the local atmospheric circulation. Mountain Waves also represent a good testing case for the linear theory as it will be described in the next chapter. Figure 1.2. shows a sketch made from observation in 1977, in one of the first research that combined observations, data and modeling of Mountain Waves. Nowadays the theory behind these phenomena is quite well understood and the research focuses more on describing the combined effects of all the dynamics at different locations in the world. More and more studies of these phenomena are becoming interdisciplinary, such as measuring mountain waves both from the ground and from satellite limb scans (Pugmire, Taylor et al. 2014). This thesis work is motivated by the pursuit of a holistic understanding of the role that Mountain Waves play in the Andes mountain range. This is where the Andes Lidar Observatory (ALO) is located, which contains a near-infrared all-sky imager that is used for observing and measuring gravity waves that perturb the nighttime airglow. The presence of Mountain Waves in this region is very significant as there is a strong wind resource especially in the winter months, making this location a prime remote sensing site for Mountain Wave studies.
Figure 1.2. Mountain Waves observations made in February 17, 1970 west of Denver, Co. (Lilly and Kennedy 1973).
Chapter 2

Linear Wave Theory

Many interesting dynamic phenomena in Physics can be described, to a first-order, by a linear theory. This is because a linearized system can predict the behavior of the dynamics when the scales are much larger than the variables themselves, for example, or when the system is in steady state with the possibility of finding special solutions, or states, in the system, such as wave harmonics or spherical harmonics. A linear theory is also a measure of our understanding of the mathematical relations within the phenomena and is often used to validate more complex dynamical nonlinear models. Whenever we observe an event that is not well described by the linear theory it can be a great opportunity to either update our current understanding or maybe justify the pursuit of more expensive computing modeling; this is with the expectation that the phenomena itself is inherently nonlinear and contains contributions from higher order terms. In any case, the linear wave theory has been essential in our intuitive understanding of what gravity waves are and under what circumstances these are more likely to be observed and measured. While there is no inclusion of self-interaction or minimal of external forces (i.e. gravity and Coriolis force) acting on the system’s state, the linear theory can still be successful in predicting sources, i.e. mountain waves, and energy transfer and energy deposition, as it will be seen in the following sections.
2.1. The Euler Equations

The mathematical treatment of gravity waves starts with the Euler equations of motion, that describe an irrotational, frictionless atmosphere. These can be written in a conservative form in terms of density $\rho$, pressure $p$, energy $E$ and the vectors of wind speed $\mathbf{u} = [u, v, w]^T$ and acceleration of gravity $\mathbf{g} = [0 \ 0 \ g]^T$:

\begin{align}
\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) &= -\nabla p - \rho \mathbf{g} \tag{2.1} \\
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &= 0 \tag{2.2} \\
\frac{\partial E}{\partial t} + \nabla \cdot [(E + p)\mathbf{u}] &= -\rho g z \tag{2.3}
\end{align}

with the corresponding state equation

\[ E = \frac{p}{(\gamma - 1)} + \frac{1}{2} \rho (\mathbf{u} \cdot \mathbf{u}) \tag{2.4} \]

where $\gamma$ is the specific heats ratio for air. Equation (2.1) is momentum conservation, Equation (2.2) is mass conservation and Equation (2.3) is Energy conservation; Equation (2.4) is the ideal gas adiabatic state equation that works very well for the MLT region of the atmosphere. An alternative form for the conservation of energy, Equation (2.4) in terms of the more useful and familiar state variable, pressure, is (Nappo 2002):
where \( c_s \) is the speed of sound in air. In an atmosphere where the density is a function of pressure and temperature, both acoustic waves and gravity waves exist. Since we are only interested in the latter, we will use the Boussinesq approximation that effectively takes the atmosphere as incompressible (constant density) (Spiegel and Veronis 1960); then the corresponding Euler equations are:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = c_s^2 \left[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) \right] \tag{2.5}
\]

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{2.6}
\]

\[
\nabla \cdot \mathbf{u} = 0 \tag{2.7}
\]

\[
\frac{\partial p}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{2.8}
\]

where density has been taken as constant. We also have an extra equation that results from the Boussinesq approximation:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{2.9}
\]

### 2.2. The Taylor–Goldstein equation

We now set out to find an equation that describes linear gravity waves. We start by writing Equations (2.6–2.9) into two-dimensional forms, horizontal \( x \) and altitude \( z \).
(the other horizontal dimension, y, can be easily extended from the preceding theory); also note that \( u \) is the horizontal wind component and it is in the direction of the propagation of the wave (Nappo 2002):

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \tag{2.10}
\]

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g \tag{2.11}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{2.12}
\]

\[
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + w \frac{\partial \rho}{\partial z} = 0 \tag{2.13}
\]

These equations can be linearized by assuming a background quantity (subscript 0) and a perturbed quantity (\( \prime \)):

\[
\mathbf{u}(x,z,t) = \mathbf{u}_o(z) + \mathbf{u}'(x,z,t) \rightarrow \begin{cases} u_o(z) + u'(x,z,t) \\ w'(x,z,t) \end{cases} \tag{2.14a,b}
\]

\[
p(x,z,t) = p_o(z) + p'(x,z,t) \tag{2.14c}
\]

\[
\rho(x,z,t) = \rho_o(z) + \rho'(x,z,t) \tag{2.14d}
\]

Background quantities are taken to be only height dependent and, as the atmosphere is assumed to be in hydrostatic equilibrium. Also, it is assumed there is no vertical background wind. This is not a unique approach to linearization, as other more complex phenomena can be added to these quantities, such as quasi-linear terms that describe additional non-linear interactions with the waves themselves (Franke and Robinson
1999). However, the linearization with respect to Equations (2.14) leads to a practical and useful wave equation. In the linearization, all second order terms are dropped (product of perturbations) (Nappo 2002):

\[
\frac{\partial u'}{\partial t} + u_0 \frac{\partial u'}{\partial x} + w' \frac{\partial u_0}{\partial z} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial x} \tag{2.15}
\]

\[
\frac{\partial w'}{\partial t} + u_0 \frac{\partial w'}{\partial x} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial z} - \frac{\rho'}{\rho_0} g \tag{2.16}
\]

\[
\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0 \tag{2.17}
\]

\[
\frac{\partial \rho'}{\partial t} + u_0 \frac{\partial \rho'}{\partial x} + w' \frac{\partial \rho_0}{\partial z} = 0 \tag{2.18}
\]

If we assume horizontal wave solutions for the perturbations, i.e. \( q' = \tilde{q}(z)e^{i(kx - \omega t)} \) so that Equations (2.15–2.18) become:

\[
-i\omega u' + iu_0 k u + \tilde{w} \frac{du_0}{dz} = -\frac{i}{\rho_0} k \tilde{p} \tag{2.19}
\]

\[
-i\omega \tilde{w} + iu_0 k \tilde{w} = -\frac{1}{\rho_0} \frac{d\tilde{p}}{dz} - \frac{\tilde{p}}{\rho_0} g \tag{2.20}
\]

\[
ik \tilde{u} + \frac{d\tilde{w}}{dz} = 0 \tag{2.21}
\]

\[
-i\omega \tilde{p} + iu_0 k \tilde{p} + \tilde{w} \frac{d\rho_0}{dz} = 0 \tag{2.22}
\]

where the partial derivatives become total derivatives. We can now introduce an intrinsic frequency, or Doppler-shifted intrinsic wave frequency (Chimonas and Hines...
1986), $\Omega = \omega - u_0 k$. This represents the wave frequency in the frame moving with the background wind flow. If a ground observer measures both the wave frequency $\omega$ and the background wind $u_0$, then this Doppler-shifted frequency may be found; note that $c_l = \Omega/k$ is the intrinsic phase speed and $c = \Omega/k + u_0$ is the apparent horizontal phase speed. Another useful quantity is the Brunt-Väisälä frequency, defined in terms of density which was introduced in the previous chapter:

\[ N \equiv \sqrt{-\frac{g}{\rho_0} \frac{\partial \rho_0}{\partial z}} \] (2.23)

We now have our system of equations:

\[ i\Omega \tilde{u} + \tilde{w} \frac{d u_0}{d z} = -\frac{i}{\rho_0} \tilde{k} \tilde{p} \] (2.24)

\[ i\Omega \tilde{w} = -\frac{1}{\rho_0} \frac{d \tilde{p}}{d z} - \frac{\tilde{p}}{\rho_0} g \] (2.25)

\[ ik \tilde{u} + \frac{d \tilde{w}}{d z} = 0 \] (2.26)

\[ i\Omega \tilde{\rho} - \tilde{w} \frac{\rho_0}{g} N^2 = 0 \] (2.27)

These are the polarization equations (Hines 1960) for the Euler equations with wave-like solutions for the parameters. From these relations we may find amplitudes and frequencies for each perturbation variables $u', w', p', \rho'$. The solution to these equations involves, as usual, an assumption of a wave like solutions, this time in the
vertical direction. In fact, Equations (2.23–2.27) can be simplified by solving for $w'$ assuming $w'(z) = Ae^{imz}$, then the solution to this equations is (Nappo 2002):

$$\frac{d^2\tilde{w}}{dz^2} - \frac{1}{\rho_0} \frac{d\rho_0}{dz} \frac{d\tilde{w}}{dz} + \left[ \frac{k^2 N^2}{\Omega^2} + \frac{k}{\Omega} \frac{d^2 u_0}{dz^2} + \frac{k}{\Omega} \frac{1}{\rho_0} \frac{d\rho_0}{dz} \frac{du_0}{dz} - k^2 \right] \tilde{w} = 0 \quad (2.28)$$

Now, we can use exponentially decreasing density in the atmosphere (an isothermal atmosphere that is infinite in vertical extent (Riegel 1992)): $\rho_0 = \rho_\infty e^{-z/H}$ with $\rho_\infty$ denoting at sea level. The scale height is $H = RT/g$, where $R$ is the specific gas constant for dry air, $T$ the temperature; it gives the height at which the state variables are reduced by a factor of $1/e$. For consistency, the new variables will be defined as:

$$\tilde{u} = \frac{\tilde{w}}{\tilde{u}} = \tilde{p} = \frac{\tilde{p}}{\tilde{p}} = e^{i2H}, \quad (2.29a,b,c,d)$$

The Taylor–Goldstein Equation can be obtained by substituting Equations (2.29b,d), that is, exponential decreasing forms for $\tilde{w}$ and $\tilde{p}$, in Equation (2.28):

$$\frac{d^2\tilde{w}}{dz^2} + \left[ \frac{k^2 N^2}{\Omega^2} + \frac{k}{\Omega} \frac{d^2 u_0}{dz^2} + \frac{k}{\Omega} \frac{1}{\rho_0} \frac{du_0}{dz} - \frac{1}{4H} \frac{d^2 u_0}{dz^2} - k^2 \right] \tilde{w} = 0 \quad (2.30)$$

where 1 is called the buoyancy term and is strongly influences the sign of the quantity in brackets, 2, the curvature term, 3 the shear term, 4 the compressibility term and
the non-hydrostatic term. We can see that now this is the familiar wave equation by casting the expression in the brackets as the constant $m^2$:

$$\frac{d^2 \hat{w}}{dz^2} + m^2 \hat{w} = 0$$

(2.31)

and if $m$ is constant we have nice and simple wave equation with solutions:

$$\hat{w} = Ae^{imz} + Be^{-imz}$$

(2.32)

where $m$ is the vertical wavenumber.

### 2.2.1. The Wave Equation with Background Wind Speed

The Taylor–Goldstein equation (2.30) can be simplified for the case of a constant $u_0$ and where the terms containing the scale height have negligible contribution (this is a good approximation for the troposphere):

$$\frac{d^2 \hat{w}}{dz^2} + \left[ \frac{k^2 N^2}{\Omega^2} - k^2 \right] \hat{w} = 0$$

(2.33)

$$\Downarrow$$

$$m^2 = \frac{k^2 N^2}{\Omega^2} - k^2$$

(2.34)
This is called the dispersion relation, that relates the frequency of the wave to its wavenumbers. If we write Equation (2.34) explicitly for the vertical wavenumber \( m \) and the wave frequency \( \omega \), we have:

\[
m = \pm \sqrt{k^2 N^2 - \frac{\omega^2}{(\omega - u_0 k)^2}}
\]

(2.35)

In the same manner, we can derive the corresponding Taylor–Goldstein equation for different scenarios. Several examples are shown in Table 2.1.

<table>
<thead>
<tr>
<th>background wind</th>
<th>Taylor–Goldstein equation</th>
<th>( m^2 )</th>
</tr>
</thead>
</table>
| \( u_0 = 0 \)   | \[
\frac{d^2 \hat{w}}{dz^2} + \left[ \frac{k^2 N^2}{\Omega^2} - k^2 \right] \hat{w} = 0
\]
| \( m^2 = k^2 \left[ \frac{N^2}{\Omega^2} - 1 \right] \) |
| \( u_0 = \text{const} \) | \[
\frac{d^2 \hat{w}}{dz^2} + \left[ \frac{k^2 N^2}{\Omega^2} - k^2 \right] \hat{w} = 0
\]
| \( m^2 = k^2 \left[ \frac{N^2}{\Omega^2} - 1 \right] \) |
| \( u_0(z) \)    | \[
\frac{d^2 \hat{w}}{dz^2} + \left[ \frac{k^2 N^2}{\Omega^2} - \frac{k}{\Omega} \frac{d^2 u_0}{dz^2} - k^2 \right] \hat{w} = 0
\]
| \( m^2 = k^2 \left[ \frac{N^2}{\Omega^2} - \frac{1}{k\Omega} \frac{d^2 u_0}{dz^2} - 1 \right] \) |

**Table 2.1.** Taylor–Goldstein equations for different values of the background wind and negligible scale height, which is suited for gravity waves in the troposphere.

We have now outlined the basic parameters of a propagating gravity wave under linear wave theory. These equations can be expanded to include other interesting effects such as the Coriolis force. A more general approach, the one used in the simulations in this work, will be described in the following section.
2.3. Wave Perturbations on a Windless Atmosphere

The wave model used in this research corresponds to perturbations on an isothermal, windless atmosphere and is described by:

$$\left( \frac{T', \rho'}{T_0, \rho_0} \right) = \Re \left\{ (\tilde{T}, \tilde{\rho}) e^{iaz + i(kx + ly + mc - \Omega t)} \right\} \tag{2.36}$$

where $T'$ and $\rho'$ are the perturbations, $T_0$ and $\rho_0$ are the unperturbed values, $\tilde{T}$ and $\tilde{\rho}$ the complex amplitude, $\Omega$ the intrinsic frequency, $k$ and $l$ the horizontal wavenumbers in $x$ and $y$ directions respectively (Liu and Swenson 2003). The vertical structure of the wave is represented by the vertical wavenumber $m$ and the imaginary eigenvalue $\alpha$. The wave parameters are related to each other by the dispersion relation for a propagating gravity wave (Zhang, Wiens et al. 1993):

$$m^2 = \frac{N^2 - \frac{\Omega^2}{\Omega^2 - f^2} k_h^2}{\frac{\Omega^2}{\gamma g H} - \frac{1}{4H^2}} \tag{2.37}$$

$$\alpha = \frac{1}{2H} \tag{2.38}$$

where $k_h = \sqrt{k^2 + l^2}$ is combined the horizontal wavenumber, and $f$ the inertial frequency (Coriolis). The relation between temperature and density perturbation is (Walterscheid, Schubert et al. 1987):

$$\hat{\rho} = \frac{1 - 2\Omega^2 H / [g(\gamma - 1)] - 2iHm}{1 - 2\Omega^2 H / [g + 2iHm]} \hat{T} \tag{2.39}$$
Gravity waves can also be evanescent waves that don’t propagate vertically (Hines 1960). The corresponding dispersion relation is (Zhang, Wiens et al. 1993):

\[ m = 0, \]

\[ \alpha = \frac{1}{2H} \pm \sqrt{\frac{1}{4H^2} - \frac{\Omega^2}{\gamma g H} - \frac{N^2 - \Omega^2}{\Omega^2 - f^2} k_h^2} \]  

and the temperature-density relation is (Walterscheid, Schubert et al. 1987):

\[ \hat{\rho} = \frac{1 - \alpha H - \Omega' H \left[ g \left( \gamma - 1 \right) \right]}{\alpha H - \Omega' H / g} \hat{T} \]  

We can now describe a 2D (in \( x \) and \( z \) direction) propagating wave using Equation (2.36) and the polarization relations (2.39) & (2.42):

\[ T'(z) = T_0(z) \Re \left\{ \hat{T} e^{i \left\{ m + i \left[ \Omega' H \right] + i (k - u) \right\}} \right\} \]  

\[ \rho'(z) = \rho_0(z) \Re \left\{ \hat{\rho} e^{i \left\{ m + i \left[ \Omega' H \right] + i (k - u) \right\}} \right\} \]  

### 2.5. Linear Wave Theory for Mountain Waves

As seen in Chapter 1, Mountain Waves are generated when air flows perpendicular into stationary obstacles (stationary in the sense much slower than the mean flow velocity of the wind) and perturbations propagate into the stratosphere. For an observer on the ground the apparent phase speed of the MWs is zero, and we’ve seen
from §2.2 that in order for this to be true, there has to be an intrinsic phase speed that is opposite to the background wind, so we have is $c_l = -u_0$. This means that

$$\omega = 0,$$  \hspace{1cm}  (2.44)

$$c_l = -u_0,$$  \hspace{1cm}  (2.45)

$$\Omega = \omega - ku_0 = -ku_0$$  \hspace{1cm}  (2.46)

where $k = |k|$ $\text{sgn}(u_0)$ is the wave vector that points towards the direction of propagation and is opposite to the background wind. These are the essential parameters of a stationary wave. The shape of MWs have strong correlation with the topography that generates them. To explore this, we start with the Taylor–Goldstein equation for constant wind, Equation (2.33), and use Equation (2.46):

$$\frac{d^2\hat{w}}{dz^2} + \left[ \frac{k^2N^2}{\Omega^2} - k^2 \right] \hat{w} = 0 \Rightarrow \frac{d^2\hat{w}}{dz^2} + \left[ \frac{N^2}{u_0^2} - k^2 \right] \hat{w} = 0; \quad m^2 = \frac{N^2}{u_0^2} - k^2$$  \hspace{1cm}  (2.47)

The standard approach for solving Equation (2.47) is transforming the equation into frequency domain (i.e. by Fourier transforms). As usual, the general solution for a particular wavenumber $m$ and $k$ (monochromatic wave) is the wave equation:

$$\hat{w}_l(z) = Ae^{-imz} + Be^{imz}$$  \hspace{1cm}  (2.48)

where we recall that hatted ($\hat{}$) quantities are Fourier Transforms of the perturbed quantities ($\hat{}$). This equation can be solved with proper boundary conditions. We recall that for an isothermal atmosphere the vertical extent is infinite. With this, we can
assume \textit{open boundary} conditions where no wave is reflected. This effectively eliminates the solution with downward propagating waves at the top boundary \((B = 0)\). The bottom boundary condition is described by

\[
w'(x,0) = u_0 \frac{dh(x)}{dx} \quad (2.49)
\]

which states that the normal component of the background flow must be zero, because the wind flow at the surface is assumed to be parallel to the surface, whose shape is given by \(h(x)\). Nonlinear boundary conditions have long been a topic of discussion and a current area of research (Smith 1977, Bao and Tan 2012, McHugh and Sharman 2012, Sharman, Trier et al. 2012, Grisogono, Jurlina et al. 2014). An illustrative example would be to use a cosine surface, so that if

\[
\hat{h}(x) = H \cos kx \quad (2.50)
\]

then we can get the solution to the boundary problem in (2.49) via Fourier transforms:

\[
w'(x,z) = \mp u_0 kH \sin (kx \pm mz) \quad (2.51)
\]

With the perturbation of Equation (2.51) and the linearized Equations (2.15–2.18) follow the perturbations of the other state variables (Nappo 2002):

\[
u'(x,z) = u_0 Hm \sin (kx \pm mz) \quad (2.52)
\]

\[
\rho'(x,z) = -\rho_0 u_0^2 Hm \sin (kx \pm mz) \quad (2.53)
\]
Figure 2.1. The shape of propagating perturbations to the vertical wind, $w'$, at different altitudes and background velocities $u_0$ for a cosine shaped surface boundary (topography). The horizontal wavelength is $\lambda_x = 2$ km, the vertical wavelength $\lambda_z = 10$ km. There is a clear offset of the wavefronts that is due to the tilt of the wave vector $k$. Notice the group velocity vector $v_g$ is along this offset.

The evanescent solution exists when $m$ is imaginary, or when

$$\frac{N^2}{u_0^2} < k^2$$

(2.54)

For a fixed background wind and buoyancy frequency, there should be a critical wavenumber, $k_c$, for which the waves shift from propagating to evanescent. The evanescent solution for the Taylor–Goldstein equation (2.48) and sinusoid topography is:

$$w'(x,z) = -u_0 H k e^{-\text{Im}\{m\}z} \sin(kx)$$

(2.55)

where $\text{Im}\{m\}$ is the imaginary part of $m$. This a non-propagating solution that decreases in amplitude with altitude as shown in Figure 2.2.
Figure 2.2. The shape of evanescent perturbations in the vertical wind, $w^\prime$, at different altitudes with a constant background wind $u_0$ and a cosine shaped surface boundary (topography). The horizontal wavelength is $\lambda_x = 2$ km.

2.5.1. Generalized Wave Equations for Mountain Waves

The previous section provided a basic cosine-shaped topography and it illustrates what kind of Mountain Waves can be expected for such a lower boundary. It is safe to assume that a mountain ridge that approximates a sinusoid topography will likely see steady Mountain Waves as the ones shown in Figures 2.1 and 2.2, provided there’s the right background wind and stable atmosphere. However, in reality the background wind is usually a function of height, and it can have a significant increase over the troposphere as seen in Figure 2.2. An appropriate form of the Taylor–Goldstein equation is taken from Table 2.1:

$$\frac{d^2 \hat{w}}{dz^2} + \left[ \frac{N^2}{u_0^2} + \frac{1}{u_0} \frac{d^2 u_0}{dz^2} - k^2 \right] \hat{w} = 0 \quad (2.56)$$
The solution of Equation (2.56) by the use of Fourier Transforms and including the open boundary condition at the top and the bottom boundary condition and for a monochromatic wave is:

\[ \hat{w}(k,z) = \hat{w}(k,0)e^{-imz}, \quad (2.57) \]
\[ m^2 = \frac{N^2}{u_0^2} + \frac{1}{u_0} \frac{d^2u_0}{dz^2} - k^2 \quad (2.58) \]

We now recall that \( \hat{w} \) is in Fourier Space (see Equations (2.19–2.22)), so that the real part of the inverse Fourier Transform gives the solution to the perturbations. In that sense we use the bottom boundary condition Equation (2.50), which is in real space, to calculate the bottom boundary condition in frequency space:

\[ \hat{w}(k,0) = \int_{-\infty}^{\infty} w'(x,0)e^{-ikx} \, dx = u_0 \int_{-\infty}^{\infty} \frac{dh(x)}{dx} e^{-ikx} \, dx \quad (2.59) \]
\[ \hat{w}(k,0) = u_0 ikh(k) \quad (2.60) \]

We can now use Equation (2.60) to find an expression for the perturbation:

\[ w'(x,z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{w}(k,z)e^{ikx} \, dk \quad (2.61) \]
\[ w'(x,z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{w}(k,0)e^{ikx} \, dk \quad (2.62) \]
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\[ w'(x,z) = \frac{u_0}{2\pi} \int_{-\infty}^{\infty} \hat{h}(k) ike^{i(zk+\xi)} \, dk \]  

(2.63)

Since the sign of \( k \) has being defined by the direction of the background wind, then we can rewrite the integral in equation (39) as

\[ w'(x,z) = \text{sgn}(k) \frac{u_0}{\pi} \Re \left\{ \int_{0}^{\infty} \hat{h}(k) ike^{i(zk+\xi)} \, dk \right\} \text{ or,} \]

\[ w'(x,z) = -\frac{u_0}{\pi} \Re \left\{ \int_{0}^{\infty} \hat{h}(k) ike^{i(zk+\xi)} \, dk \right\} \]

(2.64)

where we multiply by 2 and we have to take the real part since the asymmetric indices means the complex part of the integrand won’t cancel out. With this equation, we can now see that we can input any shape for the initial condition of the bottom boundary. Equation (2.64) also shows that the solution can be represented by a Fourier Series with bases \( \Sigma ke^{ikx} \). Indeed, the general solution to the Taylor–Goldstein Equation is a superposition of monochromatic waves with their own distinct wavenumber. Also, we’ve encountered two flavors of these waves, propagating and evanescent. The 2 cases can be easily accounted for in Equation (2.64) by recalling that there is a critical wavenumber for which one solution shifts from propagating \((k < k_c)\) to evanescent \((k > k_c)\):

\[ w'(x,z) = -\frac{u_0}{2\pi} \Re \left\{ \int_{0}^{k_c} \hat{h}(k) ike^{i(zk+\xi)} \, dk + \int_{k_c}^{\infty} \hat{h}(k) ike^{-3|m|z} \, dk \right\} \]

(2.65)

and taking the real part,
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The first integral in Equation (2.66) is the propagating wave mode, while the second one is the evanescent mode. We have now derived an expression for an arbitrary bottom boundary. We can explicitly calculate the cutoff wavenumber from the dispersion relation (2.58):

\[ k_c = \sqrt{\frac{N^2}{N^2 - \frac{d^2 u_0}{u_0 dz^2}}} \] (2.67)

As before, once \( \hat{\nu} \) is known we can find the remaining perturbed quantities using the polarization equations (2.24–2.27), \( \Omega = -ku_0 \) for MWs, and the inverse Fourier Transform. Several plots with the results of the linear theory are presented in Figures 2.3–2.5. The descriptions are given in their captions. These simulations are the integration of Equation (2.64). One important comment on these plots is that it shows both how well the linear theory can work for determining wave structures, and how it fails. Since the linear theory doesn’t take into account the actual mountain \( h(x) \), but the spectrum due to the mountain \( \hat{h}(k) \), the representation of the wavefield can only be accurate at large distances from the boundary, a.k.a. the mountain, as in Figure 2.5. Indeed, just integrating Equation (2.64) yields solutions at places where the wind should be zero, such as at the surface and inside the mountain. In any case, it represents a valuable tool in determining the type of energy content to be expected by given topography shapes and background wind profiles. Additionally, if the mountain is very
small or very far from where we’re integrating for a solution, provided the hydrostatic approximation is still valid, then the solution becomes much more accurate and reliable.

As a last comment on this linear theory development, the extension to a linear three-dimensional model for mountain waves is straightforward and follows the same procedure as above. The only main difference is the use of two-dimensional Fourier Transforms and the appearance of the second wavenumber, i.e. $k$ and $l$ in $x$ and $y$ directions respectively; the one-dimensional theory used above can be obtained from the two-dimensional one if we take the wave vector in the direction of the mean background flow and magnitude of the wavenumber vector, $k_H = \sqrt{k^2 + l^2}$, as the horizontal wavenumber. This 3D linear theory has been greatly researched and it is still an important part of (Kim and Mahrt 1992) modern day campaigns. (Blumen and McGregor 1976, Smith 1980, Hines 1988, Nappo 2002, Smith, Skubis et al. 2002, Jiang and Doyle 2004, Smyth 2009).

**Figure 2.3.** Mountain Waves predicted by the linear theory. (a) Vertical wind perturbations due to mostly propagating waves and constant wind profile of $-40$ m/s. (b) Vertical wind perturbations due to mostly propagating waves and the variable wind profile in Figure 2.6.
Figure 2.4 Vertical wind perturbations (arbitrary scale) due to two different shaped boundaries and mostly evanescent waves with the variable wind profile in Figure 2.6.

Figure 2.5. A triangle-shaped mountain with the variable wind profile in Figure 2.6. Propagating waves can be seen trailing downwind; these are called Lee waves.
Figure 2.6. Wind and temperature profile obtained from NCEP/NCAR Reanalysis wind data for (30°S, 70°W) and date 06/22/2011 00UT (a typical winter day in the Southern Hemisphere with large background wind). The temperature profile is used for obtaining the buoyancy frequency $N^2$.

Another set of profiles is presented in Figure 2.7. In these plots the atmosphere is extended up to 100 km, including the horizontal winds by the HWM07 model and temperature profile by the MSISE-00 model. Also, perturbation percentages are provided. There is a clear dependence of all mountain shapes to the background wind.
Figure 2.7. Linear Wave Mountain wave model for, from left to right, a cosine ridge, a bell shaped mountain and an arbitrary shaped mountain using the background quantities of Figure 2.8.

Figure 2.8. (Left) Horizontal Wind Model HWM07 (Drob, Emmert et al. 2008, Emmert, Drob et al. 2008) profile and (right) MSISE-00 (Hedin 1991) temperature profile for June 22, 2011.
2.5.2. Wave Stress and Energy Transfer

It was described in Chapter 1 that Mountain Waves are responsible for carrying and depositing momentum onto the middle and upper atmosphere. The wave stress represents the flow of momentum across a surface area. In case of AGWs, this is a quantity called pseudo-momentum (Mcintyre 2006), defined as

\[ p_s = \frac{\bar{E}}{\Omega} K \]  \hspace{1cm} (2.68)

with a corresponding pseudo-energy

\[ E_s = \frac{\bar{E}}{\Omega} \omega \]  \hspace{1cm} (2.69)

where \( \bar{E} \) is the mean perturbation energy per unit volume, \( K \) is the wavenumber vector and mean the quantity \( \bar{E}/\bar{\Omega} \) is called the wave action. The need for these new quantities arise since the flow’s total average momentum and energy are not conserved in the presence of a background wind. For example, it can be shown for the case of no background wind:

\[ \frac{\partial \bar{E}}{\partial t} + \nabla \cdot (v_s \bar{E}) = 0 \]  \hspace{1cm} (2.70)
However, the actual processes that determine the transfer of energy cannot be described by a simple linear equation for conservation of energy. But if instead we use the wave action, it can be proved that (Bretherton 1966)

\[
\frac{\partial \left( \frac{\mathcal{E}}{\Omega} \right)}{\partial t} + \nabla \cdot \left( v_g \frac{\mathcal{E}}{\Omega} \right) = 0
\]  

which is the conservation of wave action. The wave stress is defined as

\[
\tau(z) = -\rho_0 \overline{u_1 w_1}
\]

where the overbar represents a spatial average over some meaningful period or wavelength. Now we see that Equation (2.72) represents the flow’s momentum density. We can express Equation (2.72) in terms of the pseudo-momentum:

\[
\frac{\mathcal{E}}{\Omega} k = \frac{\rho_0 \overline{u' w'}}{w_g}
\]

where \( k \) is the horizontal wavenumber and \( w_g \) is the vertical group velocity. Equation (2.73) has units of momentum per unit volume. Additionally, if we know the gravity wave amplitude then the energy density can be calculated by:

\[
\mathcal{E} = \frac{1}{2} \rho_0 A^2 \frac{|k|^2}{k^2}
\]
The differences in pseudo-quantities and flow quantities can be subtle, but meaningful in numerical models and in the overall understanding of the energy transfer process due to mountain waves. An exemplification of the pseudo-momentum vectors in a numerical simulation is shown in Figure 2.9, which is taken from the extensive work on pseudo-momentum by (Durran 1996). In the figure, the instantaneous momentum flux vector is shown in arrows, on top of isentropes of potential temperature. A very important result from these simulations show that while the flow momentum vector points at times in both directions, the pseudo-momentum vector always points downwards, just as we expect from the continuity equation for this quantity.

Figure 2.9. The (a) perturbation momentum and (b) pseudo-momentum vectors plotted over isentropes of potential temperature. (Taken from (Durran 1996)).

Another important result of the linear wave theory is that the Energy flux vector,

\[ \mathbf{F}_E = \bar{E}(u_g \hat{x} + w_g \hat{z}) \]  

(2.76)
determines that the transfer of energy occurs in the direction of the group velocities. This was already implied in the pseudo-momentum definition, where the direction is determined by the wavenumber vector $\mathbf{K}$. Now we know from basic wave theory

$$u_g = \frac{\partial \Omega}{\partial k} \quad (2.77)$$

$$w_g = \frac{\partial \Omega}{\partial m} \quad (2.78)$$

For Mountain Waves with negligible apparent frequency we need to use the dispersion relation to figure out the intrinsic frequency $\Omega$. For the case of the Taylor–Goldstein equation with varying wind profile the dispersion relation is, from Table 2.1:

$$m^2 = k^2 \left[ \frac{N^2}{\Omega^2} - \frac{1}{k\Omega} \frac{d^2 u_0}{dz^2} - 1 \right] \quad (2.79)$$

The solution to (2.79) is given by the quadratic formula:

$$\Omega = \frac{-k(u_0'' \pm \sqrt{4K^2N^2 + u_0''^2})}{2K} \quad (2.80)$$

where $K = \sqrt{k^2 + m^2}$. We can now take the partials in (2.77) & (2.78) by considering the negative solution for the radical in (2.80) and then use the fact that $\Omega = -ku_0$ for Mountain Waves, so that we get
Chapter 2 – Linear Wave Theory

(2.81)

\[ u_g = \frac{\partial \Omega}{\partial k} = -2N^2 + u_0 \left( 2k^2 u_0 - u_0'' \right) \frac{\partial \Omega}{2N^2 / u_0 + u_0''} = -u_0 \]

(2.82)

\[ w_g = \frac{\partial \Omega}{\partial m} = \frac{2ku_0 \sqrt{N^2 - u_0 \left( k^2 u_0 - u_0'' \right)}}{2N^2 / u_0 + u_0''} \approx \frac{u_0^2 k}{N} \]

where double primes stand for double derivative with respect to \( z \). The approximate quantities in (2.81) & (2.82) represent the case of hydrostatic equilibrium, where \( k^2 \) term (called precisely the non-hydrostatic term) is neglected in the Taylor–Goldstein; this is a very good approximation for gravity waves in the tropopause with very long horizontal wavelengths, since the hydrostatic assumption holds and the vertical wind perturbations are in hydrostatic balance. We can see that only real solutions for the vertical group velocity represent propagating waves; the imaginary solutions correspond to evanescent waves, which is evident when comparing the radical to the evanescent condition underlain in the dispersion relation (2.79). Finally, the 2D total group velocity vector can be computed if the group velocity components of Equations (2.81) & (2.82) are known:

\[ \mathbf{v}_g = \frac{u_g \hat{x} + w_g \hat{k}}{\sqrt{u_g^2 + w_g^2}} \]

These results are shown in Figure 2.10, where the group velocities for propagating waves and evanescent waves are demonstrated. The vectors point entirely in the positive direction; this is an expected result for the flow of energy happens from where the wave was generated at the surface of the mountain, to the upper atmosphere. When the wave contribution is mostly by evanescent waves, as in Figure 2.10a, the group velocity goes
to infinity, as all of the energy is instantaneously transferred to the atmosphere. As a final remark, it is important to note that current research in the topic of mountain gravity waves and atmospheric gravity waves relies heavily on the use of this linear theory and it is the case for most of the references in this chapter. For gravity waves, and especially mountain waves, the linear theory provides a first-order understanding of the dynamics of this phenomena.

Figure 2.10. Group velocity directions that represent the direction of the mean energy flux vector. (a) The case of predominantly evanescent waves, where the group velocity goes to infinity, and (b) the case of only propagating waves.
Chapter 3

The All-Sky imager and the Airglow Response to Gravity Waves

The Earth’s airglow layer is a part of the MLT region that produces emission due to photochemical reactions of different molecular and atomic species. AGWs create perturbations in the airglow. Many studies have been conducted on this in an effort to further comprehend the dynamic and mean impact to the atmosphere (Krassovsky and Shagaev 1977, Takahashi, Batista et al. 1985, Myrabø, Deehr et al. 1987, Taylor, Hapgood et al. 1987, Garcia, Taylor et al. 1997). Additionally, studies have been conducted to determine the heights and emission of these layers (Roach and Meinel 1955, Sharov and Lipaeva 1973, Garcia, Taylor et al. 1997). Table 3.1 presents an overview of the predominant emissions in the 85–100 km altitude in the MLT region.

<table>
<thead>
<tr>
<th>Emission line</th>
<th>Emission Wavelength (Å)</th>
<th>Mean Layer Height (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OI</td>
<td>5577</td>
<td>96</td>
</tr>
<tr>
<td>Na</td>
<td>5892</td>
<td>90</td>
</tr>
<tr>
<td>NIR OH</td>
<td>7150 – 9300(^1)</td>
<td>87</td>
</tr>
<tr>
<td>O(_2)</td>
<td>8655</td>
<td>94</td>
</tr>
</tbody>
</table>

Table 3.1. Airglow layer emissions in the MLT region. The NIR OH imager uses a passband filter that excludes the O\(_2\) emission line.
The description of these effects on the airglow can help determine the characteristics of the originating waves, such as whether they are vertically propagating, evanescent, ducted, reflected or breaking.

In this study we are interested in determining the characteristics of observed gravity wave phenomena. It is possible to measure the airglow intensity and the rotational temperature, whose relation is a function of wave parameters often described by the Krassovsky parameter (Krassovsky and Shagaev 1977). Yet, some intrinsic wave parameters cannot be determined directly from measurement, such as vertical wavelength of the wave. These can be described by a model from distinct airglow layer measurements (i.e. to get phase differences).

3.1 Airglow Photochemistry

The task to build a model that simulates the airglow emission involves a description of the chemistry of the interactions that emit photons in the airglow layer. These models are based on recombination and quenching of exited molecules in the atmosphere. We discuss here a 2D model that uses these chemical reactions in combination with density data for the airglow layer to calculate the volume emission rate for the OH Meinel emission band. Without much description, these models are presented here with due reference. The OH Meinel emission band emissions are described by:

$$H + O_3 \xrightarrow{k_1} O_2\left(c^{1\Sigma_u^-}\right) + M, \text{ and}$$

(3.1)
where \( f_8 = 0.29 \) is fraction of emission at level 8, \( k_6^{N_2} = 5.7 \times 10^{-34} (300/T)^{2.62} \) and \( k_6^{O_2} = 5.96 \times 10^{-34} (300/T)^{2.37} \) are quenching coefficients (Liu and Swenson 2003). The brackets \([ \cdot ]\) represent number densities in unit of \( \text{cm}^{-3} \). \( \varepsilon_{O_2} \) and \( \varepsilon_{OH} \) are in unit of photons \( \text{cm}^{-3} \text{s}^{-1} \).

### 3.2 Wave Perturbations to the Airglow

As seen in §2.2, perturbations are defined with respect to different background quantities. For \( N_2 \) and \( O_2 \) this is simply

\[
\left\{ \frac{[N_2]}{[N_2]^u} \right\} = \left\{ \frac{[O_2]}{[O_2]^u} \right\} = \frac{\rho'}{\rho_u} \tag{3.3}
\]

and for \( O \) (Walterscheid, Schubert et al. 1987, Zhang, Wiens et al. 1993):

\[
\left\{ \frac{[O]}{[O]^u} \right\} = -DH \left\{ \frac{\rho'}{\rho_u} + \frac{1+DH}{\gamma-1} \frac{T'}{T} \right\} \tag{3.4}
\]

where \( D = d \ln [O]^u/dz \) is the inverse of the local scale height of unperturbed \([O]\).

Once the volume emission rate \( \varepsilon \) is calculated from Equations (3.1) and (3.2), the Integrated Volume Emission Rate (hereafter referred as IVER) is:

\[
I(t) = \int_{z_1}^{z_2} \varepsilon(z,t)dz \tag{3.5}
\]

with the perturbations given by:
\[I'(t) = I(t) - \langle I(t) \rangle\] (3.6)

where \( I(t) \) is the time-dependent IVER and \( \langle I(t) \rangle \) is the IVER temporal mean. It is also possible to obtain the rotational temperature, given by:

\[T_R(t) = \frac{1}{I(t)} \int_s^{z} \varepsilon(z,t) T(z,t) dz\] (3.7)

and its respective perturbation:

\[T_R'(t) = T_R(t) - \langle T_R(t) \rangle\] (3.8)

These are the main perturbation that make up for the visible gravity waves when observed through airglow imagers.

### 3.3 The All-Sky Imager as Airglow Emission Measuring Devices

Wave phenomena is very common in the nightly airglow. It is common to find small scale phenomena, with short periods of a few minutes (such as acoustic waves) and several hours for gravity waves. There are also planetary wide waves such as Rossby waves and atmospheric thermal tides. The All-Sky Imager (hereto referred to as ASI) is a highly sensitive cooled charge-coupled device (CCD) that is capable to observe the medium scale gravity waves by using its wide field-of-view (hereto referred as FOV) of 180°. For airglow layer heights for \(~87\) km, such as the OH band emission, the ASI is capable of capturing \(1000\) km \(\times\) \(1000\) km of the airglow layer; however, extreme FOV angles have very low resolution; details and smaller scale phenomena are not observable at these large FOV angles. A more detailed discussion of these resolution changes are
discussed in the next Chapter. Figure 3.1 shows ASI images of a typical gravity wave event in the airglow, as well as an undisturbed example.

Figure 3.1. Typical ASI images for Cerro Pachón, Chile: (a) A quiet night typical of summer months and (b) a winter night in July with both propagating waves and standing waves (orographic).

Typical temporal resolution for these instruments is 1 or 2 minutes between images, with a 60 to 90 second exposure. The ASI CCD used in this research has a $1024 \times 1024$ pixel resolution, binned down on chip to $512 \times 512$ to enhance signal-to-noise ratio and a temporal resolution of 1 minutes for years 2009–2011 and 2 minutes thereafter, all with a 60 second exposure. This is the pixel resolution that becomes the base size of every processed image and the simulated ones in the following chapters. There is another essential parameter that characterizes the angular resolution of each pixel, the lens function, $G$, which determines the zenith (or elevation) angle as a function of pixel distance from zenith. The profile of this function is usually taken as provided by the manufacturer. However, for the imager in use this information is not
readily available and a form of this function is fitted from the one used by (Garcia, Taylor et al. 1997), shown in Figure 3.2, and it is assumed to be a typical profile. This turns out to be a good approximation as it is shown in the calibration procedures of the next Chapter. If, alternatively, a linear lens function is used, then a fixed resolution of 0.35225 °/pixel can be used as a substitute (just a little larger than 180°/(512 pixels), but is a mean value that works well for the images). For simulation purposes, a fixed pixel/° resolution works well as there is not much change in the overall shape of the simulated waves; however, the lens function becomes relevant for the most accurate representation of gravity waves in actual imager data and especially important for calibration with the background stars, as discussed in the next section.

![Figure 3.2](image.png)

**Figure 3.2.** The normalized lens function $G(\theta)$ with a third degree polynomial fit.

### 3.3.1. ASI Calibration with Background Stars

Due to irregularities in the manufacturing process, as well as the installation of the imager, it cannot be assumed that the images are properly aligned. In fact, it is the case that center of the image is several pixels off from the actual zenith and the top of
the image is a couple of degrees off of North. To account for this, the only reliable way is to correlate the position of the background star field with their computed positions at the corresponding moment that the image was taken. It is easy to obtain a database of the stars position and plot them along with an individual ASI image. With this information, we can then manually adjust the location of the zenith and the required rotation of the image to match the location of the stars. Figure 3.2 shows an example of this procedure, also showcases the importance of the lens function for the stars to match. Depending on the imager, if this procedure is done manually as it is done in this work, then this has to be calibrated for different years as small changes happen throughout the lifetime of the imager (such as installation of new filters or relocation due to natural hazards).
Figure 3.3. Star calibration of an ASI image. The figure on the left uses the fitted lens function $G$, while the one on the right assumes a linear lens function (or fixed pixel/° resolution). The yellow circle are the position of the stars from astronomical database. The middle crosshair shows the calibrated position of the zenith and the large blue/white circle is the 180° FOV.
Chapter 4
All-Sky Imager Simulations

The analysis of imager data requires a computational algorithm. To validate this method it is important to develop tools to diagnose the performance of this image processing algorithm (hereto referred as IPA). Additionally, simulations provide evidence of our good understanding of the theory and model that’s currently being used to explain the phenomena we see in all-sky imagers (Schoeberl 1985, Liu and Swenson 2003, Michael P Hickey 2007, Fritts, Wang et al. 2009). So in one part we have the modeling of the engineering system, in this case the imager itself, and on the other hand we have the modeling of wave equations that describe the most important dynamics observed. This work focuses on modeling gravity waves, whose dynamics have been described above by a linear wave theory. In the particular case of Mountain Waves, the expectancy is to observe stationary waves with particular parameters that somewhat relate the topography scale heights. However, for the purpose of testing the IPA it is sufficient to model a propagating wave and use a snapshot in time of it's corresponding image in the instrument.
4.1. The Simulated Wave Structure

The 2D model presented here has been built from the ground up, but it is heavily based on the one developed by (Liu and Swenson 2003). The original MATLAB code was provided at the start of this study and adapted for a simpler and cleaner workflow. As the new 2D code was written it was important to verify it by reproducing the 1D results previously published by Liu and Swenson, at least partially. Figure 4.2 shows the 1D results that mimic those in the 2003 research for the volume emission rate for OH. The red line is the unperturbed volume emission rate and the green-yellow lines represent the perturbed volume emission rate each at 1/24 of a wave period. Additionally, the blue line is the standard deviation of the perturbed emission, which peaks at several kilometers below the unperturbed one. It is important to note that for all the proceeding discussion the demonstrated figures follow either a 1% or 10% perturbation of temperature $T'$ and the densities are obtained from the MSIS90 (Hedin 1991). The perturbations of $[O_2]$, $[N_2]$ and $[O]$ densities are calculated relative to this $T$ perturbation. This was chosen as it becomes more practical to discuss disturbances in temperature in small percentages as data for temperature is easily available and accurate; in other words, it is a common case to have these kind of perturbations. Additionally, all of the studied waves are downward propagating waves, as the other possible mode, evanescent waves provides no additional insight to this discussion; also the upward propagating waves don’t change the overall results. One final caveat is that there is no dampening in this analysis. Without dampening, it is easier to verify the behavior of the model as an undamped wave is constant in amplitude over time and this will reflect better the shapes of standing waves.
Figure 4.1. The MSIS-E-90 profiles of $[O]$, $[O_2]$ and $[N_2]$. These are the same profiles used in Liu and Swenson, 2003 (Liu and Swenson 2003).

Figure 4.2. 1D model results for the volume emission rate $\varepsilon$ of OH. The blue profile and axes are the standard deviation. The green-yellow lines represent the perturbed volume emission rate each at 1/24 of a wave period. The vertical wavelength of the perturbed wave is 25 km at 1% temperature perturbation.
Figure 4.3. Wave perturbations on the model quantities $T$, $[O_2]$, $[N_2]$ and $[O]$, based on a $10\% T'$ perturbation. The upper plots represent the unperturbed (green lines) and perturbed (blue lines) quantities. The $[O_2]$ and $[N_2]$ plots are in log space. The bottom plots are the perturbed quantities minus the unperturbed ones.

It may seem that the next natural step for expanding the model into 2D would be to readily compute the $\varepsilon$ for a $z$ vs. $x$ grid. Although that is indeed what was done for the model, it is important to further analyze the main features of the perturbations described in equation (2.43). To showcase the shape of these waves, a plot has been made in Figure 4.1 against the background (MSIS-E-90 profiles). These are $10\% T'$ perturbation plots, which is a high value, but helps for a better visualization of the effects on the background quantities. In particular, the $[O]$ profile is the one that has a greater change due to a higher perturbation as shown in equation (3.6); any negative $\varepsilon$ result is discriminated by the model. These results are part of the 2D model and that’s why a specific horizontal position is set ($x = 0$ km for Figure 3).

When expanding the model into an $x$-coordinate, the MSIS90 vertical profiles are taken to be constant along this new dimension. Recalling the wave Equation (2.43), the disturbances are expanded into 2D space and are shown in Figure 4.5. In accordance with the wave equation, the $x$ oscillation is constant in time. Also, the wave is
propagating in the downward $z$-direction and positive $x$-direction, although is not evident in the plots shown here. These undamped waves are in accordance to typical cases of ideal AGWs: no energy loss, refraction or dampening. They are well-suited for resolving the geometric nuances of the model. It is worth mentioning that all of these plots are easily animated and converted into movies by the model; a feature that might be useful for consistency check as well as informative purposes.

**Figure 4.4.** Wave perturbations on the $T$: vertical profile (a), the horizontal profile (b) and the 2D extension of the vertical profile into $x$ dimension (c). In (a) and (b) the green lines are the unperturbed temperature and the blue lines are the perturbed ones. The mesh grid in (c) represents the unperturbed temperature while the smooth-colored surface is the 2D perturbation.

**4.2. 2D Volume Emission Rate.**

Using the expanded profiles from §4.1 and the equations (3.2) and (3.4) for the ions reactions we can obtain the 2D volume emission rate for each of the species given their densities. Without any perturbation, we can obtain a uniform grid of density data. To create a perturbation, we assume there is already a wave and that it is in
thermodynamic equilibrium with the background atmosphere. This means there is a steady state wave and constant emission rate. As densities change due to the perturbation, equations (3.5) and (3.6), a new emission rate is computed from the chemical reaction equations. This process is repeated for all of the densities in the $z$ vs $x$ grid for a given time $t$. A complete flowchart can be found under §4.2.6. The volume emission rate $\varepsilon$ data is stored in a 2D matrix that can be easily visualized in a surface (3D) plot, as shown in Figure 4.6 for the $O_2$ volume emission rate and different $T$ perturbations. The wave patterns are more evident in the 10% $T$ perturbation, but the 1% $T$ perturbations are closer to representing actual AGWs. If one “slab” of the plot is taken for a specific $x$, the resulting profile will resemble the 1D model results.

Although the $\varepsilon$ data can be readily integrated along the height $z$ to get the Integrated Volume Emission Rate (hereto referenced as IVER), there is little use of performing this computation. Since the system is in cartesian coordinates, the resulting IVER would depend on the $x$-coordinate. While our ground-based airglow imaging systems indeed measure the IVER, they are localized devices with wide fields-of-view; unless an array of detectors is considered, where each of sensors measures the IVER straight up, there is no practical use in modeling IVER data for a Cartesian system. This sets up the need to describe the model in a new coordinate system that is based around the imaging detector. For this two 2D model an appropriate one is the polar coordinate system.
4.2.1. Polar Coordinate System

In order to model the IVER that an actual airglow imaging detector would measure a polar coordinate system is used. Figure 4.6 shows the geometry of the airglow-detector system. The coordinate conversion for this configuration is:

\[ x(z) = z \tan \theta \text{ [km]} \]  \hspace{1cm} (4.1)

\[ \rho = \sqrt{x(z)^2 + z^2} \text{ [km]} \]  \hspace{1cm} (4.2)

where \( x(z) \) is the horizontal coordinate at a given altitude \( z \) and angle \( \theta \) and is measured in kilometers; \( \rho \) is the distance between the imaging sensor and a point in space at the coordinate \([x(z), z]\). A system described by these equations has the property of being
defined by both $z$ and $\theta$. If these two quantities are given vectors, then the equations become:

$$\bar{x}(\bar{z}) = \bar{z} \tan \bar{\theta} \text{ [km]}$$  \hspace{1cm} (4.3)  

$$\bar{\rho} = \sqrt{\bar{x}^2 + \bar{z}^2} \text{ [km]}$$  \hspace{1cm} (4.4)

It is essential to have a system defined by the altitude $z$ as all of the MSIS90 background data is dependent on this coordinate. Additionally, having a system that is set by $\theta$ is important for the airglow imager as it is directly connected to the its resolution (i.e. a pixel corresponds to certain $\Delta \theta$). So the spatial resolutions of equations (4.3) and (4.4) are determined by the resolutions in vectors $\bar{z}$ and $\bar{\theta}$ which in turn are set by the MSIS90 height resolution and the imager resolution respectively. However, this discussion is not constrained by the imager resolution yet, so while $d\bar{z}$ is still determined by the MSIS data (100 m), $d\bar{\theta}$ is chosen by design to improve the model resolution. A further discussion about system resolution follows in §4.2.5

![Figure 4.6. Schematic of the airglow-imager system. The imager has a set FOV.](image)
The volume emission rate $\varepsilon$ in §3.2 can be directly transformed into polar coordinates and then integrated to get an IVER that depends on $\theta$ (and not on $x$ as in the cartesian coordinate system). Still, once again it is important to explore the shape and features of the $\varepsilon$ mapped in polar coordinates, partially for consistency purposes and also for an accurate visual representation of the geometric system defined in figure 4.6. The final $\theta$-dependent IVER is described beneath in §3.4.

The direct mapping of coordinates determines a new grid in the $\rho$ and $\theta$ space. The $\varepsilon$ is then calculated as in the previous section. Yet, for showing purposes of the calculated $\varepsilon$ information, a logical numerical grid (matrix) needs to be constructed. This means that interpolation is needed to create a uniformly spaced $\bar{\rho}$, and $\bar{\theta}$ is arbitrarily defined to be uniformly spaced as well. Once the interpolation is done, a new $\rho$ vs $\theta$ vs $\varepsilon$ 3D grid contains the necessary information for plotting. Figure 4.7 shows a very similar plot as the one in figure 4.5, but this time in polar coordinates. As expected from the geometry, the $\rho$ position of the $\varepsilon$ from the airglow layer “bends” towards larger $\theta$. It is worthwhile to note that the shape of the wavefronts is somewhat distorted. This is a feature that will greatly change the shape of the $\theta$-dependent IVER and will be addressed below. Figure 4.7 shows the polar plot for a 1% $T^\prime$ perturbation at a 120° FOV for both OH. Just like the 1D model results in figure 2, the OH $\varepsilon$ peak appears at around 87 km.
Figure 4.7. Volume emission rates in $\rho$ vs $\theta$ grid for OH with a small $T'$ perturbation (1%) with 120° FOV. The OH layer sits at around 87 km at center (zenith).

As it was described above, one big advantage of using polar coordinates is to allow for a visual representation of the airglow-detector system geometry. This accomplished by a cylindrical plot of the polar $\varepsilon$ data. Even though the plots in figures 8 and 9 are mapped in polar coordinates, they are still plotted in a 3D cartesian system. However, it is possible to recast this data into a “true” cylindrical plot, determined by the variables $\rho$, $\theta$ and $\varepsilon$. It is of importance to mention this is the data being mapped into a different coordinate system twice, and therefore it is expected that the visual feature of the perturbation waves are in accordance to those found in $z$ vs $x$ vs $\varepsilon$ space. This means that the “deformation” of the wavefronts in Figure 4.7 disappear, or rather, shown in their original cartesian shape. The resulting plots are shown in Figure 4.10 and are an appealing visual summary of what has been accomplished with the model so far. Additionally, Figure 4.8 depicts the would-be geometry of an 80° FOV CCD detector. All of the dark blue “empty” data represents 0’s in the $\rho$ vs $\theta$ numerical grid (matrix); so the higher the FOV the larger the grid is.
4.2.2. \( \theta \)-dependent IVER

Going a step back, before the interpolation to new coordinates that led to the plotting of Figures 4.7 and 4.8, it was stated that the data, right after being transformed into polar coordinates, was readily available for integration. The idea is to integrate along the \( \rho \) in an analogous fashion to the vertical \( z \) integration:

\[
\text{vertical integration of } \varepsilon: \quad I(x,t)/dx = \int_{z_1}^{z_2} \varepsilon(z,x,t) dz, \quad (4.5)
\]

\[
\rho \text{ integration of } \varepsilon: \quad I(\theta,t)/d\theta = \int_{\rho_1}^{\rho_2} \varepsilon(\rho,\theta,t) d\rho \quad (4.6)
\]
where \( I \) is the \( \theta \)-dependent IVER; it is of course also dependent on time. Both these equations are for fixed \( x \) and \( \theta \) in 2D space. It is not necessary to integrate for the horizontal coordinates since we want the IVER at each \( x \) and \( \theta \). Yet, we must multiply these equations by \( \Delta x \) and \( \Delta \theta \) since we need to account for the “seen” area of the imager. The equivalent numerical integrations, for the polar system, are:

\[
I(\theta,t) = \left[ \sum_{k=1}^{N} \epsilon(\rho_k,\theta,t) \right] \Delta \rho \Delta \theta, \tag{4.7}
\]

for a constant \( \rho \) (obtained from interpolation), and

\[
I(\theta,t) = \left[ \sum_{k=2}^{N} \frac{\epsilon(\rho_k,\theta,t) + \epsilon(\rho_{k-1},\theta,t)}{2} \Delta \rho_k \right] \Delta \theta \tag{4.8}
\]

for a variable \( \rho \) (resulting from direct transformation of the \( z \) vs \( x \) grid; \( k \) is a point \( \hat{\rho} \) and \( N \) is the number of points in the grid. Using these two equations the model is capable of calculating the IVER from both the interpolated data and the non-interpolated one. For this discussion, it will suffice to say that both methods were tested and compared for consistency, and the results differ by less than 0.1%. This negligible difference sets the interpolation method as only useful for allowing for a visual representation of the model. The resulting IVER is shown in Figure 4.9 for a set FOV of 160° and different vertical wavelengths \( \lambda_z \). The relative perturbation has been plotted as well; this is just the perturbed IVER minus the unperturbed IVER and it shows clearly the wave features.
Chapter 4 - ASI Simulations

Figure 4.9. O$_2$ IVERs for a 180° FOV and different wavelengths. The bottom plots represent the relative perturbation and they are the perturbed IVER minus the unperturbed IVER. The wave form can be easily observed as well as distinct cancellation effects.

One of the interesting characteristics of Figure 4.9 is that the IVER gets larger as $\theta$ grow larger. This is a feature that can be predicted from the geometry; as $\theta$ increases, $\theta$ becomes larger and there is more airglow density information in the integration path. Furthermore, the shape of this curve matches perfectly the volume emission rates $\epsilon$ shown above. An additional enhancement is due to the van Rhijn effect, and it has already been accounted for the plots of Figure 4.9. A further discussion of this can be found in §4.3.2.

4.2.3. The Cancellation Effect

Another interesting feature found in Figure 4.9 is that the plots are not symmetric. This is due to the fact that the line-of-sight (which is effectively the integration path) can “see” either positive or negative interference from the wave. This
is easily observable if different rays are traced for different angles in the plots in Figure 4.8. Due to shape of the wavefronts, strong cancellation occurs on the negative angles, while strong constructive interference occurs on the positive ones. This feature alone explains why an airglow imager might pick a AGW signal in one direction but not in the other one. This effect is stronger in the smaller wavelength AGWs and in the longer wavelength ones is easier to reconstruct a wave signal. This is an important novel result as the model is able to generate perturbations and estimate for which parameters a detector might be able to distinguish AGW from the data. Another important result is that for shorter wavelengths the wave period is easier to measure and then use to characterized the wave; for the longer wavelengths only in the zenith the period is accurate. Finally, from the 25 km wavelength example in Figure 4.9, an amplitude maximum can be discerned around 60° and then it decays faster. This is again attributed to the cancellation effect that occurs beyond this point. The combination of all these features allows for an accurate characterization of the ideal AGW. By looking at the relative IVER, it is easy to tell the direction of the propagation of the wave (towards positive angles), the frequency and also get an estimate of the wavelength, depending on how clear the period is.

4.2.4. The Cancellation Factor

The observed amplitudes of the perturbed intensity $I'$ are largely dependent on the vertical wavelength of the waves. Because of the thickness of the emission layers, airglow and temperature perturbations induced by waves with small $\lambda_z$ would have a strong cancellation effect and consequently a smaller amplitude in $I'$. This cancellation effect can be measured by the ratio of the relative intensity amplitude of to the relative
temperature amplitude of the perturbing AGW. A Cancellation Factor (CF) can thus be defined for the airglow intensity as

\[
CF = \frac{\max(I'/\langle I \rangle)}{\max(T'_z/\langle T_z \rangle)}
\]  

(4.9)

where \( T_z \) is the temperature at a certain altitude and \( \langle \cdot \rangle \) means the background (unperturbed) quantity (Liu and Swenson 2003). Figure 4.10 shows the variation of cancellation factor for \( I' \) at each viewing angle for different wave parameters (4.12 (a): upward and rightward propagating wave, 4.10 (b: evanescent wave). As the viewing angle grows positively larger so does the cancellation factor; this is due to the amplitude ratio increasing as shown in Figure 4.9. Additionally Figure 4.12 shows how this CF also increases as \( \lambda z \) increases. The top values lie between 3 and 5 for \( \lambda z > 10 \) km for the propagating mode and it increases largely towards the extreme angles for the evanescent mode and large vertical wavelengths.
4.2.5. Model Resolution

It was described above that for an actual airglow imager system both FOV and $\Delta \theta$ are set. This will then limit the modeling resolution of the data. The above discussion proceeded having set in the model a uniformly spaced $\tilde{\theta}$, that is, a constant $\Delta \theta$. This resembles the prescribed conditions for a CCD imager. However, it is always possible to change the resolution in the model. For example, if instead of using a linearly spaced $\tilde{\theta}$ a linearly spaced $\tilde{x}$ is used to define the grid. Then $\tilde{\theta} = \tan^{-1}(\tilde{x}/z)$ where this time $z$ is a scalar. When $x$ sets the grid spacing, the resolution at large $\theta$s will be much higher, whereas when they’re small, resolution will be poor. Figure 4.11 shows these 2 resolution schemes, plus a hybrid one where the grid gets larger towards large $\theta$s in the same fashion the IVER curve does.
Figure 4.11. Different examples of resolutions. (a) $\theta$ sets the resolution, (b) $\xi$ sets the resolution and (c) is a hybrid resolution that uses a finer grid spacing at shorter wavelengths.

4.3. The 3D Extension and Imager Simulation

The previous description was an implementation of a 2D model that accounted for the volume emission rates $\varepsilon$ of a two-dimensional wave, where the integration path, or the observing path of an imager situated on the surface yields a one-dimensional plot of the intensity of the airglow emissions. This same algorithm can be extended to account for three-dimensional waves, with two horizontal wavenumbers and a vertical one. The effective integration of these waves will yield a resulting 2D image much like what is seen by imagers. However, an additional geometry will be introduced here that
accounts for the curving of the airglow along with the curved shape of the Earth; a spherical Earth will be taken as a sufficient approximation.

4.3.1. The Curved Airglow Geometry

An observer standing on the surface of the Earth that observes a point $P$ in the airglow layer that is curving with the radius of the Earth $R_E$ makes an angle with zenith $\theta$; this is shown in Figure 4.12. If the length of the airglow from the observer’s zenith is called $x_{arc}$, the distance from the observer to the point $P$ in the airglow is $\rho$, the airglow layer height is $z$ and if the zenith from the center of the Earth is aligned with the observer’s zenith, then there is a zenith angle $\alpha$ from the center of the Earth to the point $P$ and the geometry relations are:

\[
x = \rho \sin \theta \tag{4.10}
\]
\[
y = \rho \cos \theta + R_E \tag{4.11}
\]
\[
z = \sqrt{x^2 + y^2} - R_E = \sqrt{\rho^2 + 2R_E \rho \cos \theta + R_E^2 - R_E} \tag{4.12}
\]
\[
\rho = \sqrt{R_E^2 \cos^2 \theta + z^2 + 2R_E z - R_E \cos \theta} \tag{4.13}
\]
\[
\alpha = \cos^{-1} \left[ 1 - \frac{\rho^2 - z^2}{2R_E (R_E + z)} \right] \tag{4.14}
\]
\[
x_{arc} = (R_E + z) \alpha \tag{4.15}
\]
Figure 4.12. The geometry of the model. The airglow (red) is curved around the Earth (blue). The angle $\theta$ is the zenith angle and represents FOV/2 of an imager placed on the surface of the Earth.

At the zenith, $\theta = 0$, and $\rho = z$; at the horizon, $\theta = \pi/2$, and $\rho = z^2 + 2R_E z$; at the point on the other side of the earth, $\theta = \pi$, and $\rho = 2R_E + z$. The horizontal arc distance from zenith $x_{arc}$ is used to determine wave perturbations and we can now create a new artificial gravity wave that curves with the radius of the Earth and it is shown in Figure 4.13.
Figure 4.13. The shape of a (a) flat perturbed airglow and (b) a curved perturbed airglow (with the radius of the Earth).

4.3.2. The van Rhijn Effect

There’s another artificial enhancement in the IVERs due to geometry and that is the van Rhijn effect. It is an important relation that has to be accounted for in the simulations especially since there is significant enhancement from this, so much in fact that it has been used to determine the height of the airglow itself (Roach and Meinel 1955). Using the geometry derived in the previous section, the formula is

\[
V(z, \alpha) = \frac{1}{\sqrt{1 - \left(\frac{R}{R_e + z}\sin \alpha\right)^2}}
\]

This enhancement is shown in Figure 4.14. It’s important to mention that this has been properly subtracted from all the simulation results presented throughout this chapter.
4.3.3. Simulated Imager

The geometry relations (4.10–4.15) can be used to construct a two-dimensional image by using two zenith angles, $\theta$ and $\phi$, that each respectively determine, through the geometry transformations, a position in the plane of this 2D image. So, for a pixel at a location $(i,j)$ in the image, there is a correspondence with $(\theta, \phi)$ such that:

$$i = (R+z)\alpha(\theta)$$  \hspace{1cm} (4.17)

$$j = (R+z)\alpha(\phi)$$  \hspace{1cm} (4.18)

where the zenith angle $\alpha$ follows from (4.12–4.14). It is now clear that the extension to 3D (in $x$, $y$ and $z$) of the perturbation model can be solved by considering two-dimensional cases for each row of pixels, and then scanning through the other dimension. In this regard, the analysis for the 2D code still applies for this 3D model. An example of the results of the simulated IVER in an imager are shown in Figure 4.15.
**Figure 4.15.** A synthesized airglow imager showcasing two distinct waves with parameters defined in Table 4.1.

**Table 4.1.** Simulated wave parameters of the simulated gravity wave shown in Figure 4.17.
Chapter 5

Spectral Analysis and Image Processing Algorithm

As mentioned before, the ASI is an ideal instrument to observe and study gravity waves due to their large FOV. The prime data processing technique is spectral analysis, wherein there are many possible algorithms depending on the types of assumptions made on the observed data. Assuming the energy spectrum of the perturbations is mostly monochromatic, or a superposition of distinct specific modes, Fourier Analysis, which is based on the sinusoidal basis functions, can easily pick out the wave parameters; this has been the standard image processing procedure on imager data in numerous research (Garcia, Taylor et al. 1997, Criddle, Taylor et al. 2011, Criddle, Taylor et al. 2012, Mangognia, Swenson et al. 2012, Taylor and Garcia 2012). For the case of more complex types of waves, such as those that change amplitude and phase over time, other types of spectral analysis should be employed and most likely some sort of modeling. When these parameters are determined from the spectral analysis, a hypothesis can be made on the type of wave and possible sources; then, they can be assimilated into a simulation model so that a well-rounded description of the phenomena can be achieved (and even prove the hypothesis). In this research, a 2D Fourier Analysis on image data has been implemented as Mountain Waves with phase velocities close to zero.
5.1. The Fourier Transform & Spectral Analysis

The Fourier transform of a function $I$ in the spatial domain $(x, y)$ to the transformed function $\hat{I}$ in the frequency domain $(k, l)$ is:

$$\hat{I}(k, l) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(x, y) e^{-i(kx+ly)} dx dy$$  \hspace{1cm} (5.1)$$

with the corresponding inverse Fourier transform

$$I(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{I}(k, l) e^{i(kx+ly)} dk dl$$  \hspace{1cm} (5.2)$$

where the hat (\(^{\hat{}}\)) represents the Fourier transform (FT) of the function, $k = 2\pi f_x$ and $l = 2\pi f_y$ are the wavenumbers in $x$ and $y$ and $f_x$ and $f_y$ are the coordinates in frequency domain that correspond to $x$ and $y$ respectively. When the function $I$ has the form of a monochromatic wave, i.e. $I(x, y) = e^{i(k_0x + l_0y)}$, then the Fourier transform in spatial domain is simply the Dirac delta function $\hat{I} = 4\pi^2 \delta(k - k_0) \delta(l - l_0).$ This means that the delta function ‘picks out’ the frequency corresponding to the monochromatic wave. This is the fundamental purpose of the Fourier transform. When the function $I$ is described by a superposition of modes, i.e. $I(x, y) = \Sigma_i \Sigma_j e^{i(k_ix + l_jy)}$, then the Fourier transform is the spectrum of frequencies contained in $I$; if there are no predominant modes in $I$, then no single frequency will be picked out by the transform. The extreme case is when $I$ is constant, for which the Fourier transform is a superposition of Dirac deltas at every single possible frequency contained in $I$; this is called a continuum in frequencies.
the spatial domain is not infinite, such as data in an image that has a finite number of $N \times M$ pixels in size, then the FT and inverse FT are approximated by a discrete version:

\[
\hat{I}(k,l) = \sum_{j=1}^{M} \sum_{i=1}^{N} I(x,y)e^{-i(\frac{k}{N}x + \frac{l}{M}y)}
\]  

\[
I(x,y) = \frac{1}{MN} \sum_{j=1}^{M} \sum_{i=1}^{N} \hat{I}(k,l)e^{i(\frac{k}{N}x + \frac{l}{M}y)}
\]

In practice, these transforms are calculated using the Cooley–Tukey or Fast Fourier Transform algorithm (FFT) that takes advantage of the orthogonality of the exponential basis functions to reduce the standard $N^2$ multiplication in (5.3) & (5.4) to just $N \log N$ multiplications (Cooley and Tukey 1965). We can obtain the amplitude of the wave (for each wavenumber) by computing the complex magnitude of the FT, this is usually called a *periodogram*. Additionally, we can calculate from the complex $\hat{I}$ the phase angle, or phasor angle, which is exactly the wave phase. The wave’s wavelength can be obtained by the definition of wavenumber. Finally, if we use two consecutive images (in time) then we can find a phase difference between them and calculate a phase velocity. For Mountain Waves, these velocities should be very close to zero as the wave should be stationary. In reality, waves will drift back and forth to within $\pm 5$ m/s. In any case, the phase velocity becomes an important criteria for determining mountain wave events in ASI data. Table 5.1 presents a summary of the equations used to extract the wave parameters from the Fourier transform.
Table 5.1. Equations for obtaining wave parameters from Fourier transform (or FFT algorithm). The wave phase velocity requires two different consecutive images, where \( n \) is the temporal index.

\[
\begin{array}{lcl}
\text{wave parameters in real space} & \text{wave parameters from frequency space} \\
I(x, y) & \hat{I}(f_x, f_y) \\
\text{wavenumber} & K = \sqrt{k^2 + l^2} & k = 2\pi f_x \\
& l = 2\pi f_y \\
\text{wavelength} & \lambda = \left(f_x^2 + f_y^2\right)^{1/2} & \lambda_x = f_x^{-1} \\
& \lambda_y = f_y^{-1} \\
\text{wave phase} & \phi = \arg(\hat{I}) \\
\text{wave amplitude} & A = \abs{\hat{I}} \\
\text{wave phase velocity} & \frac{\Delta \phi}{\Delta t} = \frac{\phi_{n+1} - \phi_n}{t_{n+1} - t_n}
\end{array}
\]

5.1.1. Image Conditioning for Spectral Analysis

A usual practice for spectral analysis includes the use of filtering windows to better eliminate the noise around the sidelobes of the frequency peaks in a periodogram. These sidelobes are the result of considering continuous functions over a finite domain, so that discontinuities at the edges produce periodic signals in the FT. By smoothing the input signal, i.e. applying a window, at the edges of the domain these undesired ‘noise’ frequencies can be greatly reduced. There are many different types of windows for
different purposes and they have been extensively studied in (Harris, 1978) (Harris 1978). For this research a Gaussian window was chosen, but certainly other such as a Hanning or a Hamming window would suit just fine too. MATLAB provides an interactive tool that provides further insight as shown in Figure 5.1.

![Figure 5.1. An example of the window selection tool in MATLAB. This is shown here for a Gaussian window, along its corresponding FT with the unwanted sidelobes, to exemplify the basic properties of windowing. A user can use this tool to interactively create a suiting window.](image)

In addition to using windows for reducing undesired sidelobes, it’s also very important that the images contain sufficient energy content for each particular wave frequency. In other words, there needs to be enough wavelengths within the image: the more wavelengths within the image, the larger the frequency peak in the periodogram. However, by increasing the area of analysis in real data, the noise content also increases. In this sense, there has to be a manual discrimination of what would be the best amount of pixels to process. In fact, much of the data selection includes a manual selection of the best parameters and there are many factors to account for, such as resolution.
concerns, the expected wavelengths and orientations, the average intensity of the image and even the appearance of the moon. At the very basic, the FFT algorithm needs to be tested with a simulated image to prove its accuracy. An example of this validation is shown for the ASI simulation model of Chapter 3 in Figures 5.2–5.4 and Tables 5.2 & 5.3. The percentage errors are found to be very small.

Figure 5.2. Simulated ASI image using the model described in Chapter 3. Two distinct colormap schemes are provided for better visualization, a uniform grayscale colormap and a rainbow colormap.

Table 5.2. Simulated wave parameters for the two waves in Figure 5.2.
Figure 5.3. The image conditioning steps for the simulated ASI image of Figure 5.2. (a) The unwarped image, (b) a selected zone around zenith and (c) selected zone with a gaussian window.
Figure 5.4. Periodogram of the image in Figure 5.3c. The axes are the dimensionless wavenumber $k$ (horizontal) and $l$ (vertical). An efficient peak finder algorithm has been used to identify the frequency coordinates of each peak.

Table 5.3. Results of the FFT algorithm. The percentage error is the difference between the true parameters of Table 5.2 and the ones found in Figure 5.4. A multiple replications of this algorithm results in percentage errors less than 0.3%–0.8%.
Figure 5.5. Different tests for the simulated image 5.3a. The periodograms are in logarithmic scale, revealing the sidelobes in frequency space due to discontinuities at the edges of the finite spatial domain. There is a small crosshair at the center of the original images. This crosshair has the longitude of the found wavelength and the orientation; it is a visual that helps corroborate the accuracy of the results. The peak threshold is a parameter of the peak finding algorithm. The zonal and meridional coordinates represent the x and y respectively.
5.2. The Algorithm

In this section a detailed description of the processing algorithm for ASI images will be outlined in an effort to make it clear the amount of modifications necessary before obtaining wave parameters. A MATLAB suite has been created with all the necessary codes for each of the steps in the processing. Some of the steps require previous interactive determination of the parameters, such as the star calibration, star removal, moving average size and zone selection; other parts are automatically determined such as the best resolution for visualization and wave discrimination for spectral analysis. The structure of the code allows itself for easy readability and therefore making suitable for different users to implement. A clear hierarchy of “set parameter–perform step” is utilized to provide flexibility in changing the key parameters for each of the steps. There are also individual byproducts for each step, all of which are progressively discarded to save system memory, as a typical image can size up to 2GB of raw matrix data; however, the clear differentiation for each of the processing makes it easy to diagnose and tweak parameters. This code serves the double purpose of producing visually attractive images to catalogue and display in the ALO website\(^1\) and also extract wave parameters for scientific purposes. These two different motivations complement each other since climatology studies benefit from the added insight obtained for visually well-represented images; the accuracy of the image processing, along with its distribution over different resources such as the website, conference posters or publications allow other researchers to better compare data and collaborate. In this

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\(^1\) Andes Lidar Observatory – [http://lidar.erau.edu](http://lidar.erau.edu)
regard, great care has been put into both the accurate determination of wave parameters as well as the creation of a comprehensive resource for mountain waves for the ALO imager. A flow chart of this algorithm is presented in Appendix B.

5.2.1. Data Read

The original ASI data is a nightly collection of .FIT files for every night of observation. The Flexible Image Transport System (FITS) format is designed to contain essential information, called *extensions*, related to each individual image. The imager records in these extensions the intensity from the CCD and the observed date in UT time, among other technical parameters. MATLAB contains a built-in library for handling these files. Once the data has been read and scanned for errors or corrupt files, everything else is processed in standard MATLAB double-precision matrix variables.

5.2.2. Zenith alignment, Cropping and Star removal

The method for aligning the ASI images using the background starfield has been described in §3.3.1. This is carried out for every image and is always the first step to be done. The next part involves the cropping of the original image to a smaller FOV around the zenith. There are three main reasons to do this: first, since the resolution decreases at large angles the imager is not able to resolve the type of waves we’re interested in. Secondly, part of the mountain range blocks the sky at these large angles. Lastly, the less pixels there are within the image, the less the program takes to run for reasons explained in the following sections. Once the image is cropped, then it is
processed for star removal. This is an important step since stars become streaks of light when projected onto geographic coordinates. The star-removal algorithm employed is an efficient peak finder where each individual star represents an intensity spike several orders of magnitude higher than its surrounding, usually with a width no larger than 3 square pixels. Once the peaks are identified, a median filter in combination with a gaussian window is employed to remove the pixels. This process is shown in Figure 5.6. While this process is simple enough it is not perfect, as very bright stars or those in the vicinity of the Milky Way sometimes fail to be recognized. This does not affect in any significant way the subsequent analysis, for most of the stars are effectively removed.

![Figure 5.6](image_url). First steps in processing a sample ASI image: (a) the original image, (b) the aligned image with extra padding, (c) the cropped image to a smaller FOV, and (d) the image with stars removed. Image date: Jun 22 2011.
5.2.3. Image Unwarping and Flat Fielding

In a process analogous, but inverse, to the one outlined in §4.3.1, the geometry of the imager-airglow problem is employed to unwarp the original images. This process is in essence a projection of a pixel as seen by the ASI onto the geographic coordinates (in kilometers). For an efficient unwarping of the images, a scattered interpolant is created; this is a class type object in MATLAB that essentially maps a matrix whose coordinates are determined by a given spatial resolution, which might be constant, linear or nonlinear, and applies a transformation (i.e. linear interpolation) onto another spatial resolution. So, if the original matrix is in ASI coordinates, there is an accurate mapping (i.e. Equations (4.10–4.15)) that relates the nonlinear original coordinates (angular) into the linearly spaced geographic coordinates (in kilometers). The choice of using a builtin function for the unwarping allows for great flexibility in mapping any set of coordinates onto another as long as the relationships between the different coordinates are known.

However, before any unwarping is done, a consideration has to be made about the variation of the background intensity, which may be a function of the zenith/elevation angle. This includes the possibility of removing the van Rhijn effect (4.16); The van Rhijn effect is an enhancement dependent on the zenith/elevation angle and that’s why it needs to be accounted for before unwarping. This is an optional step as sometimes it is desired to visualize the raw intensity of the image especially for the purpose of creating high resolution videos. The accounting and removal of the intensity that does not come from the perturbation of the airglow by gravity waves or the background emissions is referred to as “flat-fielding”, where the main idea is remove intensity enhancements at large FOVs. Besides removing the van Rhijn effect, there are other
types of techniques that can be used for flat-fielding, such as fitting a 2nd or 3rd degree two-dimensional polynomial or the one employed in this research, which is the use of a *temporal moving average* (with equal weights) and will be described in the following section. Part of the flat-fielding process also includes the removal of large linear enhancements of intensity due to tides or Rossby waves, all of which can be accounted for by a simple linear detrending (fitting a plane).

![Figure 5.7](image)

**Figure 5.7.** The unwarping of the starless image: (a) without cropping to a smaller FOV, and (b) with cropping. The mapping to geographic coordinates reveals large structures in the airglow layer. The Andes are seen on the edges of the image.

### 5.2.4. Background Removal

The ASI imager signals are composed of emissions from many different sources exciting the airglow. It is important to try and quantify all of the emissions that are not due to the wave perturbations. So, if the signal $S$ from the airglow layer is composed by
where \( \langle I \rangle \) is the background intensity, \( \delta I \) the perturbation displacement by the gravity waves, \( N_{\text{sky}} \) the intensity (noise) from the night sky for sources other than airglow emissions and \( N_{\text{stars}} \) the intensity (noise) from stars, then the data \( D \) recorded by the imager is:

\[
D = V(i, j)S + N_{\text{elec}} = V(i, j)(\langle I \rangle + \delta I + N_{\text{sky}} + N_{\text{stars}}) + \eta_{\text{elec}}
\]  

(5.6)

where \( V(i, j) \) is a vignetting function in ASI coordinate \( i \) and \( j \) that is associated with flat-fielding and \( \eta_{\text{elec}} \) is the electron noise in the CCD. The data recorded by imager that is associated with background is:

\[
B = V(i, j)(N_{\text{sky}} + N_{\text{stars}}) + \eta_{\text{elec}}
\]  

(5.7)

Now, if we use a star removal algorithm then the set of Equations (5.5–5.7) become:

\[
S = \langle I \rangle + \delta I + N_{\text{sky}}
\]  

(5.8)

\[
D = V(i, j)(\langle I \rangle + \delta I + N_{\text{sky}}) + \eta_{\text{elec}}
\]  

(5.9)

\[
B = V(i, j)N_{\text{sky}} + \eta_{\text{elec}}
\]  

(5.10)
In an airglow imager the noise due to electrons is mostly DC (Garcia, Taylor et al. 1997), so it can be assumed to be constant. To extract the wave perturbations we can subtract the background from the data on the imager:

\[ C = D - B = V(i,j)(\langle I \rangle + \delta I) \]  \hspace{1cm} (5.11)

If we average this background-corrected image then the wave perturbations get filtered out (provided the average is larger than the period of the wave or wavelength for a temporal or spatial averaging respectively):

\[ \langle C \rangle = V(i,j)\langle I \rangle \] \hspace{1cm} (5.12)

And we can find the relative perturbations using (5.11) & (5.12) as:

\[ \frac{C}{\langle C \rangle} = \frac{\delta I + \langle I \rangle}{\langle I \rangle} \Rightarrow I' = \frac{C}{\langle C \rangle} - 1 \] \hspace{1cm} (5.13)

where we define the relative intensity perturbation as

\[ I' \equiv \frac{\delta I}{\langle I \rangle} \] \hspace{1cm} (5.14)

Equation (5.13) shows that if we are able to directly measure the background noise \( N_{\text{sky}} \), i.e. measurements at lines or bands other than the OH (Garcia, Taylor et al. 1997), then we can easily estimate the wave perturbations. If we don’t have this measurement, then
we must make estimate what the background noise is. Assume a 30% of the background intensity is due to sky noise (continuum emission, aerosol scattering, etc.) (Sternberg and Ingham 1972, Maihara, Iwamuro et al. 1993):

\[
D = V(i,j)(0.7\langle I \rangle + \delta I + 0.3\langle I \rangle) + N_{\text{elec}} \\
B = V(i,j)(0.3\langle I \rangle) + N_{\text{elec}} \\
C = D - B = V(i,j)(0.7\langle I \rangle + \delta I)
\]

(5.15)  
(5.16)  
(5.17)

At this point we can use any flat-fielding technique to eliminate the vignetting function, such as the ones shown in Figure 5.9:

\[
F = 0.7\langle I \rangle + \delta I
\]

(5.18)

and finally, using Equation (5.14):

\[
I' = \frac{F - 0.7\langle I \rangle}{0.7\langle I \rangle}
\]

(5.19)

With this equation we can now process the flat-fielded images to obtain the gravity waves in the airglow images. In the previous section, a moving average filter was used to remove the enhancing effects at large zenith angles and effectively flat-field the image. There is another more important use to this filter: background estimation. It is clear from Equation (5.19) that some can of averaging must be done. This can be accomplished by using a single image and averaging over the space around the wave.
structures; this is not convenient due to the presence of mountains and the milky way
and the variations in background intensity across the FOV. A more practical averaging
is the aforementioned temporal moving average. The moving average operation takes a
single pixel of the image throughout an entire night of data and smooths it out with a
given temporal window (filter) size. By setting different sizes we can even filter out any
waves whose periods are out of the window boundaries. Figure 5.8 shows two different
filters where mountain waves become apparent, as their stationary features with a
duration up to several hours.

![Two different temporal moving averages centered on 07/23/2011 04:01:58 UT, one with a 25-minute filter (left) and the other with a 90-minute filter (right). Mountain Waves come out as can last up to several hours in a single night.](image)

Once the perturbations are extracted with the filters, the final processed image is
a combination of cropping, unwarping, flat-fielding, averaging over time and detrending.
Figure 5.9 shows profiles that demonstrate flat-fielding, averaging and detrending.
Figure 5.9. Profiles of a horizontal slice through zenith of an airglow imager: (a) the original image, (b) a 25-minute moving average, (c) a 90-minute moving average, (d) a flat-fielded (background removed) image using the 25-minute filter, and (f) a flat-fielded (background removed) image using the 90-minute filter. The intensity $I$ is in arbitrary units. The peak at the center is the Milky Way. A 2nd order polynomial has been fitted to show what a polynomial fit to the angular enhancement would look like.
A very useful feature of the algorithm is the ability to filter out both Mountain Waves to enhance smaller-scale propagating waves (most of them with periods < 25 mins); then the filter itself becomes the extracted mountain waves, as seen in Figure 5.8. With this, a new filter (with 90+ mins window) can be used to estimate the background to be used for flat-fielding and background removal for the Mountain Waves. While Mountain Waves can last up to many hours in a night, there is a small phase shift, usually less than ± 5 m/s. This is why the waves don’t get completely removed by a 90 min filter and makes it a good estimator of the background emissions.

![Figure 5.10](image_url)

**Figure 5.10.** Two final processed images on 07/23/2011 04:01:58 UT, one enhancing propagating waves (left) and the enhancing Mountain Waves (right). A distinct custom colormap has been created to show all of the processed data.

### 5.2.4. Zone Selection and Spectral Analysis

The final processed images shown in Figure 5.10 are too large to be practical for spectral analysis. The main problem is that there are many waves with many energy
distributions across the image. Smaller parts of the sky make up for more reliable processing. The idea is to select zones within the larger FOV with clear wave structures, so that the energy content of certain waves is better represented in the Fourier transform. Figure 5.11 shows an arbitrarily chosen zone distribution; the choice was made as it is expected to find structures there throughout many nights of data.

![Figure 5.11. Four distinct zones have been extracted from the processed image.](image)

Once the zones have been selected, then the spectral analysis of §5.1 can be performed. Once the peaks have been determined, there is an additional masking done, to isolate the energy content related to that peak (Tang, Kamalabadi et al. 2014). Figure 5.12 shows the periodogram for one of the zones. As before, a crosshair is included as a visual validation of the found parameters; the length of the crosshair is the same as the wavelength and it is in the direction of the found orientation. The masking of the peaks appears on the bottom periodogram.
**Figure 5.12.** The periodogram of one of the zones: the periodogram in logarithmic scale (top) and the masked peaks with their appropriate energy content (bottom). A crosshair is overlain onto the original image, as a visual validation: the length of the crosshair equals the found wavelength and its alignment is with the found orientation.
Chapter 6

The Andes Lidar Observatory

Situated at an altitude of approximately 2800 ft, the Andes Lidar Observatory sits atop Cerro Pachón in the Chilean Andes (30S, 71W & GMT +5), the largest continental mountain range in the world. It is an NSF Upper Atmosphere Facility, a member of the Consortium of Resonance and Rayleigh Lidars. With one of the darkest skies, in part thanks to Chile’s legislation against excessive light pollution, it is a prime location for astronomy and aeronomy. The long mountain range, combined with strong easterly winds, makes the whole region an important source of momentum deposition for the upper atmosphere and has been extensively studied (Criddle, Taylor et al. 2011, Criddle, Taylor et al. 2012, Fritts, Janches et al. 2012, Mangognia, Swenson et al. 2012, Mangognia, Swenson et al. 2013, Pugmire and Taylor 2013, Pugmire, Taylor et al. 2013, Fritts, Pautet et al. 2014), with many studies using ALO data (Hecht, Walterscheid et al. 1994, Hecht, Walterscheid et al. 2011, Li, Liu et al. 2011, Lu, Liu et al. 2011, Taylor, Pautet et al. 2011, Vargas, Swenson et al. 2012, Cao, Liu et al. 2013, Li, Liu et al. 2014).
The data available to this research spans from 2009 to the present. However, not all of the years had great data and only data through 2013 was analyzed. The discrimination of good data for processing begins with the creation of Keograms. These are plots of pie lines than run through the middle of an image (zenith) along horizontal (zonal or East-West) and vertical (meridional or South-North). With the appropriate intensity scaling, these plots are really useful in determining the content of a night’s worth of data and to help determine if nights are cloudy, have predominantly propagating waves or stationary ones, or perhaps show if there are errors in the data. Stationary waves are easy to pick out from a Keogram, they appear as a clear horizontal line. Just as the images themselves, the Keograms can be plotted in ASI coordinates or unwarped linear ones. Figure 6.1 shows an example of a Keogram showcasing propagating waves in ASI coordinates. Figure 6.2 shows a Keogram with a clear standing wave signature in unwarped coordinates. Keograms are an excellent tool to probe a large database in search for Mountain Waves signatures. However, they fail to show any signature outside the pixels they sample, making them a limited tool for complete determination of MW parameters. In this research, a large repository of these Keograms were made and archived. Then, a manual survey across the entire Keogram database discriminated the corrupted and cloudy nights from the ones with clear wave signature and evidence of Mountain Waves, yielding candidates for processing using the Image Processing Algorithm of the previous section. The results from this survey for Mountain Waves are shown in Table 6.1.
Figure 6.1. Example of meridional and zonal Keograms in ASI coordinates. The curved lines are propagating waves across the FOV.

Figure 6.2. Example of meridional and zonal unwarped Keograms. The horizontal structures represent standing Mountain Waves.
6.1.1. Manual Survey of MW Parameters

A manual survey for the year 2011 was made for determining the wave parameters of Mountain Waves. This survey has served as a sort of statistical validation for the results that have been produced with the IPA and spectral analysis. Figure 6.3 contains the result of this survey. It shows that most of the waves tend to the appear at the start of the observation, around 18 LT (11 UT). Most of the wavelengths are within 20 and 30 kms while their duration can be anywhere from 25+ mins to up to 9 hours. Figure 6.4 shows the orientations and locations in the night sky. While MWs can be found localized all across the sky, they can frequently spread all across the FOV and even superimpose with other MW. The distribution of the large-scale MWs show an orientation that aligns with the mountain range. It is important that this data should be read with caution, as it is hard to define in a manual survey what constitutes a standing wave as there is no clear measurement of phase speeds. Also, scales can be subjective and these results might change somewhat if a different person takes the measurements.

<table>
<thead>
<tr>
<th>Year</th>
<th>Available Data</th>
<th>MW Signature from Keogram</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>45</td>
<td>2</td>
</tr>
<tr>
<td>2010</td>
<td>157</td>
<td>16</td>
</tr>
<tr>
<td>2011</td>
<td>180</td>
<td>55</td>
</tr>
<tr>
<td>2012</td>
<td>83</td>
<td>8</td>
</tr>
<tr>
<td>2013</td>
<td>79</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 6.1. Days of available ASI imager data at ALO. The MW signature days were determined by using Keograms.
from the ASI images. Still, it provides a great insight into the MW occurrence over the Andes Lidar Observatory.

**Figure 6.3.** Manual MW survey for 2011. Start times are in Local Time. Approximate uncertainties: wavelength: ± 2 km, duration: ± 4 min, start time: ± 4 min.
6.2. Using Wind Data

Understanding the wind resource at Cerro Pachón is an important part of the MW survey. As seen in Chapter 2, MWs only exist when strong background wind flows over topography, which averages around 5 km/s for the Andes. To get a sensible picture of the background wind at the location of the ALO, we must look at wind data. Low-level wind data from measurement typically comes radiosonde data. Unfortunately, there is no radiosonde station where ALO is. The closest available radiosonde station is at approximately 400 km to the South, the station SCSN Santo Domingo of the University of Wyoming’s Department of Atmospheric Science Upper Air Soundings. The location of this station is shown in Figure 6.5.
Figure 6.5. Location of the radiosonde station SCSN from the University of Wyoming’s Department of Atmospheric Science Upper Air Soundings, along with an elevation map. The location of the ALO is also highlighted in the map.

Figure 6.6 shows the wind measurements through 2011. There is a strong enhancement of the wind intensity from Jun-Sept (austral winter). The wind is mostly from the eastward as it is shown in the June-July monthly means to the right.

Figure 6.6. Radiosonde data for 2011 from station SCSN. The maximum wind value occurs in the months of Jun-August with a peak around 80 m/s at 13 km.
Figure 6.7. 3D visualization of the total horizontal wind for the SCSN data, July 23, 2011. The wind is mostly eastward reaching up to 80 m/s at a 15 km height. At 5 km the wind is already around 40 m/s.

Given the distance from the SCSN station to the ALO, an alternative low-level wind data repository was also researched. Figures 6.8 and 6.9 show the data from NCAR/NCEP Reanalysis data at the location of the ALO. Expanded information on this dataset can be found in the Appendix B. Just as with the SCSN station, we can see the strong winterly enhancement of the horizontal wind.
Figure 6.8. NCEP/NCAR horizontal wind data for 2011 at ALO (30°S, 71°W).

Figure 6.9. NCEP vertical wind data map for Jun-July 2011 mean.
Chapter 7

Results and Conclusions

The task of describing the upper atmosphere has increasingly become a multidisciplinary science and many of the most important scientific pursuits in the region require a collaboration between different observatories and institutions. Part of the results of this thesis is to catalogue and quantify the most important events so that it can be compared with other findings. The shared goal between all of the different research conducted in the area is to get a complete description of the dynamics of the atmosphere, which itself plays an important role in the global atmospheric circulation. This, of course, serves many deep and intricate purposes that go all the way from simple meteorological forecasting to a complete Global Circulation Model (GCM). This research plays a part in the understanding of the just how much orography influences the middle-upper atmosphere, and that is unique to this zone. That is precisely why we need other remote sensing data, such as wind and temperature, to be able to describe the dynamics over the ALO. It all begins with laying a foundation, which in this case has been done by many preceding this research. The new an important contribution of this research to such effort has been the creation of a simple but powerful framework dedicated to finding Mountain waves. It builds upon previous ideas (Liu and Swenson 2003, Li, Liu et al. 2011, Tang, Kamalabadi et al. 2014) and expands unto a coherent
single-platform program that is able to process images, extract propagating gravity waves and stationary gravity waves and also do some statistical analysis. In addition, a linear theory model has been built to accompany the description of these events.

The development of this thesis has been shown in multiple conferences on atmospheric sciences. The airglow simulation algorithm was shown in a poster in the NSF sponsored Coupling, Energetics and Dynamics of Atmospheric Regions (CEDAR) 2013 workshop. In the CEDAR 2014 workshop the basics of the spectral analysis, along with a manual Mountain wave survey, were presented in a poster. In the American Geophysical Union (AGU) 2014 Fall Meeting a more advance algorithm, along with a statistical survey and correlation with wind data was presented in a poster.

7.1. Preliminary Survey and Statistics

The final form of the program described in Chapter 5 has been recently completed and therefore more powerful statistical tools can be developed. However, with basic statistical analysis capabilities already developed and along with earlier manual surveys have yielded a clear picture of the relationship between wind, seasonality and Mountain Wave occurrence, a sort of preview climatology. Figure 7.1 shows what is a typical distribution of wavelengths for a Mountain Wave event during ‘high’ season (ustral winter). It is clear from the histogram that there is a strong signal that is intermittent and centered on ~35 and ~37 km. While the program is able to produce these results, there is still a necessity of providing the most appropriate statistical interpretation to these results. For example, the two distinct peaks in wavelengths can either be attributed to: two different waves, or a single intermittent wave centered at a mean wavelength. To be able to correctly identify this, there needs to be a sensible
correlation to the other found parameters, that is, orientation and phase speed. Additionally, this result from one of the zones may be correlated to a result from other zones, since the types of Mountain Waves we observe have a sufficient large scale to be observed across the entire FOV.

Figure 7.1. The distribution of wavelengths for an individual zone throughout a night. There are two clear peaks in the histogram, one for a wavelength centered at ~35 km and the other at ~38 km.

Another statistical tool that was implemented was fitting a distribution function to the parameters, in this case the found wavelength and using a Weibull distribution. Figure 7.2 shows an example for this and includes a more detailed statistical analysis, that includes using the standard deviation of the fitted distribution as a filtering value. Then the occurrence of waves with corresponding parameters are plotted in time, along with their orientation. If the orientation is roughly the same, then we can conclude it is the same wave event.
Figure 7.2. Fitting a Weibull distribution function to the found wavelengths, for two different zones in the same night. The confidence interval is established by using the standard deviation, where all the occurrences within this standard deviation, where the orientations of the wave are roughly the same, are considered to be the same event.
For a comprehensive analysis of the data collected throughout the years, we considered only the result with higher frequency for each parameter, that is, the mode of a nightly histogram. Also, waves with phase speeds higher than 5 m/s are neglected. These results are shown in Figures 7.3.–7.7. The green dashed lines represent days where data was available but no Mountain Wave was detected. The colored distributions are actually histograms for the wave with largest energy content. This means that there could’ve actually been more than one wave in the data, but only one has been plotted. Another important caveat to this results is that there is no information about the duration of the events.

![Figure 7.3. Wave parameters histograms for the year 2009.](image-url)
Chapter 7 - Results and Conclusions

Figure 7.4. Wave parameters histograms for the year 2010.

Figure 7.5. Wave parameters histograms for the year 2011.
Figure 7.6. Wave parameters histograms for the year 2012.

Figure 7.7. Wave parameters histograms for the year 2013.
The most immediate correlation that can be made is with the wind data. Figures 7.8–7.12 where created by overlaying the yearly histograms onto the wind data found in Appendix B. One important result from doing this is the possibility of there being a connection to the appearance of Mountain Waves when there is either (or both): a strong positive wind single below 5 km (approximately the mean Andes mountain height), or a strong positive wind at the heights altitudes. There seems to be a suggestion that there is a relation between them and this initial findings lay such hypothesis.

**Figure 7.8.** Histogram of 2009 data of Figure 7.3 on top of NCEP data B.1.

**Figure 7.9.** Histogram of 2010 data of Figure 7.4 on top of NCEP data B.2.
Figure 7.10. Histogram of 2011 data of Figure 7.5 on top of NCEP data B.3.

Figure 7.11. Histogram of 2012 data of Figure 7.6 on top of NCEP data B.4.

Figure 7.12. Histogram of 2013 data of Figure 7.7 on top of NCEP data B.5.
7.2. Future Work

One of the main goals to be completed in the future is an expanded, detailed statistical analysis on the findings; this includes a reasonable error estimation for all the processing that has been done. In this regard, the code should be expanded to handle every step of the processing, including the reading and diagnosis of data to the creation of a Keogram and video catalog, and finally producing statistical results for all of the findings. Part of the statistical description of the Mountain Wave events requires the assimilation of wind data, which has been explained in Chapter 2 is the most important parameter for the creation of Mountain waves. Low-level wind data (radiosonde) is available from near stations as seen in Chapter 6, while wind data at the airglow height is available from meteor radars; for the space in between there are no available instruments, so data from the Horizontal Wind Model (Drob, Emmert et al. 2008, Emmert, Drob et al. 2008) can be used.

The previous is in regard to enhancements to the program itself. There are also other scientific pursuits in plan that will use the capabilities of this framework. One of them is an in-depth look at a particularly strong case of a Mountain wave, such as the one shown in Figure 7.13. The amplitudes along this wave feature are significantly strong and deserve a deeper analysis, one that includes predicting under what kind of conditions it originated. Such study should also provide an estimation of the momentum flux and energy transfer that can be expected from these kind of events. Some of these ideas will require collaboration to obtain temperature data and even other imager information. Another possible project will be include the analysis of other airglow layers, especially the Na and O5 emissions, for which there is data available from airglow
imagers and Na Lidars. These multiple-layer study will help determine with certain accuracy the vertical characteristics of the waves.

**Figure 7.13.** Details of a strong mountain wave feature with perturbation intensity up to 30\% of the mean (spatial) intensity. A cut through vertical pixel 90 is plotted to show the distinct profile of the perturbation.
Appendix A

Flow Charts

A.1. Airglow perturbation model

A general flow chart of the simplified model is shown in Figure A.1. It is a simple forward process that doesn’t include any of the more complex cylindrical plots or any interpolation at all shown in Chapter 4. This is all that is needed to produce an IVER in polar coordinates.

![Flow Chart](image)

**Figure A.1.** The simplified flowchart of the 2D airglow perturbation model.
A.2. Image Processing Algorithm

The Image Processing Algorithm IPA is capable of reading ASI data and process it with several manually set parameters, while others are done automatically. Processes that use manually set parameters include: aligning, cropping, moving average filters and zone selection; all other processes are set to work with optimal parameters. The simplified flowchart is given in Figure A.2.

![Flowchart of IPA and spectral analysis algorithm](image)

**Figure A.2.** The simplified flowchart of the IPA and spectral analysis algorithm.
Appendix B

NCEP/NCAR Wind Data

B.1. Yearly plots for vertical wind perturbations

The NCEP/NCAR wind data is a modeled based on reanalysis, that is, it assimilates past measurements to update the model. It contains data from 1948 to present with daily or 4 times daily availability. Among the quantities available are air temperature, Geopotential height, horizontal and vertical winds, all referenced to pressure levels. The data can be found at its site part of NOASS website: <http://www.esrl.noaa.gov/psd/data/gridded/data.ncep.reanalysis.html>. Here presented are the horizontal and vertical winds for the approximate location of the ALO (30°S, 71°W). A uniform colormap for negative (blue) and positive (red) has been used for better differentiation of the directionality of the wind, which is a very important trait for Mountain Waves, as easterly winds that flow over the Andes are the usual generators of these waves.
Figure B.1. NCEP/NCAR wind data for the year 2009 at ALO (30°S, 71°W).

Figure B.2. NCEP/NCAR wind data for the year 2010 at ALO (30°S, 71°W).
Figure B.3. NCEP/NCAR wind data for the year 2011 at ALO (30°S, 71°W). (Top plot same as Figure 6.8)

Figure B.4. NCEP/NCAR wind data for the year 2012 at ALO (30°S, 71°W).
Figure B.5. NCEP/NCAR wind data for the year 2013 at ALO (30°S, 71°W).

Figure B.6. NCEP/NCAR wind data for the year 2014 at ALO (30°S, 71°W).
B.2. Wind Maps for the Austral Winter Months

Here presented are maps of the wind distribution at different heights for the winter months, when we expect to see more Mountain Wave events. The location of the ALO can be seen as a cross plotted on the maps. The mean plots for June-July are especially interesting since they show this strong constant eastward horizontal wind.

Figure B.7. NCEP/NCAR horizontal wind data map for July 23, 2011.
Figure B.8. NCEP/NCAR vertical wind data map for Jun-July 2011 mean. (Same as Figure 6.9)

Figure B.9. NCEP/NCAR vertical wind data map for July 23, 2011.
Figure B.10. NCEP/NCAR vertical wind data map for Jun-July 2011 mean.


Cao, B., A. Liu and Z. Li (2013). Gravity wave Duration and Intermittency Observed by Airglow Imagers at Maui and Andes. AGU Fall Meeting Abstracts.


Li, Z., A. Liu, S. Franke, G. Swenson and X. Lu (2011). Gravity wave characteristics observed with airglow imager at the Andes Lidar Observatory. AGU Fall Meeting Abstracts.


Vargas, F., G. Swenson and A. Liu (2012). The observation of a circular gravity wave in the OH and O (1S) airglow emissions at the Andes Lidar Observatory, Chile. AGU Fall Meeting Abstracts.

