Case Studies of IMF Driven Auroral Emissions

Ryan James Soldin
Embry-Riddle Aeronautical University - Daytona Beach

Follow this and additional works at: http://commons.erau.edu/db-theses
Part of the The Sun and the Solar System Commons

Scholarly Commons Citation

This thesis is brought to you for free and open access by Embry-Riddle Aeronautical University – Daytona Beach at ERAU Scholarly Commons. It has been accepted for inclusion in the Theses - Daytona Beach collection by an authorized administrator of ERAU Scholarly Commons. For more information, please contact commons@erau.edu.
Case Studies of IMF Driven Auroral Emissions

By
Ryan James Soldin

A thesis submitted to the Physical Sciences Department
In Partial Fulfillment of the Requirements of
Master of Science in Engineering Physics

Embry-Riddle Aeronautical University
Daytona Beach, FL 32114
Fall 2009
The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleed-through, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.
Copyright by Ryan James Soldin 2009

All Rights Reserved
Case Studies of IMF Driven Auroral Emissions

A thesis submitted to the Physical Science Department
in partial fulfillment of the requirements for the
Degree of
Master of Science in Engineering Physics
by
Ryan James Soldin

Thesis Committee:

Dr. Irfan Azeem, Chair

Dr. Bereket Berhane, Member

Dr. John Hughes, Member

Dr. Katariina Nykyri, Member

Dr. P. Erdman, Program Coordinator, MSEP

Dr. J. I. Olivero, Department Chair

Dr. J. Cunningham, Associate Provost
Acknowledgements

I could not have written this thesis or made it to a point in my life where I had the opportunity to write this without the help of many different people. I would like to first express my thanks to those who directly contributed to the writing of this thesis. First and foremost to my advisor, Dr. Azeem, for guiding me along this whole process and for taking me to the CEDAR conference in Santa Fe which was an amazing experience. I would also like to thank him for staying on as my advisor through his change of employment from Embry-Riddle to ASTRA LLC. I am appreciative to Dr. Sivjee for providing much of the SPS data and for brutalizing me in his classical and quantum mechanics classes. Dr. McEwen from the University of Saskatchewan also helped in providing the data. I would also like to express my gratitude to the thesis committee, Dr. Berhane, Dr. Hughes, and Dr. Nykyri, for their thoughtful input and reading the thesis despite my Yoda-like writing style.

I am grateful for the support of my parents, who continue to put up with my antics even to this day. My life would also not be the same if it were not for my brothers, Steve, Nick, and Jeff, who have always been my partners in crime and always will be. To Evan and John I thank you for providing me with a safe haven in GVille whenever I needed it. My cousin Andrew I can not thank you enough for providing me with mental stability over years. My close friends back in RFD, Jason and Kris, you guys might as well be family to me, a person could not ask for better or closer friends. Jim and Kelly, I do not know what to even say, other than the Barrington was a blast
and I will always miss having you two around everyday. For all those mentioned and for those not mentioned thank you.

December ’09

R. J. Soldin
Abstract

The topic of solar wind - magnetosphere - ionosphere coupling has become increasingly important in recent years, as it deals with the energy transfer from the Sun to the Earth. The solar wind plasma has direct entry into Earth’s ionosphere at the polar cusp. At the cusp Earth’s magnetic fields lines are open and connect directly to the magnetic field lines of the solar wind. The energy from the solar wind particles precipitating into the ionosphere are dissipated as the aurora. The purpose of this thesis was to present case studies of the coupled relationship between the periodic fluctuations in the interplanetary magnetic field and the auroral emissions of southern hemisphere dayside polar cusp. The specific emissions used are the 6300 Å and 8446 Å, as these are due to direct electron impact. Data was obtained via a meridian scanning photometer, CCD spectrograph, and NASA’s WIND & ACE satellites. The wavelet power spectrums of the auroral emissions and satellite data were compared to identify events of interest. The observed time lags from the power spectrums were then compared to the calculated time lags. There were 6 events on 4 days, May 6 2008, May 10 2007, August 14 2006, and August 15 2006. Location relative to the ecliptic plane appeared to be of importance. For all events the satellites were never further than 13 $R_E$ from the ecliptic plane. The observed time lag can be reasonably explained by the physical travel time it takes the solar wind plasma to reach the polar cusp ionosphere. The confidence levels on the power spectrums ranged from 60% to as high as 99.98%.
# Contents

Acknowledgments v  
Abstract vi  
List of Figures ix  

## 1 Introduction 1  
1.1 Solar Wind Magnetosphere Ionosphere 1  
1.2 The Aurora 5  
1.2.1 $O(^1D)$ 6300 Å Emission 6  
1.2.2 $O(^3P)$ 8446 Å Emission 8  
1.3 Previous Studies 9  
1.4 Thesis Statement 10  

## 2 Data Description 11  
2.1 Meridian Scanning Photometer 15  
2.2 CCD Spectrograph 15  
2.3 Satellites: WIND & ACE 17
3 Methodology

3.1 Wavelet Spectrum Analysis .................................. 24
3.2 Time Lag Determination ........................................ 32

4 Results

4.1 Case Study 1: May 6 2008 ..................................... 38
4.2 Case Study 2: May 10 2007 ..................................... 39
4.3 Case Study 3: August 14 2006 .................................... 41
4.4 Case Study 4: August 15 2006 .................................... 42
4.5 Discussion of Results .............................................. 43

5 Concluding Remarks

A MATLAB Code

A.1 Wavelet Code .................................................. 63
A.2 Time Lag .......................................................... 86
A.3 Support Functions ............................................... 91
## List of Figures

1.1 Solar Corona Solar Wind [Walker, 2001] ................................................. 3
1.2 Earth's Bow Shock & Magnetosphere [Chaisson, 2003] ......................... 4

2.1 Nasa's ACE spacecraft, the triaxial fluxgate magnetometers are located at the end of the booms depicted in the diagram and are labeled MAG. WIND's magnetometer is identical to one on ACE [Smith et al., 2001]. 12
2.2 The southern hemisphere is shown above with the center being the SPS. The typical location of polar cap aurora is shown relative to the SPS. Diagram was obtained through private communication with Dr. Irfan Azeem, ERAU, originating from Dr. D. J. McEwen, University of Saskatchewan. ............................................................... 14
2.3 MSP auroral observations for May 10, 2007. The 6300 Å line is shown in red ................................................................. 16
2.4 CCD spectrograph auroral observations for May 10, 2007, full image of all wavelengths with false color. The color bar displays brightness in Rayleighs. ................. 18
2.5 CCD spectrograph auroral observations for May 10, 2007, only showing the 8446 Å emission 20
2.6 Polar cusp auroral spectrum from the CCD spectrograph on May 10 2007. 20
2.7 L1 lissajous orbit for ACE & SOHO. SOHO was not used in this thesis, WIND was used. The diagram is used to depict the orbit [Roberts, 2002]. 22
2.8 Geocentric Solar Magnetospheric Coordinate System [Frank, 1970]. 23
3.1 Y-component of the magnetic field strength at ACE, original signal before the WFT or Wavelet transform. 26
3.2 Morlet Localization in time and frequency 28
3.3 Windowed Fourier Transform (WFT) of ACE IMF By data for May 10 2007 (Top) Wavelet Power Spectrum of ACE IMF By data for May 10 2007 (Bottom). 33
4.1 Solar wind parameters observed by WIND on May 6 2008. 49
4.2 Wavelet power spectrums for the MSP (top) and WIND IMF Bxy (bottom) on May 6 2008. There is a 12 minute period oscillation at 11.50 UT in WIND and a 15 minute oscillation at 12.25 UT (hours) in the MSP. 50
4.3 Wavelet power spectrums for the MSP (top) and WIND IMF Bxy (bottom) on May 6 2008. There is a 6 minute period oscillation at 15.05 UT in WIND and a 7 minute oscillation at 16.10 UT (hours) in the MSP. 51
4.4 Solar wind parameters observed by ACE on May 10 2007.

4.5 Wavelet power spectrums for the MSP (top), CCD spectrograph (middle), and ACE IMF By (bottom) on May 10 2007. ACE shows an 11 minute period oscillation at 16.30 UT (hours), followed by a 10 minute and 12 minute period oscillation in the MSP at 17.10 UT (hours) and in the CCD spectrograph at 17.30 UT (hours), respectively.

4.6 Solar wind parameters observed by WIND on August 14 2006.

4.7 Wavelet power spectrums for the MSP (top), CCD spectrograph (middle), and WIND IMF Bx (bottom) on August 14 2006. WIND shows a 13 minute period oscillation at 14.45 UT (hours), followed by 12 minute period oscillations in both the MSP at 15.85 UT (hours) and in the CCD spectrograph at 15.85 UT (hours), respectively.

4.8 Wavelet power spectrums for the MSP (top), CCD spectrograph (middle), and WIND IMF Bx (bottom) on August 14 2006. WIND shows an 8 minute period oscillation at 16.80 UT (hours), followed by a 12 minute period oscillation in the CCD spectrograph at 15.85 UT (hours).

4.9 Solar wind parameters observed by WIND on August 15 2006.

4.10 Wavelet power spectrums for the MSP (top), CCD spectrograph (middle), and WIND IMF Bx (bottom) on August 15 2006. WIND shows an 13 minute period oscillation at 17.75 UT (hours), followed by a 11 minute period oscillations in the MSP at 19.25 UT (hours) and in the CCD spectrograph at 19.45 UT (hours), respectively.
4.11 Satellite locations for WIND and ACE on the dates of the case studies presented. All locations are within 13 $R_E$ of the ecliptic plane.
Chapter 1

Introduction

1.1 Solar Wind - Magnetosphere - Ionosphere

The auroral emissions of the upper atmosphere are driven by the solar wind. The aurora provides an ideal location to study the solar wind - magnetosphere - ionosphere coupling process. This coupling process has become an area of intense research as it deals with how and where energy from the Sun is dissipated in regards to Earth’s upper atmosphere. To begin understanding this process it is first necessary to know what the solar wind, magnetosphere, and ionosphere are. The solar wind is plasma. Plasma is created when enough energy is present to strip the outer electrons away from the host atom/molecule. This is referred to as ionization. Once ionized, a fluid of roughly equal numbers of positively and negatively charged particles exists. As a whole the plasma is neutral in charge, or nearly neutral in charge. With freely moving charged particles, plasmas respond to electric and magnetic fields. This can lead to
unexpected and non-intuitive behavior.

The solar wind plasma originates from the Sun and is composed of mainly protons and electrons. The largest component is ionized hydrogen. At Earth, about 1 AU (148 598 000 km), the solar wind has a typical speed of ~400 km/s, density of 5 cm$^{-3}$, and magnetic field magnitude of ~5 nT [Russel, 2001]. The solar wind originates in the upper atmosphere of the Sun, specifically in the layer known as the corona. The corona resides above the photosphere, the visible "surface", and chromosphere. The temperature of the corona is on the order of 10$^6$ K. This large of a temperature will give roughly 50% of the electrons the necessary energy to escape the gravity of the Sun, while only 1% of the protons will have the necessary energy. In order for Sun to maintain neutrality of charge, the outflow of electrons creates an electric field that accelerates the protons outward as well [Parks, 2004]. This expanding corona is the solar wind and encompasses the interplanetary space of the solar system. Figure 1.1 shows the expanding corona which is only visible during solar eclipses. The solar wind velocity is supersonic and creates a bow shock around the Earth. Even though the solar wind is considered a collisionless plasma, the shock is communicated via magnetic and electric fields.

The solar wind carries with it a "frozen-in" magnetic field. The term "frozen-in" means that the magnetic field travels along with, and at the same speed as the solar wind plasma. This magnetic field is referred to as the interplanetary magnetic field (IMF). The direction of the IMF is broken down into cartesian coordinates. The X-direction is towards the Sun, the Z-direction is North in regards to Earth, and Y is perpendicular to both of these while pointing towards the dusk side of the Earth.
Chapter 1: Introduction

The coordinate systems used in this thesis will be discussed in more detail in later chapters.

It has been well known for many years that the Earth has an inherent magnetic field. All the details of the generation of Earth's magnetic field are not completely understood, but it is currently described by dynamo theory. At low altitudes the Earth's magnetic field can be approximated as a tilted dipole. As the altitude increases Earth's magnetic field becomes compressed on the dayside, stretched on the nightside, and generally deformed away from being a dipole field through its interaction with the IMF. The area contained within Earth's magnetic field is referred to as the magnetosphere. The major components of the magnetosphere are all shown in Figure 1.2. The first boundary the solar wind plasma encounters is the bow shock.
The bow shock is a result of the solar wind traveling at supersonic velocity, and despite the fact that the solar wind is considered collisionless the shock is communicated through electromagnetic forces. The second boundary that will be encountered is the magnetopause. The magnetopause is the edge of Earth's magnetic field, separating what is inside Earth's field and what is outside. Also shown in the diagram is the location of the polar cusps. The southern polar cusp was focused on in this thesis.

The atmosphere of Earth at low altitudes (< 60 km) is composed primarily of neutral particles, but starting at altitudes of around 60 km and above exists the ionosphere. The ionosphere is characterized by the stable existence of freely moving electrons and ions (plasma). The ionization of the upper atmosphere is a result
of solar ultraviolet rays and energetic particles impacting neutral particles and the
density of the particles being low enough that collisions are infrequent, allowing the
ions and electrons to exist for considerable length before recombination occurs. The
ionosphere consists of layers D, E, F₁, and F₂ with altitude ranging from 60-90 km,
105-160 km, 160-180 km, and 200-400 km, respectively.

Solar wind, ring current, and geomagnetic tail particles are all able to enter to
the ionosphere and interact. This interaction between the energetic particles and the
ionosphere results in the optical emissions known as the aurora.

1.2 The Aurora

The aurora is a direct result of the IMF Magnetosphere Ionosphere coupling
process. It results from the injection of energetic particles that originate directly
from the solar wind or from various locations in magnetosphere. The polar cusp,
shown in Figure 1.2, is the only location where the solar wind plasma has direct
entry into the ionosphere. Earth's magnetic field lines are open, connecting directly
to the IMF. This allows the solar wind plasma direct entry into the upper ionosphere,
resulting in the polar cusp aurora. This thesis will specifically focus on the southern
dayside polar cusp aurora.

There are two prominent emissions in the aurora, the 5577 Å (green line) and
the 6300 Å (red line). The green line is the brighter of the two the emission and is
favored on the night side aurora where energetic particles tend to originate from the
geomagnetic tail. The red line is favored on the dayside aurora, where the particles
causing the aurora originate from the dayside magnetopause and direct entry of the solar wind. Typical auroral emissions are O(\(^1\)D) 6300 Å, O(\(^1\)S) 5577 Å, O(\(^3\)P) 8446 Å, O(\(^5\)P) 7774 Å, O\(^+\) 7990 Å with brightness 1-10 kR, 0.1-1 kR, 0.2-3 kR, 0.02-0.8 kR, 0.02-0.2 kR, and 0.015-0.4 kR, respectively [Sandholt et al., 2002; Christensen et al., 1983; Sivjee et al., 1984, Smith et al., 1982]. This thesis will focus specifically on the O(\(^1\)D) 6300 Å and O(\(^3\)P) 8446 Å. These emissions are dominated by direct electron impact [Christensen et al., 1978, Solomon, 1988].

1.2.1 O(\(^1\)D) 6300 Å Emission

The 6300 Å emission is emitted when excited atomic oxygen, O(\(^1\)D), returns to its ground state of O(\(^3\)P). O(\(^3\)P) is actually a triplet state of atomic oxygen and results in three different emissions at 6300, 6364, and 6392 Å. The 6364 and 6392 Å are very faint, as the emission of O(\(^1\)D) is dominated by the 6300 Å. The emission is also considered to be "forbidden" because electric dipole transitions give this emission a very low probability of occurrence. However, electric quadrupole and magnetic dipole transitions do lead to possible emissions from this state. This "forbidden" emission has a relatively low excitation energy above its ground state, 1.96 eV (via \(E = h\nu\)). Due to the fact that it is not a favored electric dipole transition it has a statistically long lifetime of around 110 s. This long lifetime ensures that the emission will peak above ~200 km. Below this altitude significant quenching of the atomic oxygen will occur with neutral components of the atmosphere.

The production mechanisms for the O(\(^1\)D) have been summarized in Solomon [1988] and are listed in Equations 1.1 to 1.9.
The production mechanisms of $O(^1D)$ are dominated by Equation 1.1, which is direct electron impact on atomic oxygen [Solomon, 1988]. For the dayside southern polar cusp the electrons that directly impact the atomic oxygen have their origins in the solar wind plasma that is directly deposited here. In Solomon [1988] the loss mechanisms of $O(^1D)$ are also summarized, given in Equations 1.10 to 1.16.
Chapter 1: Introduction

\[ O(1D) + O(3P) \rightarrow O(3P) + O(3P) \]  \hspace{1cm} (1.12)

\[ O(1D) + e \rightarrow O(3P) + e \]  \hspace{1cm} (1.13)

\[ O(1D) \rightarrow O(3P) + h\nu \ [6300 \ \text{Å}] \]  \hspace{1cm} (1.14)

\[ O(1D) \rightarrow O(3P) + h\nu \ [6364 \ \text{Å}] \]  \hspace{1cm} (1.15)

\[ O(1D) \rightarrow O(3P) + h\nu \ [6392 \ \text{Å}] \]  \hspace{1cm} (1.16)

The loss mechanism resulting in the 6300 Å emission is given in Equation 1.14.

1.2.2 \(O(3P)\) 8446 Å Emission

The 8446 Å emission is a result of the excited atomic oxygen, \(O(3P)\), returning to its ground state, \(O(3S^0)\). Unlike the 6300 Å emission the 8446 Å emission is an electric dipole allowed transition. It also has a very short statistical lifetime of \(10^{-6}\) to \(10^{-7}\) s. The excitation energy is 1.46 eV above the ground state. It is widely accepted that this emission is also due to direct electron impact on atomic oxygen and dissociative excitation of \(O_2\) [Christensen et al., 1978]. Observational and modeling work done by Christensen et al. [1978] has shown less than 5% of the total emission rate of 8446 Å to be from dissociative excitation of \(O_2\). The major production of the 8446 Å emission is given by Equations 1.17 and 1.18.

\[ O(3S^0) + e \rightarrow O(3P) + e \]  \hspace{1cm} (1.17)

\[ O(3P) \rightarrow O(3S^0) + h\nu \ [8446 \ \text{Å}] \]  \hspace{1cm} (1.18)
Chapter 1: Introduction

With the 8446 Å emission being an allowed transition its production and destruction is not nearly as complicated as the "forbidden" 6300 Å emission.

1.3 Previous Studies

The aurora is the direct result of the solar wind magnetosphere ionosphere coupling process. The energy from the solar wind, whether it has direct entry or traverses different parts of the magnetosphere, is the driver of this phenomenon. The night side aurora is driven from particles entering the ionosphere from inside of the magnetosphere. The polar cusp dayside aurora is driven by the direct entry of solar wind plasma into the ionosphere. The cusp is characterized by having open magnetic field lines that map down onto the dayside magnetopause. As the solar wind particles precipitate to ionospheric heights their energy is absorbed and one form of release is the aurora. Many recent studies have previously been published on the driving force of the IMF to affect the auroral outputs, with emphasis on Alfven waves. Alfven waves have been shown to power the aurora [Pekka et al., 2006], generate polar cap patches [Prkryl et al., 1999], and create pulsed ionospheric flows [Prkryl et al., 2002]. This thesis does not focus on the Alfvénic fluctuations but rather just short lived fluctuations in the IMF. The polar cusp aurora have been linked to the IMF and solar wind plasma conditions [Sandholt et al., 1998; Sandholt et al., 2003].
1.4 Thesis Statement

This thesis will present 4 case studies of Solar Wind - Magnetosphere - Ionosphere coupling. Specifically the coupled relationship that exists between the fluctuations in the IMF and the southern hemisphere dayside polar cusp auroral emissions. Short term fluctuations (5 to 30 minutes) in any one or combination of IMF components will be linked to the southern hemisphere dayside polar cusp auroral emissions, 6300 Å and 8446 Å, of the same period through the use of wavelet transform to analyze power spectrums and theoretical time lag determinations. It is hypothesized that the IMF fluctuation will modulate the electrons in the solar wind plasma, which has direct entry at the polar cusp. These electrons, which have direct entry at the polar cusp, will precipitate down to the $F$ region of the ionosphere. Here it will excite the metastable atomic oxygen, that will radiate the excess energy. The emissions produced from the atomic oxygen are the 6300 Å and 8446 Å emissions because they are dominated by direct electron impact on atomic oxygen. The idea being tested here is that if the electrons driving the emissions are modulated at some period, then the emissions that result from them will show the same modulation.
Chapter 2

Data Description

In order to be able to study the coupled relationship between the solar wind and the southern hemisphere dayside polar cusp aurora, data needs to be collected from interplanetary space outside Earth’s bow shock and from South Pole. The solar wind data used in this thesis are comprised of \textit{in situ} measurements from the satellites Global Geospace Science WIND and ACE (Advanced Composition Explorer). The \textit{in situ} measurements being used from WIND and ACE are the position, magnetic field vector measurements, and upstream dynamic solar wind pressure. Magnetic field vector measurements are made with the satellite’s magnetometers. The magnetometers of both satellites are identical, with ACE’s magnetometer being the flight spare for WIND [Smith et al., 1998]. There are dual magnetometers mounted at the end of 12 m booms, shown in Figure 2.1. The magnetometers are triaxial fluxgate magnetometers.

The auroral data used in this thesis was collected remotely using a meridian
Figure 2.1: Nasa's ACE spacecraft, the triaxial fluxgate magnetometers are located at the end of the booms depicted in the diagram and are labeled MAG. WIND's magnetometer is identical to one on ACE [Smith et al., 2001].
scanning photometer (MSP) and CCD spectrograph. Both instruments are ground based and located at the South Pole Station (SPS). The MSP and CCD spectrograph are automated instruments operated by the Space Physics Research Lab (SPRL) at Embry-Riddle Aeronautical University (ERAU).

The 6300 Å and 8446 Å auroral emissions are best observed during the austral winter, April through September. During this time period the dayside polar cusp aurora can be observed from SPS during the hours of about 10 to 18 hr (UT). These auroral emissions are better observed during these months because the height of the solar shadow. The solar shadow first dips below the horizon on March 22 and does not rise above the horizon until September 22. It reaches a maximum altitude of ~600 km, which far exceeds the altitude of the auroral emissions, ~200 km. The location of SPS in regards to the polar cap boundary makes it an appropriate location for observing the polar cusp aurora. It is located inside of the polar cap boundary, meaning it will not likely observe the more intense aurora that occur at this boundary. The location of SPS and the polar cap boundary aurora are shown in Figure 2.2. Also increasing the visibility of these auroral emissions is the low humidity at South Pole, tending to minimize the effects of atmospheric absorption of the near infrared. The high polar latitude, extremely low temperature, and very low humidity makes the skies over the SPS the most consistently cloudless skies on the planet, making it ideal for optical observations of the aurora.
Figure 2.2: The southern hemisphere is shown above with the center being the SPS. The typical location of polar cap aurora is shown relative to the SPS. Diagram was obtained through private communication with Dr. Irfan Azeem. ERAU. originating from Dr. D. J. McEwen, University of Saskatchewan.
2.1 Meridian Scanning Photometer

As stated earlier the MSP is located at the SPS. It is a sky-mapper photometer system that is sensitive to very, very low levels of airglow and auroral emissions. The system consists of a programmable turntable on which rests a meridian-scanning, multi-channel photometer. The multi-channel photometer has 2" diameter narrow band-pass filters with transmission peaks at $N_2^+1NG$ (0,1) 4278 Å, $H_\beta$ 4863 Å, [OI] 5577 Å and [OI] 6300 Å and 1° field-of-view (FOV). The detector is a TE (Thermo-electric) cooled GaAs (Gallium-Arsenide) PMT (photomultiplier tube) with a very high quantum efficiency (QE, QE > 15%) at all these wavelengths and dark noise of < 1 s$^{-1}$ at -30° C. Thus the signal-to-noise ratio of each 100 ms duration measurement of very weak signals (~1 R) (in each channel) exceeds 100. The instrument scans the entire sky along a meridian line. The data used in this thesis was at an elevation angle of 150° (at 5° average resolution, 2° at each side), and 2 minute averaging over time (about 3 scans). This was done so that the data from the MSP corresponded to the same portion of sky as the CCD spectrograph data also being used. Although only the 6300 Å emission is utilized in this thesis, because it is a result of direct electron impact upon atomic oxygen, the MSP also records emissions at 4278, 4863, 5577, and 5890 Å. Figure 2.3 shows the MSP data for May 10 2007 for all emissions.

2.2 CCD Spectrograph

The CCD spectrograph, located at SPS, is used for observations of wavelengths ~6500 Å to ~9000 Å. It has a wavelength range of about ~2500 Å, with the av-
Chapter 2: Data Description

Figure 2.3  MSP auroral observations for May 10, 2007. The 6300 Å line is shown in red.
average full width at half maximum over the wavelength range being 8 Å. The CCD spectrograph used is a very high throughput, very fast, half meter focal length, modified Czerny-Turner spectrometer. It is fitted with a 50 mm arc length curved entrance slit and several serially replaceable 1200 groves/mm gratings; each grating is blazed at a different wavelength to maximize efficiency in the preferred spectral range and spectral order. It is fitted with a 3-stage TE-cooled, thin, back-illuminated very high QE (QE_{\text{max}} > 80\%) 1024 \times 1024 pixel scientific grade CCD detector. Each pixel is 24 \mu m square; the dark current is < 1 e/pixel/sec and the slow read out rate (50 kHz) reduces the read out noise to less than 4 e/sec. The CCD is fitted with an 80 mm aperture, f/1.2 compound lens which produces a very flat field image of all emissions within a 12° circular FOV [Svjee et al., 1999]. Figure 2.4 shows an image of the CCD with false color applied to the brightness. The data recorded by the CCD spectrograph is in the first order and has an optical order sorting filter to prevent wavelengths shorter than 6500 Å from entering the instrument [Svjee et al., 1999]. The data of just the 8446 Å line is used in this thesis and is isolated and displayed Figure 2.5. Figure 2.6 shows the polar cusp auroral spectrum at a specific time, ~17 UT. The strongest peak present in the brightness is the 8446 Å emission.

2.3 Satellites: WIND & ACE

The magnetic field vector (GSM), satellite position vector (GSM), solar wind velocity (GSM), and upstream dynamic pressure are necessary information regarding the solar wind. The solar wind data was obtained from NASA satellites ACE and WIND. ACE
Figure 2.4: CCD spectrograph auroral observations for May 10, 2007. Full image of all wavelengths with false color. The color bar displays brightness in Rayleighs.
Figure 2.5: CCD spectrograph auroral observations for May 10, 2007, only showing the 8446 Å emission
Figure 2.6. Polar cusp auroral spectrum from the CCD spectrograph on May 10 2007.
and WIND were both designed to study the solar wind and interplanetary medium. WIND was launched on November 1, 1994 and ACE on August 25, 1997. Both satellites are currently in a Lissajous orbit around the L\(_1\) Lagrange point. The L\(_1\) lissajous orbit, also referred to as libration point 1, is an unstable equilibrium point between the Sun and the Earth. With active thrust control satellites can be "parked" at this location. Figure 2.7 shows the Lissajous orbit for ACE and SOHO, although SOHO was not used in this thesis it provides a visual of the type of orbit.

The satellite data is taken in the Geocentric Solar Magnetic System (GSM). Figure 2.8 illustrates the GSM coordinate system. The coordinates are centered on the Earth with the X-axis being the Sun-Earth line. The Y-axis is defined to be perpendicular to the Earth's magnetic dipole so that the X-Z plane contains the dipole axis. The positive Z-axis is chosen to be in the same sense as the northern magnetic pole. This coordinate system was chosen because since early studies on the response of the magnetosphere to changes in the IMF it has been shown to be more physically reasonable than the Geocentric Solar Ecliptic System (GSE) [Rostoker et al., 1983]. NASA satellite data is made available for public use and was obtained through NASA's CDAWeb. For WIND the data used was averaged at 30 s, ~2 min, ~2 min, and 1 min for magnetic field vector, position vector, solar wind velocity, and upstream dynamic pressure, respectively. ACE data used was averaged 16 s, ~1 min, ~1 min, and 1 min for magnetic field vector, position vector, solar wind velocity, and upstream dynamic pressure, respectively. The satellite data files often contained bad data points. For example the data files would show a value of -1.000000E+31 in the magnetic field vector data. The bad data points were removed and interpolated.
Figure 2.7: L1 lissajous orbit for ACE & SOHO. SOHO was not used in this thesis. WIND was used. The diagram is used to depict the orbit [Roberts, 2002]
Figure 2.8: Geocentric Solar Magnetospheric Coordinate System [Frank, 1970]
Chapter 3

Methodology

3.1 Wavelet Spectrum Analysis

Power spectrum analysis is a method of analyzing localized variations of power within a signal. The signal is quite often discrete, and sampled at regular intervals. In the past power spectrum analysis was performed using the windowed Fourier transform (WFT). The discrete Fourier transform (DFT) of a signal, \( x_n \), is given by Equation 3.1.

\[
\hat{x}_k = \frac{1}{N} \sum_{k=0}^{N-1} x_n e^{-2\pi i kn/N}
\] (3.1)

In the DFT, Equation 3.1, \( N \) is the total number of data points in the signal, \( n \) is the current index number of the signal, and \( k \) is referred to as the frequency index. The relationship to angular frequency commonly used in the DFT are defined by
Equation 3.2, where $\delta t$ is the sampling rate of the signal.

$$
\omega_k = \begin{cases} 
\frac{2\pi k}{N\delta t} & \text{if } k \leq \frac{N}{2} \\
-\frac{2\pi k}{N\delta t} & \text{if } k > \frac{N}{2}
\end{cases} 
$$

(3.2)

In order to get a view of localized power variations within a signal a window is applied with this method. The window can be of arbitrary shape, with two common shapes being the boxcar and the Gaussian window. The boxcar contains the Heaviside function, while the Gaussian window weighs the center heavier. The window of length $T$ is then slid over the signal containing $N$ data points. The total length of the signal is $N\delta t$. The frequencies computed will range from $T^{-1}$ to $(2\delta t)^{-1}$. This is a result of Nyquist sampling theory. It states that a continuous signal must be sampled at a frequency twice that of the highest frequency present in the original signal.

$$
f_n = \frac{f_s}{2}
$$

(3.3)

Where, $f_n$ is the Nyquist frequency, which is the highest frequency discernable from a discretely sampled signal and $f_s$ is the sampling frequency.

The localized time of the power variations is found by assigning the time to be at the center of the window. While this is not the only method it is the most common. Two other methods would be to assign the time to the left or right hand side of the window. The raw magnetic field strength data, Y-component, for ACE on May 10 2007 is shown in Figure 3.1. The top panel of Figure 3.3 shows the WFT of the same IMF data from ACE. In order to show resolution comparable to the wavelet
Figure 3.1: Y-component of the magnetic field strength at ACE, original signal before the WFT or Wavelet transform.

transform, shown in the bottom panel of Figure 3.3, the raw data was passed through a highpass butterworth filter, of the $5^{th}$ order. The largest period desired to be resolved is 30 minutes. This corresponds to the smallest frequency that is discernible, $T^{-1}$, and sets the window length to 30 minutes. The highest frequency is set by the resolution of the data file, $2(\delta t)^{-1}$, here $\delta t$ is $\sim$16 seconds. The range of periods resolved in this WFT is 0.5 minutes to 30 minutes.

The WFT is a very powerful tool when analyzing a signal for the frequencies that
Chapter 3: Methodology

contain power. It has the advantage of being an orthogonal transform which can take a signal from time domain to frequency domain and back to the time domain with no loss to the actual signal. Computationally however there can be drawbacks, one of them being that it can be inefficient [Kaiser, 1994, Torrence et al., 1998]. The frequencies resolved are dependent on the size of the window, so with a physical signal with an unknown range of frequencies present many different window sizes need to be applied to find the dominant frequencies. Additionally, $T/(2\delta t)$ frequencies must be computed at each time stepped window. Wavelet analysis has been shown to be more efficient [Kaiser, 1994] computationally and to exhibit a better balance between resolution in frequency and localization in time. For these reasons wavelet analysis will be used in the case studies presented, Figure 3.3 illustrates a comparison between the WFT and the wavelet analysis on magnetic field data used in this thesis.

The wavelet transform can be used to analyze time series that contain nonstationary power at many different frequencies [Daubechies, 1990]. To introduce wavelet analysis there must be an understanding of what a wavelet is. A wavelet may be any arbitrary shape as long as specific criteria are met. A wavelet must have a mean value of zero in time space. The wavelet must also be normalized. The two previous statements are expressed mathematically in Equation 3.4 and Equation 3.5.

\[
\int_{-\infty}^{\infty} \psi(\eta) \, d\eta = 0 \quad \text{(3.4)}
\]

\[
\int_{-\infty}^{\infty} |\psi(\eta)|^2 \, d\eta = 1 \quad \text{(3.5)}
\]

Here the wavelets, $\psi(\eta)$, are a function of a nondimensional time parameter. Wavelets
in common use are Morlet, Paul, and DOG [Torrence et al., 1998].

The wavelet chosen for use in this thesis is the Morlet wavelet. The Morlet wavelet is plane wave that has been modulated by a Gaussian function, defined in Equation 3.6 [Torrence et al., 1998]. In Equation 3.6, as stated earlier, to be admissible as a wavelet, it must be localized in both time and frequency. To be localized in time means that in time space the wavelet function must go to zero as time approaches ±∞. To be localized in frequency means that the wavelet's representation in frequency space must fall off to zero as the frequency goes from ±∞. The Morlet wavelet is shown to meet this criteria in Figure 3.2. Here it can be seen that both the real and imaginary components of the Morlet wavelet do fall off to zero. It is also shown that the Morlet wavelet falls off to zero in frequency space.
There are several criteria involved in choosing a wavelet. The main criteria to keep in mind are orthogonal/nonorthogonal, complex/real, width, and shape. The Morlet wavelet is nonorthogonal. This makes it a valid wavelet for performing both continuous and discrete wavelet transforms. Nonorthogonality is also useful when smooth, continuous variations in amplitude are expected. It also has both real and complex components in time space. This allows the Morlet wavelet to determine both the amplitude and phase of a transformation. The complex nature is better for determining the oscillatory behavior of a signal. Torrence et. al. [1998] define the width of a wavelet to be the $e$-folding time of the wavelet amplitude. The $e$-folding time is the time span near the edge, beginning or end, of the power spectrum where a discontinuity from the signal starting or stopping can result in a false power peak. The $e$-folding time is chosen so that the wavelet power for a discontinuity at the edge drops by a factor $e^{-2}$ [Torrence et al., 1998]. This determines the balance between time resolution and frequency resolution. A wavelet that has very high time resolution will have very low frequency resolution and vice versa. For Morlet this is adjusted by changing the nondimensional frequency, $\omega_0$. The shape of a wavelet should resemble the features of the signal.

It is usually desirable to pad a discrete signal with zeros. Zeros are added until the length of the signal, $N$, has reached the next power of 2. This is done to decrease edge effects. Edge effects arise from discontinuities at the beginning and end of a signal. The padded zeros will help to smooth out the discontinuities, but the signal is still

$$\psi_0(\eta) = \pi^{-\frac{1}{4}} e^{i\omega_0 \eta} e^{-\eta^2/2}$$

(3.6)
discrete and there will be a sharp change in data point values where the padding is added to the signal. Edge effects can appear as false power peaks near the beginning and end of the power spectrums. With the signal padded with zeros and the fact that wavelet power spectrum assumes the signal to cyclic, this leads to a discontinuity at the edges of the signal. The region of spectrum that is affected by edge effects is termed the *cone of influence* (COI). The COI is defined to be one e-folding width into the signal. This width is dependant on the current scale and for the Morlet is defined to be \( \sqrt{2} s \).

The wavelet transform is defined by Equation 3.7. Here \( \psi_0 \) has been normalized, scaled, and translated. The (*) indicates the complex conjugate.

\[
W(s) = \sum_{n'=-\infty}^{\infty} x_n^* \frac{(n'-n)\delta t}{s} \]

The mother wavelet, \( \psi_0 \), must be normalized and scaled to create the daughter wavelet, \( \psi \). The wavelets are normalized to have unit energy. The daughter wavelets to use with Equation 3.7 are calculated from Equation 3.8.

\[
\psi \left[ \frac{(n'-n)\delta t}{s} \right] = \left( \frac{\delta t}{s} \right)^{1/2} \psi_0 \left[ \frac{(n'-n)\delta t}{s} \right] \]

It is very slow to use the convolution definition of the wavelet transform. The calculations can be performed much faster in Fourier space, and then transforming back using the inverse Fourier transform. This second definition of the wavelet transform
is given by Equation 3.9.

\[ W_n(s) = \sum_{k=0}^{N-1} \hat{x}_k \hat{\psi}^* (s\omega_k) e^{i\omega_k n\delta t} \]  

(3.9)

In Fourier space the convolution of the signal and the wavelet is just multiplication. Equation 3.1 is used on both the signal and the wavelet. The Morlet wavelet in Fourier space is given by Equation 3.10.

\[ \hat{\psi}_0(s\omega_k) = \pi^{-1/4} H(\omega) e^{- (s\omega - \omega_0)/2} \]  

(3.10)

The normalization of the mother wavelet must now be performed in Fourier space so that each scaled wavelet has unit energy.

\[ \hat{\psi}(s\omega_k) = \left( \frac{2\pi s}{\delta t} \right)^{1/2} \hat{\psi}_0(s\omega_k) \]  

(3.11)

Every unscaled mother wavelet must also be normalized to have unit energy, Equation 3.12. This will force all wavelet transforms at every scale \( s \) to be comparable to one another. It will also ensure that the wavelet transformation is only weighted by its Fourier coefficients, \( \hat{x}_n \), and not by the wavelet transform itself. Both of the normalizations lead to Equation 3.13.

\[ \int_{-\infty}^{\infty} |\hat{\psi}_0(\omega')|^2 d\omega' = 1 \]  

(3.12)

\[ \sum_{k=0}^{N-1} |\hat{\psi}(s\omega_k)|^2 = N \]  

(3.13)
Finally the wavelet power spectrum is defined by Equation 3.14. Shown for comparison to the WFT the wavelet power spectrum for ACE IMF By data from May 10 2007 is given in the bottom panel of Figure 3.3. Here the range of periods is chosen ahead of time to be 0.5 minutes to 30 minutes. This defines the set of scales used in the wavelet transform, with the increments of the scales being 0.25. The nondimensional frequency used here was 15, to provide a decent balance between time and period resolution.

\[
P = | W_n(s) |^2
\]  

(3.14)

With the power spectrum resolved for the wavelet transformation, the last thing to discuss is the relation between the scale \( s \) and the Fourier period (frequency). In Torrence et al. [1998] it gives a closed form conversion between scale \( s \) and Fourier wavelength \( \lambda \), here shown as Equation 3.15. For this thesis the desired period was 0.5 min to 30 min and the necessary scales were determined from that range.

\[
\lambda = \frac{4\pi s}{\omega_0 + \sqrt{2 + \omega_0^2}}
\]  

(3.15)

### 3.2 Time Lag Determination

In order to determine if the emissions of either the 6300 Å or 8446 Å lines are begin modulated by fluctuations in the solar wind, it is first necessary to be able to explain the time delay from the observed location of the IMF fluctuation (WIND or ACE) to the observed time of the auroral fluctuation. The time delay from observing satellite
Figure 3.3: Windowed Fourier Transform (WFT) of ACE IMF By data for May 10 2007 (Top) Wavelet Power Spectrum of ACE IMF By data for May 10 2007 (Bottom).
to the observing auroral instrument is broken into several parts. The calculation assumes that the velocity of a fluid parcel in the solar wind upstream of Earth’s bow shock is unaccelerated and directed anti-sunward [Lester et al., 1993]. It is also being assumed that the spacecraft observation represents a phase front disturbance in the solar wind.

The first of these parts is the delay from the spacecraft to the subsolar bow shock. This time lag is shown in Equation 3.16 [Lester et al., 1993].

\[
\tau_1 = \frac{X - (1 - \beta)D + L \tan \phi}{v_\infty}
\]

In equation 3.16 \(X\) is the geocentric distance to the spacecraft and \((1 - \beta)D\) is the geocentric distance to the subsolar bow shock, where \(\beta\) is a result of a gas dynamic model developed by Spreiter and Stahara [1980] and \(D\) is the geocentric distance to the subsolar magnetopause, \(L\) is the orthogonal distance the spacecraft is from the Sun-Earth line, \(\phi\) is the angle between the Sun-Earth line and the phase front disturbance in the IMF, and \(v_\infty\) is the local solar wind velocity. The short term IMF fluctuations in these events exist for about an hour or so. For example in Figure 3.3 the event about 14 minutes in period that occurred just after 14 hours UT has power spread out over about an hour. In the time lag calculations the necessary solar wind and satellite measurements were averaged over the time the event occurred.

The second part of the time delay is from the subsolar bow shock to the subsolar magnetopause. To estimate the time to traverse the magnetosheath Lester et al. [1993] employed results of the Spreiter and Stahara [1980] gas-dynamic model for the
flow of magnetized solar wind around a magnetosphere. The second time lag is given in Equation 3.17.

\[ \tau_2 = \frac{8\beta D}{\nu_\infty} \]  

(3.17)

Here, as above, \( \beta \) is a result of the Spreiter and Stahara [1980] model and ranges from 0.20 to 0.25. The two time lags are now combined to obtain a single time lag from the space craft to the subsolar magnetopause.

\[ \tau = \frac{(X + \frac{L \tan \phi}{\cos \beta} + (7\beta - 1)D)}{\nu_\infty} \]  

(3.18)

In equations 3.16 3.17 and 3.18, \( D \) is based on steady state Newtonian pressure balance [Mead et al., 1964] and is calculated by Equation 3.19.

\[ D = \left( \frac{\alpha^2 B_{eq}^2}{2\mu_0 P_\infty} \right)^{1/6} R_E \]  

(3.19)

In equation 3.19 \( \alpha \) is a geometrical compression factor due to the Chapman-Ferraro magnetopause current, \( B_{eq} \) is the strength of the equatorial magnetic field (31 000 nT), \( \mu_0 \) is the permeability of free space (4\( \pi \)\( 10^{-7} \) N/A\(^2 \)), \( P_\infty \) is the dynamic pressure of the upstream solar wind, and \( R_E \) is the radius of the Earth (6378 km). The values for \( \alpha \) range from 2 to 3 (6), but \( \alpha = 2.44 \) was chosen based on Nishitani et al. [1999].

The last part of the total time lag is the delay from the subsolar magnetopause to the location of the auroral emissions. The auroral emissions of the 6300 Å and 8446 Å lines are based on direct electron impact with atomic oxygen. The time taken for the pulsed electrons to travel from the subsolar magnetopause to the location of
the auroral emissions is estimated to be about $18 \pm 10$ minutes [Zhang et al., 1998]. The calculated time delays from WIND and ACE to the auroral emissions range from about 60 min to 100 min. As stated earlier the fluctuations in the solar wind last around an hour or so. The error in the satellite measurements was taken to be the standard deviation over the length of the fluctuation. This error was then propagated through the calculations and then with the 10 minute error associated with travel time of solar wind plasma from the subsolar magnetopause to the auroral emission. The 10 minute error was the dominating factor in the calculation of the total error on the time lag. The typical error for all time lags calculated was approximately 10 minutes.
Chapter 4

Results

Four case studies were identified on the dates of May 6 2008, May 10 2007, August 14 2006, and August 15 2006. The process of selection for the case studies presented began with observations of the cusp aurora from SPS. The dates initially selected for study were chosen based on the particularly strong cusp aurora, specifically the 6300 Å and 8446 Å. The second criteria was that the fluctuation in the cusp aurora was due to fluctuations in auroral brightness and not due to the location of cusp aurora moving in and out of the instrument's field of view. It was determined that the cusp aurora was stationary by viewing the complete scans of MSP. With strong and stationary cusp auroral conditions satisfied, WIND and ACE satellite data was analyzed via wavelet analysis for similar fluctuations in IMF magnitude roughly an hour prior to the auroral brightness fluctuations. As a rule of thumb it takes roughly an hour for a disturbance in the solar wind to reach Earth. Time lag calculations were then used to relate IMF magnitude fluctuations to cusp auroral brightness fluctuations.
4.1 Case Study 1: May 6 2008

The magnitude of the IMF components and total magnetic field are shown along with the solar wind speed and dynamic pressure in Figure 4.1. The solar wind began the day at higher than average speed, around 650 km/s, but slowed down to just over 500 km/s by the end of the day. The speed of the solar wind has a direct effect on the time lab calculation as the IMF fluctuations travel at this speed. The solar wind dynamic pressure was initially around 2 nPa but during the time of the two events to be described in the following paragraphs the dynamic pressure was around 1 nPa. Examination of Figure 4.1 showed that one of the components of the IMF is not dominating in strength in comparison to the other components. The most influential component in controlling the cusp aurora is the Z-component, as it effects the location at which magnetic reconnection takes place. During a southward Z-component reconnection occurs around the equator on the dayside magnetopause. A northward Z-component shifts the location of reconnection tailward of the polar cusp. Regardless of the location of reconnection the polar cusp remains the location where solar wind plasma has direct entry in to the upper ionosphere. The Z-component is mainly northward for the entire day, resulting in a stable cusp location that is verified by the full scan of the MSP. It should be restated here that the Z-component controls the location of reconnection and hence the location of the cusp aurora. The thesis examined electron fluctuation caused by oscillations in any component or combination of components of the IMF.

Figure 4.2 shows the wavelet power spectrum for the date of May 6 2008. The
Chapter 4: Results

CCD spectrograph data was not available for this date, so only the MSP and WIND data are shown. In the top half of Figure 4.2 shows a 15 minute period oscillation in the intensity of the 6300 Å emission at 12.25 UT (hours). The bottom half of Figure 4.2 shows a 12 minute period oscillation in the combined X and Y components of the IMF at WIND that occurred at 11.50 UT (hours). The white lines in each half of Figure 4.2 signify the confidence levels. The confidence levels for the MSP and WIND are 65% and 98%, respectively. The observed time lag from the power spectrum is 45.0 minutes, while the calculated time lag is 62.4 ± 10.1 minutes. The satellite is located at 234 \( R_E \), 96 \( R_E \), and 11.5 \( R_E \) in GSE coordinates X, Y, and Z, respectively. The satellite location is being recorded in GSE coordinates so that satellite location relative to the ecliptic plane could checked to determine if satellite location played a role in its chance of observing an event.

Later on May 6 2008 there was another event, shown in Figure 4.3. In the WIND spectrum at around 15.05 UT (hours) there is a 6 minute oscillation in the IMF, still the combined X and Y components. At around 16.10 UT (hours) there is a 7 minute period oscillation in MSP spectrum. The observed time delay between WIND and the MSP is 63.0 minutes, and the calculated time delay is 65.1 ± 10.1 minutes. The confidence levels are 55% and 95% for the MSP and WIND, respectively.

4.2 Case Study 2: May 10 2007

The second case study took place on May 10 2007 and Figure 4.4 shows the IMF magnitudes and solar wind speed and dynamic pressure for the day. The solar wind
had a speed closer to 450 km/s. This lead to longer calculated time lags of this case in comparison to the first case study, where the speed was significantly faster. The dynamic pressure was around 1 nPa but increased to around 2 nPa as the day progressed. However during the event observed by ACE, about 16 UT to 17 UT, the pressure was relatively stable around 1.5 nPa. Also the Z-component of the IMF fluctuated between northward and southward throughout the day, which lead to the speculation that the polar cusp itself may be moving also. For the time period in question, 16 UT to 17 UT, the Z-component was relatively constant and southward in direction, which lead to a stable location of the cusp aurora. The stability of the cusp location was verified with the full MSP scans.

The wavelet power spectrums for the MSP, the CCD spectrograph, and the Y component (GSM) of the IMF at ACE are shown in Figure 4.5 on the top, middle, and bottom, respectively. At 17.10 and 17.30 UT (hours) there were periodic oscillations at 10 minutes and 12 1.8 minutes recorded by the MSP and CCD spectrograph. ACE located at 245 $R_E$, $-26.8$ $R_E$, and $-12.7$ $R_E$ in GSE X, Y, and Z components, respectively, showed an 11 ± 1.8 minute periodic oscillation in the Y component (GSM) of the IMF at 16.30 UT (hours). The confidence levels for the MSP, CCD spectrograph, and ACE are 99%, 99.98%, and 90%, respectively. The observed time lag from ACE to the MSP is 48.0 minutes, while from ACE to the CCD spectrograph it is 60.0 minutes. The calculated time to both the MSP and CCD spectrograph is 78.7 ± 10.0 minutes.
4.3 Case Study 3: August 14 2006

The solar wind and IMF conditions for the date of August 14 2006 and shown in Figure 4.6. The solar wind speed was just over 300 km/s for the entire day and the dynamic had some large variations during the beginning of the day. It went from around 1 nPa to about 2 nPa but steadied out at around 1 nPa after 12 UT. The Y-component of the IMF dominated in the earlier part of the day but as the day progressed its magnitude dropped to values that more closely compared the other two components. The two events shown in Figures 4.7 and 4.8 took place at around 14 UT to 15 UT and 16.5 UT to 17 UT, respectively, occurred when the Z-component was transitioning from southward to northward. Despite this transition in the IMF the cusp was shown to be stable through MSP scans for the day.

Two events occurred on August 14 2006. The first of these events appeared in the X component (GSM) of the IMF at WIND. It is shown in Figure 4.7. The IMF spectrum shows a 13 minute period oscillation at 14.45 UT (hours). This is followed by 12 minute and 12 minute period oscillation in the MSP and CCD spectrograph, respectively, at 15.85 UT (hours). The observed time lag from WIND to the MSP and CCD spectrograph observations is 84.0 minutes and 84.0 minutes, respectively, compared to the calculated time lag of 98.2 ± 10.0 minutes. The confidence levels of this event in the MSP, CCD spectrograph, and WIND were 65%, 85%, and 80%, respectively. During these events WIND is located at 262 $R_E$, $-98.7$ $R_E$, and 4.2 $R_E$ in GSE X, Y, and Z components, respectively.

The second event, shown in Figure 4.8, also appears as an oscillation in the X
component of the IMF at WIND. There is an 8 minute period oscillation that occurs at 16.80 UT (hours). Later at around 18.20 UT (hours) there is an 8 minute period oscillation in the CCD spectrograph. It should be noted that in this specific event did not show up in the MSP spectrum. The observed time lag between the IMF oscillation at WIND and the polar cusp auroral emission measured by the CCD spectrograph is 84.0 minutes, the calculated time lag was 97.0 ± 10.0 minutes. The confidence level on the CCD spectrograph was 99% and for WIND it was 60%.

4.4 Case Study 4: August 15 2006

The last case study presented in this thesis took place on August 15 2006. The solar wind and IMF conditions for the day are shown in Figure 4.9. The solar wind speed was roughly 300 km/s for the entire day. The event occurred just prior to 18 UT which corresponded to an increase in dynamic pressure from around 2 nPa to almost 4 nPa. The flat spot in the dynamic pressure graph between 15 UT and 18 UT was due to bad data points in the satellite data file, so when computing the time lag these values were left out and only the data points after the interpolated points were used. The Z-component during the event was northward, so along with the MSP scan indicate the location of the cusp was stationary.

At 17.75 UT (hours) there was a 13 minute period oscillation in the X component of the IMF at WIND’s location. The satellite was located at 230 $R_E$, $-99.3$ $R_E$, and 3.5 $R_E$ in GSE X, Y, and Z components, respectively. A similar periodic oscillation is observed in the MSP data at around 19.25 UT (hours). This is an observed time
lag of 90.0 minutes. The CCD spectrograph power spectrum also shows an 11 minute period oscillation at a time of 19.45 UT (hours). Thus the observed time lag was 102.0 minutes. The calculated time lag for this event was $98.2 \pm 10.0$ minutes. The respective confidence levels for the MSP, CCD spectrograph, and WIND are 75%, 95%, and 85%.

4.5 Discussion of Results

The case studies presented in this thesis began with the days of May 6 2008, May 10 2007, and August 14-20 2006. Originally these dates were selected for the particularly strong polar cusp aurora that took place at the SPS. Polar cusp aurora here specifically refers to the 6300 Å and 8446 Å emission lines from the southern polar dayside cusp. Using the outlined approach, based on Lester et al. [1993], for calculating the time lag, a more general lag was calculated for the entirety of the day. The rough estimation for the lag was then used to visually observe the wavelet power spectrums of the IMF from WIND and ACE to be compared with the spectrums from the MSP and CCD spectrograph. It allowed for the first determination of events to be focused on. With events occurring at defined time intervals, the time lag was calculated for each event’s time interval and compared to the observed time lag from the visually analyzed wavelet spectrums. The significance levels of the events in the IMF, MSP, and CCD spectrograph was the last piece to be examined for the inclusion or exclusion of an event. The dates choosen for the case studies were May 6 2008, May 10 2007, August 14 & 15 2006, shown in Figures 4.2 to 4.8.
Chapter 4: Results

One of the first things considered after analyzing the wavelet power spectrums of WIND and ACE in comparison to the MSP and CCD spectrograph, was to check the physical location of the satellite in reference to the Sun. The position of the satellite will inherently affect the calculation of the time lag, but it is the effect of the position on whether or not the event will reach the polar cusp and hence modulate the auroral emissions. The position of WIND and ACE for the dates examined is shown in Figure 4.11. This was done in GSE coordinates to find the perspective satellites location relative to the Sun-Earth line and also to the ecliptic plane. The X component of the satellite’s position is not thought to have any effect on whether or not the event in question will modulate the auroral emissions in question, due to the fact that any event will have to traverse the length of the Sun-Earth line. However the X components of WIND and ACE range from ~230 \( R_E \) to ~270 \( R_E \). The satellite’s position in the X-Y plane, distance off the Sun-Earth line, was considered. Based on the 4 case studies this relative position to the Sun-Earth line in the X-Y plane is not thought to have a large affect on the events ability to modulate the polar cusp aurora. On May 6 2008 WIND’s Y component is ~96 \( R_E \), and on August 15 2006 the Y component is ~-99 \( R_E \), while for the one date ACE was used, the Y component was ~-27 \( R_E \). The large variation in the distance from the Sun-Earth line in the X-Y plane from event to event lead to the conclusion that it may not play a large roll in the coupling process. The Z component of the satellite’s position, or distance off the ecliptic plane in the X-Z plane, is hypothesized to have the largest effect on whether the event in question may or may not modulate the auroral emissions. For all case studies presented the distance above or below the ecliptic plane never exceeded 13 \( R_E \).
Explicitly for the dates of May 6 2008, May 10 2007, August 14 2006, and August 15 2006 this distance was $11.5 \, R_E$, $-12.7 \, R_E$, $4.2 \, R_E$, and $3.5 \, R_E$, respectively.

The periodicities in the IMF do not perfectly match the auroral emissions. This is to be expected as the solar wind plasma encounters the bow shock, the magnetopause, and the magnetosheath. Even though the solar wind is effectively collisionless, the mean free path of a solar wind particle being about $1.5 \times 10^8$ km, across the bow shock there is a change in momentum and temperature of the plasma. As the solar wind continues towards the magnetopause, Earth’s magnetic field begins to alter the direction of flow of the solar wind. Both of these boundaries where the conditions on the solar wind are changed could have an effect on the periodic behavior of the electrons that had been modulated by the IMF causing a shift in period. However it is hypothesized in this thesis that these have minimal effect on the period of oscillation of the electrons in the solar wind plasma. This is because the polar cusp is the region where the solar wind has direct entry into Earth’s ionosphere, with the least interaction with the planet’s magnetosphere. Instead the majority of the shifts in periodicity from IMF to auroral emission can be explained by the uncertainty in the periods themselves. The uncertainty is based on full width at maximum (FWHM) of the peak in the wavelet power with respect to the period. For a discretely sampled signal the power will never show up at a single period, or frequency. There will always be some spread in the power over multiple periods. This can be seen in all case studies, Figures 4.2 to 4.10, where the power of the events is not singular in period. For the first case study there were two events shown in Figures 4.2 and 4.3. For the first event the period at WIND was centered on 12 minutes but spread across
periods of about 10 minutes to 14 minutes compared to the MSP which was centered
15 minutes and spread across periods of about 14 minutes to 17 minutes. The second
event being centered at 6.6 minutes and spread over periods of about 5 minutes to
7 minutes at WIND and 7 minutes spread over periods of 6 minutes to 8 minutes at
the MSP. The uncertainties in the period at both locations for both events overlap,
giving validity to the explanation of the shifts in period being expalainable.

The uncertainty in the observed time lags is viewed similarly to the uncertainty
in the period. The events presented in the case studies were localized in time and in
the power spectrums appeared to last for about an hour. The event on May 6 2008,
shown in Figure 4.2, was spread out in time. It was possible to know when the power
peaked more precisely in time, but this would make the period resolution much worse.
The balance between period resolution and time resolution can be adjusted by the
non-dimensional frequency in the wavelet transform. There will always be a trade off
between and event’s uncertainty in period and its localization in time. This is inherent
to the mathematics of taking a one dimensional time series and transforming it into a
diffuse two dimensional time-frequency, or time-period, image [Torrence et al., 1998].
When the observed time lag was compared to the calculated time lag some events
corresponded better than others. The worst comparison was for the first event where
the observed time lag was 45.0 minutes and the calculated time lag was 62.4 minutes.
For the same day the second event had an observed time lag of 63.0 minutes and the
calculated was 65.1 minutes. The events themselves are not completely localized in
time and the uncertainty associated with when an event actually occured is thought
to reasonably explain the difference between the observed and calculated time lags.
Chapter 4: Results

It should be mentioned that the uncertainty in the calculated time lag of all events tends to be around 10 minutes. The time lag was from satellite to auroral emission and had two major components. The components being, the lag from satellite to the subsolar magnetopause and from the subsolar magnetopause to the auroral emission. The first component was based on Lester et al. [1993] as discussed earlier, the second on the work of Zhang et al. [1998]. The uncertainty of the first lag was calculated by propagating the error of the in situ satellite measurements, where the error in the measurement was taken to be the standard deviation for the length the event lasted. These were generally quite small, approximately 1 to 2 minutes. The time lag from magnetopause to auroral emission was the dominating term of the uncertainty, being ±10 minutes. Zhang et al. [1998] showed this time lag by correlating pressure enhancements in the solar wind to polar auroras during times of southward IMF. During southward IMF conditions this would correspond to direct entry of solar wind particles into the polar ionosphere, a similar situation being studied in this thesis.

The confidence levels are based on an assumed red noise background spectrum. With a red noise background the power increases with decreasing frequency, or increasing period. A red noise background is appropriate for many geophysical phenomena [Torrence et al., 1998], and appeared appropriate for solar wind and auroral emissions, as the longer periods tend to carry more power than the smaller periods. The wavelet power spectrum for IMF and auroral emissions was compared to the wavelet spectrum of the red noise background to identify confidence levels. For the first case study on May 6 2008 the confidence levels for the MSP were low in both events, 65% and 55% for the first and second events, shown in Figures 4.2 and 4.3,
respectively. The confidence levels on WIND for these two events were much higher, 98% and 95%. The second case study on May 10 2007 had the highest overall confidence levels for IMF, MSP, and CCD spectrograph of all days. The levels were 99%, 99.98%, and 90% for IMF (ACE), MSP, and CCD spectrograph, respectively. As in the first case study, cases 3 and 4 have mixed confidence levels between the IMF, MSP, and CCD spectrograph, ranging from 60% to 99%. The second event of August 14 2006, shown in Figure 4.8, shows up significantly in WIND IMF spectrum and in the CCD spectrograph spectrum. It does not appear in the MSP emission. With both emissions being due to direct electron impact it makes the event questionable, however the power of the wavelet spectrum for the CCD spectrograph is considerably larger than the MSP. It may be that the MSP just did not significantly show the power of the emission. In order to show a strong coupled relationship between the IMF fluctuations and direct electron impact auroral emissions, it is necessary to have higher confidence intervals for all spectrums on more days.
Figure 4.1: Solar wind parameters observed by WIND on May 6 2008.
Figure 4.2: Wavelet power spectrums for the MSP (top) and WIND IMF Bxy (bottom) on May 6 2008. There is a 12 minute period oscillation at 11.50 UT in WIND and a 15 minute oscillation at 12.25 UT (hours) in the MSP.
Chapter 4: Results

Figure 4.3: Wavelet power spectrums for the MSP (top) and WIND IMF Bxy (bottom) on May 6 2008. There is a 6 minute period oscillation at 15.05 UT in WIND and a 7 minute oscillation at 16.10 UT (hours) in the MSP.
Figure 4.4: Solar wind parameters observed by ACE on May 10 2007.
Figure 4.5: Wavelet power spectrums for the MSP (top), CCD spectrograph (middle) and ACE IMF By (bottom) on May 10 2007. ACE shows an 11 minute period oscillation at 16.30 UT (hours), followed by a 10 minute and 12 minute period oscillation in the MSP at 17.10 UT (hours) and in the CCD spectrograph at 17.30 UT (hours), respectively.
Figure 4.6: Solar wind parameters observed by WIND on August 14 2006.
Figure 4.7: Wavelet power spectrums for the MSP (top), CCD spectrograph (middle), and WIND IMF Bx (bottom) on August 14 2006. WIND shows a 13 minute period oscillation at 14.15 UT (hours), followed by 12 minute period oscillations in both the MSP at 15.85 UT (hours) and in the CCD spectrograph at 15.85 UT (hours), respectively.
Figure 4.8: Wavelet power spectrums for the MSP (top), CCD spectrograph (middle), and WIND IMF Bx (bottom) on August 14 2006. WIND shows an 8 minute period oscillation at 16.80 UT (hours), followed by a 12 minute period oscillation in the CCD spectrograph at 15.85 UT (hours).
Figure 4.9: Solar wind parameters observed by WIND on August 15 2006.
Figure 4.10: Wavelet power spectrums for the MSP (top), CCD spectrograph (middle), and WIND IMF Bx (bottom) on August 15 2006. WIND shows a 13 minute period oscillation at 17.75 UT (hours), followed by a 11 minute period oscillations in the MSP at 19.25 UT (hours) and in the CCD spectrograph at 19.45 UT (hours), respectively.
Figure 4.11: Satellite locations for WIND and ACE on the dates of the case studies presented. All locations are within 13 $R_E$ of the ecliptic plane.
Chapter 5

Concluding Remarks

The solar wind - magnetosphere - ionosphere coupling process is likely to become more and more important not only to academic researchers but also to private companies and the military. The Sun has recently passed its solar minimum and will be entering solar maximum sometime in 2011 to 2013. During solar maximum the energy output of the Sun will increase dramatically. The increased solar activity can negatively impact satellite electronics affecting any and all systems. Solar activity has even been violent enough to affect the power grids on the surface of the Earth, most notably during the solar storm of March 1989 which resulted in the power outages of 6 million people in Quebec, Canada. In order to protect power grids, research satellites, civilian satellites, and military satellites it has become extremely important to understand how energy is transferred from the Sun to the Earth, or the solar wind - magnetosphere ionosphere coupling process. The aurora presents a perfect opportunity for studying this coupled relationship. The polar cusp aurora allows a unique location to study how...
the solar wind can directly drive the behavior of the ionosphere. It has been shown in multiple studies that the solar wind can drive certain polar auroral phenomena. Zhang et al. [1998] have shown that polar aurora are observed to follow sudden enhancements in solar wind pressure. Alfvén waves in the solar wind have been the focus of current research in driving aurora. With Alfvén waves being shown to power the aurora, generate polar cap patches, and create pulsed ionospheric flows. The polar cusp aurora have also been shown to be linked to the IMF and solar wind plasma conditions. The solar wind and polar ionosphere are coupled. The question posed in this thesis was, do fluctuations in any component or combination of components of the IMF drive fluctuations in the southern hemisphere dayside polar cusp aurora?

The approach taken to attempt to answer this question was to analyze the wavelet power spectrums of IMF data from WIND and ACE and compare it to the wavelet power spectrums of MSP and CCD spectrograph data, while analyzing observed and calculated time lags for any significant power. It is thought that these case studies provided evidence for the direct coupling of the IMF to the 6300 Å and 8446 Å emissions of the polar cusp. With the exception of May 10 2007 all case studies had mixed confidence levels. While the confidence levels on this day were high, so was the uncertainty in the observed time lag. So while it is thought that evidence showing the coupled nature of the emissions is presented, it is by no means completely conclusive evidence. This can be accomplished as SPRL is continuing to operate the high resolution instruments at SPS and both WIND and ACE will be in operation for many years to come.

In order to either strengthen or dismiss this relationship between the IMF and
polar cusp further study is needed. The largest problem was the lack of days for which there was significantly strong polar cusp aurora. This is in part due to the fact that it can only be observed during austral winter and then only during the hours of about 10 to 18 hours (UT). There is limited time during the year and limited time on those days to observe the dayside polar cusp aurora, so more data needs to be collected.

There is always room to expand the techniques used to analyze the data, but not usually enough time. For analyzing power spectrum one technique that may be useful to expand into, is the use of the Stockwell transform, which is supposed to have less attenuation at higher frequencies than the wavelet transform. Also it was originally intended to use multiple spacecraft to obtain the orientation of the solar wind phase front. However a lack of available satellites on the days in question led to using a single satellite approach.

In conclusion there is enough evidence of solar wind IMF coupling to the southern hemisphere dayside polar cusp auroral emissions to warrant further study even though concrete evidence was not presented. Further research through expanded techniques of analyzing data and collecting more useable data when the polar cusp aurora is strong is necessary.
Appendix A

MATLAB Code

A.1 Wavelet Code

The following code was used for processing the wavelet power spectrums. It was provided courtesy of Christopher Torrence and Gilbert P. Compo University of Colorado, program in Atmospheric and Oceanic Sciences. It was obtained through the website http://paos.colorado.edu/research/wavelets/.

Wavelet.m

close all
clear all
clc

%****************************************************
%FIRST WAVELET ANALYSIS - MERIDIAN SCANNING PHOTOMETER
%*******************************************************************************

% Add Paths for functions and data files
%*******************************************************************************
addpath('C:\Users\RMFS\Documents\MATLAB\Thesis\Functions')
addpath('C:\Users\RMFS\Documents\MATLAB\Thesis\Data\Ionosphere\MSP ..

63
%Load scanning photometer data file
%********************************************************************
[time,nm428,nm630,nm558,nm589,nm486]=textread('C:\Users\RMFS\...
Documents\MATLAB\Thesis\Data\Ionosphere\MSP\Modified...
...MSP007510.dat',',

sst = nm630;

% WAVELET ANALYSIS BEGIN *****************

c = 15;
fo = c/(2*pi);
variance = std(sst)^2;
n = length(sst);
sst = sst - mean(sst);
sst = sst/sqrt(variance);

% These parameters should be changed when tuning the power spectrum
% to a desired resolution

dt = (time(2)-time(1))*60;% sampling rate, units of minutes
k0 = 8; % parameter value
period = [0.5:0.25:30]; %Period I am looking for, my choice
fourier_factor = (4*pi)/(k0 + sqrt(2 + k0^2));
scale_in = period/fourier_factor;

% ********************************************************************

% These parameters should be changed when tuning the power spectrum
% to a desired resolution

pd = 1; % pad the time series with zeroes (recommended)
dj = 0.01; % this will do 4 sub-octaves per octave
so = 1*dt; % this says start at a scale of 6 months
j1 = length(period) - 1; % do 7 powers-of-two with dj sub-octaves
lagl = 0.72; % lag-1 autocorrelation for red noise background
mother = 'Morlet';

% Wavelet transform:
Chapter A: Concluding Remarks

[wave, period, scale, coi] = wavelet_c(sst, dt, pad, dj, s0, j1, mother, k0, ... 
...scale_in);
power = (abs(wave).^2); % compute wavelet power spectrum
amp = sqrt(power)*sqrt(2);
% Significance levels: (variance=1 for the normalized SST)
[signif, fft_theor] = wave_signif(1.0, dt, scale_in, 0, lag1, 0.99, -1, ... 
...mother, k0);
sig95 = (signif')*(ones(1, n)); % expand signif --> (J+1)x(N) array
sig95 = (power./ sig95)/10; % where ratio > 0.1, power is signif
% WAVELET ANALYSIS END **************************************

% Plot Wavelet
a = find(time < 10, 1, 'last');
b = find(time > 20, 1, 'first');

figure(1)
subplot 211
contourf(time(a:b), period, power(:, a:b), ...
...[0, 0.5*max(max(power(:, a:b))):0.01:max(max(power(:, a:b)))];
xlabel('Time UT (hours)')
ylabel('Period (minutes)')
shading flat;
colorbar;
hold on;
contour(time(a:b), period, sig95(:, a:b),[0, 0.1:0.1:max(max(sig95))],...
...'w');
title('MSP (6300) May 10 2007')
grid on

%%%%%%%%%%%%%%%%%%%%%%%%
%SECOND WAVELET ANALYSIS - CCD
%%%%%%%%%%%%%%%%%%%%%%%%

clear all
%Load ccd data file
%%%%%%%%%%%%%%%%%%%%%%%%
load('dccd_SP070510.mat');
time = b.num_times*24;
Chapter A: Concluding Remarks

```matlab
sst = b.data(:,852)*1000;

% WAVELET ANALYSIS BEGIN *******************
c = 15;
fo = c/(2*pi);
variance = std(sst)^2;
n = length(sst);
sst = sst - mean(sst);
sst = sst/sqrt(variance);

% These parameters should be changed when tuning the power spectrum
% to a desired resolution

dt = (time(2)-time(1))*60; % sampling rate, units of minutes
k0 = 8; % parameter value
period = [0.5:0.25:30]; % Period I am looking for, my choice
fourier_factor = (4*pi)/(k0 + sqrt(2 + k0^2));
scale_in = period/fourier_factor;

% Pad the time series with zeroes (recommended)
dj = 0.01; % this will do 4 sub-octaves per octave
s0 = 1*dt; % this says start at a scale of 6 months
j1 = length(period) - 1; % do 7 powers-of-two with dj sub-octaves
lag1 = 0.72; % lag-1 autocorrelation for red noise background
mother = 'Morlet';

% Wavelet transform:
[wave,period,scale,coi] = wavelet_c(sst,dt,pad,dj,s0,j1,mother,k0,...
    ...scale_in);

power = (abs(wave).^2); % compute wavelet power spectrum
amp = sqrt(power)*sqrt(2);

% Significance levels: (variance=1 for the normalized SST)
[signif,fft_theor] = wave.signif(1.0,dt,scale_in,0,lag1,0.9998,-1,...
    ...mother,k0);
sig95 = (signif')*(ones(1,n)); % expand signif --> (J+1)x(N) array
sig95 = (power./ sig95)/10; % where ratio > 0.1, power is signif
```

7. WAVELET ANALYSIS BEGIN *******************
7. These parameters should be changed when tuning the power spectrum
7. to a desired resolution

7. These parameters should be changed when tuning the power spectrum
7. to a desired resolution

7. These parameters should be changed when tuning the power spectrum
7. to a desired resolution

dt = (time(2)-time(1))*60; % sampling rate, units of minutes
k0 = 8; % parameter value
period = [0.5:0.25:30]; % Period I am looking for, my choice
fourier_factor = (4*pi)/(k0 + sqrt(2 + k0^2));
scale_in = period/fourier_factor;

% Pad the time series with zeroes (recommended)
dj = 0.01; % this will do 4 sub-octaves per octave
s0 = 1*dt; % this says start at a scale of 6 months
j1 = length(period) - 1; % do 7 powers-of-two with dj sub-octaves
lag1 = 0.72; % lag-1 autocorrelation for red noise background
mother = 'Morlet';

% Wavelet transform:
[wave,period,scale,coi] = wavelet_c(sst,dt,pad,dj,s0,j1,mother,k0,...
    ...scale_in);

power = (abs(wave).^2); % compute wavelet power spectrum
amp = sqrt(power)*sqrt(2);

% Significance levels: (variance=1 for the normalized SST)
[signif,fft_theor] = wave.signif(1.0,dt,scale_in,0,lag1,0.9998,-1,...
    ...mother,k0);
sig95 = (signif')*(ones(1,n)); % expand signif --> (J+1)x(N) array
sig95 = (power./ sig95)/10; % where ratio > 0.1, power is signif
% WAVELET ANALYSIS END *******************************

% Plot Wavelet
a = find(time < 10,1,'last');
b = find(time > 20,1,'first');

figure(1)
subplot 312
contourf(time(a:b),period,power(:,a:b),...
...[0,0.5*max(max(power(:,a:b))):0.01:max(max(power(:,a:b))))]);
xlabel('Time UT (hours)')
ylabel('Period (minutes)')
shading flat;
colorbar;
hold on;
contour(time(a:b),period,sig95(:,a:b),[0,0.1:0.1:max(max(sig95))],...
...'w');
title('CCD (8446) May 10 2007')
grid on

%THIRD WAVELET ANALYSIS - SATELLITE IMF
%********************************************************************
clear all
%Load satellite data file
[time,Bx,By,Bz]=textread('C:\Users\RMFS\Documents\MATLAB\...
...Thesis\Data\Satellite\ACE\05_10_2007\AC_H0_MFI_8706.txt',...'
...'s%f%f%f');
time = TIME(time);
Bx = INTRP(Bx,time,-1.00000E+31);By = INTRP(By,time,-1.00000E+31);...
...Bz = INTRP(Bz,time,-1.00000E+31);
% sst = Bx;
% sst = By;
% sst = Bz;
% sst = sqrt(Bx.^2+By.^2);
% sst = sqrt(Bx.^2+Bz.^2);

7. Plot Wavelet
% sst = sqrt(By.^2+Bz.^2);
% sst = sqrt(Bx.^2+By.^2+Bz.^2);

% WAVELET ANALYSIS BEGIN **********************
c = 8;
fo = c/(2*pi);
variance = std(sst).^2;
n = length(sst);
sst = sst - mean(sst);
sst = sst/sqrt(variance);
% *******************************************************************
% These parameters should be changed when tuning the power spectrum
% to a desired resolution

dt = (time(2)-time(1))*60; % sampling rate, units of minutes
k0 = 10; % parameter value
period = [0.5:0.25:30]; % Period I am looking for, my choice
fourier_factor = (4*pi)/(k0 + sqrt(2 + k0^2));
scale_in = period/fourier_factor;

% *******************************************************************
pad = 1; % pad the time series with zeroes (recommended)
dj = 0.01; % this will do 4 sub-octaves per octave
s0 = 1*dt; % this says start at a scale of 6 months
j1 = length(period) - 1; % do 7 powers-of-two with dj sub-octaves
lag1 = 0.72; % lag-1 autocorrelation for red noise background
mother = 'Morlet';

% Wavelet transform:
[wave,period,scale,coi] = wavelet_c(sst,dt,pad,dj,s0,j1,mother,k0,...
...scale_in);
power = (abs(wave).^2); % compute wavelet power spectrum
amp = sqrt(power)*sqrt(2);
% Significance levels: (variance=1 for the normalized SST)
[signif,fft_theor] = wave_signif(1.0,dt,scale_in,0,lag1,0.90,-1,...
...mother,k0);
sig95 = (signif')*(ones(1,n)); % expand signif -->(J+1)x(N) array
Chapter A: Concluding Remarks

\[ \text{sig95} = (\text{power./ sig95})/10; \]  
\% where ratio > 0.1, power is signif

% WAVELET ANALYSIS END ****************************************

% Plot Wavelet
a = find(time < 10,1,'last');
b = find(time > 20,1,'first');

figure(1)
subplot 212
contourf(time(a:b),period,power(:,a:b),...
...[0,0.5*max(max(power(:,a:b))):0.01:max(max(power(:,a:b)))]);
xlabel('Time UT (hours)')
ylabel('Period (minutes)')
shading flat;
colorbar;
hold on;
contour(time(a:b),period,sig95(:,a:b),[0,0.1:0.1:max(max(sig95))],...
...'w');
title('ACE IMF By May 10 2007')
grid on
wavelet.m

%WAVELET  1D Wavelet transform with optional singificance testing
%
% [WAVE,PERIOD,SCALE,COI] = wavelet(Y,DT,PAD,DJ,SO,J1,MOTHER,...
% ...PARAM,SCALE_IN)
%
% Computes the wavelet transform of the vector Y (length N),
% with sampling rate DT.
%
% By default, the Morlet wavelet (k0=6) is used.
% The wavelet basis is normalized to have total energy=1
% at all scales.
%
% INPUTS:
%
% Y = the time series of length N.
% DT = amount of time between each Y value, i.e. the sampling time.
%
% OUTPUTS:
%
% WAVE is the WAVELET transform of Y. This is a complex array
% of dimensions (N,J1+1). FLOAT(WAVE) gives the WAVELET amplitude,
% ATAN(IMAGINARY(WAVE),FLOAT(WAVE) gives the WAVELET phase.
% The WAVELET power spectrum is ABS(WAVE)^2.
% Its units are sigma^2 (the time series variance).
%
% OPTIONAL INPUTS:
%
% *** Note *** setting any of the following to -1 will cause the
% default value to be used.
%
% PAD = if set to 1 (default is 0), pad time series with enough
% zeroes to get N up to the next higher power of 2. This
% prevents wraparound from the end of the time series to the
% beginning, and also speeds up the FFT's used to do the
Chapter A: Concluding Remarks

wavelet transform. This will not eliminate all edge effects
(see COI below)

DJ = the spacing between discrete scales. Default is 0.25.
A smaller # will give better scale resolution, but be slower
to plot.

SO = the smallest scale of the wavelet. Default is 2*DT.

J1 = the # of scales minus one. Scales range from SO up to
SO*2^(J1*DJ). to give a total of (J1+1) scales. Default is
J1 = (LOG2(N DT/S0))/DJ.

MOTHER = the mother wavelet function.
The choices are 'MORLET', 'PAUL', or 'DOG'

PARAM = the mother wavelet parameter.
For 'MORLET' this is k0 (wavenumber). default is 6.
For 'PAUL' this is m (order), default is 4.
For 'DOG' this is m (m-th derivative), default is 2.

OPTIONAL OUTPUTS:

PERIOD = the vector of "Fourier" periods (in time units) that
corresponds to the SCALEs.

SCALE = the vector of scale indices, given by SO*2^(j*DJ), j=0...J1
where J1+1 is the total # of scales.

COI = if specified, then return the Cone-of-Influence, which is a
vector of N points that contains the maximum period of useful
information at that particular time. Periods greater than
this are subject to edge effects. This can be used to plot
COI lines on a contour plot by doing:

contour(time,log(period),log(power))
plot(time,log(coi),'k')
Chapter A: Concluding Remarks

%---------------------------------------------------------------------------------------
% Copyright (C) 1995-1998, Christopher Torrence and Gilbert P. Compo
% University of Colorado, Program in Atmospheric and Oceanic Sciences
% This software may be used, copied, or redistributed as long as it
% is not sold and this copyright notice is reproduced on each copy
% made. This routine is provided as is without any express or implied
% warranties whatsoever.
%
% Notice: Please acknowledge the use of this program in any
% publications:
% 'Wavelet software was provided by C. Torrence and G. Compo,
% and is available at URL:
% http://paos.colorado.edu/research/wavelets/'.
%
% Notice: Please acknowledge the use of the above software in any
% publications:
% 'Wavelet software was provided by C. Torrence and G. Compo,
% and is available at URL:
% http://paos.colorado.edu/research/wavelets/'.
%
% Wavelet Analysis. <I>Bull. Amer. Meteor. Soc.</I>, 79,
% 61-78.
%
% Please send a copy of such publications to either C. Torrence
% or G. Compo:
%
% Dr. Christopher Torrence
% Advanced Study Program
% National Center for Atmos. Research
% P.O. Box 3000
% Boulder CO 80307-3000, USA.
% E-mail: torrence@NCAR.noaa.gov
%
% Dr. Gilbert P. Compo
% NOAA/CIRES Climate Diagnostics Center
% Campus Box 216
function [wave, period, scale, coi] = wavelet_c(Y, dt, pad, dj, s0, Jl, mother, param, scale_in);

if (nargin < 8), param = -1;, end
if (nargin < 7), mother = -1;, end
if (nargin < 6), Jl = -1;, end
if (nargin < 5), s0 = -1;, end
if (nargin < 4), dj = -1;, end
if (nargin < 3), pad = 0;, end
if (nargin < 2)
    error('Must input a vector Y and sampling time DT')
end

n1 = length(Y);

if (s0 == -1), s0 = 2*dt;, end
if (dj == -1), dj = 1./4.;, end
if (Jl == -1), Jl = fix((log(n1*dt/s0)/log(2))/dj);, end
if (mother == -1), mother = 'MORLET';, end

%....construct time series to analyze, pad if necessary
x(1:n1) = Y - mean(Y);
if (pad == 1)
    base2 = fix(log(n1)/log(2) + 0.4999); % power of 2 nearest to N
    x = [x, zeros(1, 2^(base2+1)-n1)];
end
n = length(x);

%....construct wavenumber array used in transform [Eqn(5)]
k = [1:fix(n/2)];
k = k.*(2.*pi)/(n*dt));
k = [0., k, -k(fix((n-1)/2):-1:1)];
%...compute FFT of the (padded) time series
f = fft(x);  % [Eqn(3)]

%...construct SCALE array & empty PERIOD & WAVE arrays
%scale = s0*2.^((0:J1)*dj);
%period = scale;

scale = scale_in;
wave = zeros(J1+1,n);  % define the wavelet array
wave = wave + i*wave;  % make it complex

% loop through all scales and compute transform
for al = 1:J1+1
[daughter,fourier_factor,coi,dofmin]=wave_bases(mother,k,scale(al),...
...param);
%wave(al,:) = (100./al).*ifft(f.*daughter);  % wavelet trans[Eqn(4)]
  %toot = sum((daughter/sqrt(daughter)).^2)
  wave(al,:) = ifft(f.*daughter);
%subplot 211,plot(100./max(daughter).*daughter);
%subplot 212,plot(daughter/max(daughter));
%pause(.01);
end

period = fourier_factor*scale;
coi = coi*dt*[1E-5,1:((n1+1)/2-1),...
...flipr((1:(n1/2-1)),1E-5);  % COI [Sec.3g]
wave = wave(:,1:n1);  % get rid of padding before returning

return
wavelet_bases.m

%WAVE_BASES 1D Wavelet functions Morlet, Paul, or DOG
%
% [DAUGHTER,FOURIER_FACTOR,COI,DOFMIN] = ...
% wave_bases(MOTHER,K,SCALE,PARAM);
%
% Computes the wavelet function as a function of Fourier frequency,
% used for the wavelet transform in Fourier space.
% (This program is called automatically by WAVELET)
%
% INPUTS:
%
% MOTHER = a string, equal to 'MORLET' or 'PAUL' or 'DOG'
% K = a vector, the Fourier frequencies at which to calculate the
% wavelet
% SCALE = a number, the wavelet scale
% PARAM = the nondimensional parameter for the wavelet function
%
% OUTPUTS:
%
% DAUGHTER = a vector, the wavelet function
% FOURIER_FACTOR = the ratio of Fourier period to scale
% COI = a number, the cone-of-influence size at the scale
% DOFMIN = a number, degrees of freedom for each point in the
% wavelet power
% (either 2 for Morlet and Paul, or 1 for the DOG)
%
%------------------------------------------------------------------------
% Copyright (C) 1995-1998, Christopher Torrence and Gilbert P. Compo
% University of Colorado, Program in Atmospheric and Oceanic Sciences
% This software may be used, copied, or redistributed as long as it is
% not sold and this copyright notice is reproduced on each copy made.
% This routine is provided as is without any express or implied
% warranties whatsoever.
%------------------------------------------------------------------------

function[daughter,fourier_factor,coi,dofmin] = ...
    wave_bases(mother,k,SCALE,PARAM);
mother = upper(mother);
n = length(k);

if (strcmp(mother,'MORLET')) %----------------------------- Morlet
    if (param == -1), param = 6.;, end
    k0 = param;
    %expnt = -0.07.*(scale.*k - k0).^2/2.*k > 0.;
    expnt = -(scale.*k - k0).^2/2.*(k > 0.);
    norm = sqrt(scale*k(2))*(pi^(-0.25))*...
          ...sqrt(n); % total energy=N[Eqn(7)]
    daughter = norm.*exp(expnt);
    %daughter = (scale.^(-4/10)).*norm*exp(expnt);
    %daughter = (100./sqrt(scale)).*norm*exp(expnt);
    daughter = daughter.*(k > 0.); % Heaviside step function
    fourier_factor = (4*pi)/(k0 + ... ...
                      ...sqrt(2 + k0^2)); % Scale-->Fourier [Sec.3h]
    coi = fourier_factor/sqrt(2); % Cone-of-influence [Sec.3g]
    dofmin = 2; % Degrees of freedom
elseif (strcmp(mother,'PAUL')) %----------------------------- Paul
    if (param == -1), param = 4.;, end
    m = param;
    expnt = -(scale.*k).*k > 0.);
    norm = sqrt(scale*k(2))+(2^-m/sqrt(m*prod(2:(2*m-1))))*sqrt(n);
    daughter = norm*((scale.*k).^m).*exp(expnt);
    daughter = daughter.*(k > 0.); % Heaviside step function
    fourier_factor = 4*pi/(2*m+1);
    coi = fourier_factor*sqrt(2);
    dofmin = 2;
elseif (strcmp(mother,'DOG')) %----------------------------- DOG
    if (param == -1), param = 2.;, end
    m = param;
    expnt = -(scale.*k).^2 ./ 2.0;
    norm = sqrt(scale*k(2)/gamma(m+0.5))*sqrt(n);
    daughter = -norm*(i^-m)*((scale.*k).^m).*exp(expnt);
    fourier_factor = 2*pi*sqrt(2./(2*m+1));
    coi = fourier_factor/sqrt(2);
    dofmin = 1;
else
error('Mother must be one of MORLET,PAUL,DOG')
end
return
wavelet_signif.m

% WAVE_SIGNIF Significance testing for the 1D Wavelet transform WAVELET
% [SIGNIF,FFT_THEOR] = ...
% wave_signif(Y,DT,SCALE,SIGTEST,LAG1,SIGLVL,DOF,MOTHER,PARAM)
% INPUTS:
% Y = the time series, or, the VARIANCE of the time series.
% (If this is a single number, it is assumed to be the variance...)
% DT = amount of time between each Y value, i.e. the sampling time.
% SCALE = the vector of scale indices, from previous call to WAVELET.
% OUTPUTS:
% SIGNIF = significance levels as a function of SCALE
% FFT_THEOR = output theoretical red-noise spectrum as fn of PERIOD
% OPTIONAL INPUTS:
% ** Note ** setting any of the following to -1 will cause the default
% value to be used.
% SIGTEST = 0, 1, or 2. If omitted, then assume 0.
% If 0 (the default), then just do a regular chi-square test,
% i.e. Eqn (18) from Torrence & Compo.
% If 1, then do a "time-average" test, i.e. Eqn (23).
% In this case, DOF should be set to NA, the number
% of local wavelet spectra that were averaged together.
% For the Global Wavelet Spectrum, this would be NA=N,
% where N is the number of points in your time series.
% If 2, then do a "scale-average" test, i.e. Eqns (25)-(28).
% In this case, DOF should be set to a
% two-element vector [S1,S2], which gives the scale
% range that was averaged together.
e.g. if one scale-averaged scales between 2 and 8, then DOF=[2,8].

LAG1 = LAG 1 Autocorrelation, used for SIGNIF levels. Default is 0.0

SIGLVL = significance level to use. Default is 0.95

DOF = degrees-of-freedom for signif test.

IF SIGTEST=0, then (automatically) DOF = 2 (or 1 for MOTHER='DOG')

IF SIGTEST=1, then DOF = NA, the number of times averaged together.

IF SIGTEST=2, then DOF = [S1,S2], the range of scales averaged.

Note: IF SIGTEST=1, then DOF can be a vector (same length as SCALEs) in which case NA is assumed to vary with SCALE.

This allows one to average different numbers of times together at different scales, or to take into account things like the Cone of Influence.

See discussion following Eqn (23) in Torrence & Compo.

function [signif,fft_theor] = wave_signif(Y,dt,scalel,sigtest,lagl,siglvl,dof,mother,param);

if (nargin < 9), param = -1;, end
if (nargin < 8), mother = -1;, end
if (nargin < 7), dof = -1;, end
if (nargin < 6), siglvl = -1;, end
if (nargin < 5), lagl = -1;, end
if (nargin < 4), sigtest = -1;, end
if (nargin < 3)
Chapter A: Concluding Remarks

error('Must input a vector Y, sampling time DT, and SCALE vector')
end

n1 = length(Y);
J1 = length(scalel) - 1;
scale(1:J1+1) = scalel;
s0 = min(scale);
dj = log(scale(2)/scale(1))/log(2.);

if (n1 == 1)
variance = Y;
else
variance = std(Y)^2;
end

if (sigtest == -1), sigtest = 0;, end
if (lagl == -1), lagl = 0.0;, end
if (siglvl == -1), siglvl = 0.95;, end
if (mother == -1), mother = 'MORLET';, end

mother = upper(mother);

% get the appropriate parameters [see Table(2)]
if (strcmp(mother, 'MORLET'))
    k0 = param;
    fourier_factor = (4*pi)/(k0 + sqrt(2 + k0^2)); % Scale-->[Sec.3h]
    empir = [2.0, -1.0, -1.0];
    if (k0 == 6), empir(2:4) = [0.776, 2.32, 0.60];, end
    elseif (strcmp(mother, 'PAUL'))
        m = param;
        fourier_factor = 4*pi/(2*m+1);
        empir = [2.0, -1.0, -1.0, -1.0];
        if (m == 4), empir(2:4) = [1.132, 1.17, 1.5];, end
        elseif (strcmp(mother, 'DOG'))
            m = param;
            fourier_factor = 4*pi/(2*m+1);
            empir = [2.0, -1.0, -1.0, -1.0];
            if (m == 4), empir(2:4) = [1.132, 1.17, 1.5];, end
            elseif (strcmp(mother, 'DOG'))
                m = param;
fourier_factor = 2*pi*sqrt(2./(2*m+1));
empir = [1.,-1,-1,-1];
if (m == 2), empir(2:4) = [3.541,1.43,1.4];, end
if (m == 6), empir(2:4) = [1.966,1.37,0.97];, end
else
error('Mother must be one of MORLET,PAUL,DOG')
end

period = scale.*fourier_factor;
dofmin = empir(1); % Degrees of freedom with no smoothing
Cdelta = empir(2); % reconstruction factor
gamma_fac = empir(3); % time-decorrelation factor
dj0 = empir(4); % scale-decorrelation factor

freq = dt ./ period; % normalized frequency
fft_theor = ((1-lag1^2) ./ (1-2*lag1*cos(freq*2*pi)+...
...lag1^2)); % [Eqn(16)]
fft_theor = variance*fft_theor; % include time-series variance
signif = fft_theor;
if (dof == -1), dof = dofmin;, end

if (sigtest == 0) % no smoothing, DOF=dofmin [Sec.4]
dof = dofmin;
chisquare = chisquare_inv(siglvl,dof)/dof;
signif = fft_theor*chisquare; % [Eqn(18)]
elseif (sigtest == 1) % time-averaged significance
if (length(dof) == 1), dof=zeros(1,J1+1)+dof;, end
truncate = find(dof < 1);
dof(truncate) = ones(size(truncate));
dof = dofmin*sqrt(1 + (dof*dt/gamma_fac ./ scale).^2 ); % [Eqn(23)]
truncate = find(dof < dofmin);
dof(truncate) = dofmin*ones(size(truncate)); % minimum DOF is dofmin
for al = 1:J1+1
chisquare = chisquare_inv(siglvl,dof(al))/dof(al);
signif(al) = fft_theor(al)*chisquare;
end
elseif (sigtest == 2) % time-averaged significance
if (length(dof) == 2)
error('DOF must be set to [S1,S2], the range of scale-averages')
end
if (Cdelta == -1)
error(['Cdelta & dj0 not defined for ',mother, ...'
' with param = ',num2str(param)])
end
s1 = dof(1);
s2 = dof(2);
avg = find((scale >= s1) & (scale <= s2));  % scales between S1 & S2
navg = length(avg);
if (navg == 0)
error(['No valid scales between ',num2str(s1),' and ',num2str(s2)])
end
Savg = 1./sum(1 ./ scale(avg));  % [Eqn(25)]
Smid = exp((log(s1)+log(s2))/2.);  % power-of-two midpoint
dof = (dofmin*navg*Savg/Smid)*sqrt(1 + (navg*dj/dj0)^2);  % [Eqn(28)]
fft_theor = Savg*sum(fft_theor(avg) ./ scale(avg));  % [Eqn(27)]
chisquare = chisquare_inv(siglvl,dof)/dof;
signif = (dj*dt/Cdelta/Savg)*fft_theor*chisquare;  % [Eqn(26)]
else
error('sigtest must be either 0, 1, or 2')
end
return
chisquare_inv.m

function X = chisquare_inv(P,V);
%CHISQUARE_INV Inverse of chi-square cumulative distribution
% function (cdf).
% %
% % X = chisquare_inv(P,V) returns the inverse of chi-square cdf with V
% % degrees of freedom at fraction P.
% % This means that P*100 percent of the distribution lies
% % between 0 and X.
% %
% % To check, the answer should satisfy:  P==gammainc(X/2,V/2)
%
% Uses FMIN and CHISQUARE_SOLVE
%
% Written January 1998 by C. Torrence

if (nargin < 2), error('Must input both P and V');, end
if ((1-P) < 1E-4), error('P must be < 0.9999');, end

if ((P==0.95) & (V==2)) % this is a no-brainer
X = 5.9915;
return
end

MINN = 0.01; % hopefully this is small enough
MAXX = 1; % actually starts at 10 (see while loop below)
X = 1;
TOLERANCE = 1E-4; % this should be accurate enough
    vers = version;
    vers = str2num(vers(1));

while ((X+TOLERANCE) >= MAXX) % should only need to loop thru once
    MAXX = MAXX*10.;
    % this calculates value for X, NORMALIZED by V
    % Note: We need two different versions, depending upon the version
    % of Matlab.
    if (vers >= 6)
\[ X = \text{fminbnd('chisquare\_solve',MINN,MAXX,...} \\
\text{...optimset('TolX',TOLERANCE),P,V);} \\
\text{\textbf{else}} \\
X = \text{fmin('chisquare\_solve',MINN,MAXX,\begin{footnotesize}[0,TOLERANCE]\end{footnotesize},P,V);} \\
\text{\textbf{end}} \\
\text{MINN} = \text{MAXX;} \\
\text{\textbf{end}} \\
X = X*V; \text{ \% put back in the goofy V factor}
chisquare_solve.m

function PD = chisquare_solve(XGUESS, P, V);

%CHISQUARE_SOLVE Internal function used by CHISQUARE_INV
%
%   PD = chisquare_solve(XGUESS, P, V) Given XGUESS, a percentile P,
%   and degrees-of-freedom V, return the difference between
%   calculated percentile and P.
%
% Uses GAMMAINC
%
% Written January 1998 by C. Torrence

% extra factor of V is necessary because X is Normalized
PGUESS = gammainc(V*XGUESS/2, V/2); % incomplete Gamma function

PD = abs(PGUESS - P); % error in calculated P

TOL = 1E-4;
if (PGUESS >= 1-TOL) % if P is very close to 1 (i.e. a bad guess)
    PD = XGUESS; % then just assign some big number like XGUESS
end
A.2 Time Lag

sfasd

time_lab.m

clear all
clc

addpath('C:\Users\RMFS\Documents\MATLAB\Thesis\Functions')
addpath('C:\Users\RMFS\Documents\MATLAB\Thesis\Data\Ionosphere\...
...MSP\Modified')
addpath('C:\Users\RMFS\Documents\MATLAB\Thesis\Data\Ionosphere\CCD')
addpath('C:\Users\RMFS\Documents\MATLAB\Thesis\Original_Code')

[datel,time1,bx,by,bz] = textread('C:\Users\RMFS\Documents\MATLAB...
...\Thesis\Data\Satellite\WIND\08_14_2006\WI_HO_MFI_11834.txt',...
...'s%s%f%f%f');
[datel,time2,x,y,z,vx,vy,vz] = textread('C:\Users\RMFS\Documents\...
MATLAB...\Thesis\Data\Satellite\WIND\08_14_2006...
...WI_K0_SWE_23366.txt','%s%s%f%f%f%f%f%f');
[datel,time3,p] = textread('C:\Users\RMFS\Documents\MATLAB...
...\Thesis\Data\Satellite\WIND\08_14_2006\OMNI_HRO_1MIN_14616.txt',...
...'s%s%f');

%Load time vectors
timeB = TIME(time1);
timeV = TIME(time2);
timeP = TIME(time3);

%Interpolate bad data points
bx = INTRP(bx,timeB,-1.00000E+31);
by = INTRP(by,timeB,-1.00000E+31);
bz = INTRP(bz,timeB,-1.00000E+31);
vx = INTRP(vx,timeV,-1.00000E+31);
vy = INTRP(vy,timeV,-1.00000E+31);
vz = INTRP(vz,timeV,-1.00000E+31);
p = INTRP(p,timeP,99.9900);
%Constant values
alpha = 2.44; %Unitless; varies 2 to 3
beta = 0.25; %Unitless; varies 0.2 to 0.3
Beq = 31000*10^-9; %T
mu_o = (4*pi)*10^-7; %H/m
Re = 6378; %km

%Time interval of event
%************************
interval = [17 18.5];
dt_B = timeB(2) - timeB(1);
i = floor(interval(1)/dt_B);
j = ceil(interval(2)/dt_B);
dt_V = timeV(2) - timeV(1);
ii = floor(interval(1)/dt_V);
jj = ceil(interval(2)/dt_V);
dt_P = (timeP(2) - timeP(1));
iii = floor(interval(1)/dt_P);
jjj = ceil(interval(2)/dt_P);

bx = bx(i:j);
by = by(i:j);
bz = bz(i:j);
x = x(ii:jj);
y = y(ii:jj);
z = z(ii:jj);
vx = vx(ii:jj);
vy = vy(ii:jj);
vz = vz(ii:jj);
p = p(iii:jjj)*10^-9;

%Calculations
%*************
By = mean(by);
%By_std = mean(by)/length(by);
$By_{\text{std}} = \text{std}(by)$;
$Bx = \text{mean}(bx)$;
$%Bx_{\text{std}} = \text{mean}(bx)/\text{length}(bx)$;
$Bx_{\text{std}} = \text{std}(bx)$;

%Solar wind normal orientation
for n = 1:length(bx)
    if $Bx > 0$
        $\phi = \frac{90}{180}\pi - \text{abs(atan2}(By,Bx))$;
    elseif $Bx < 0$
        $\phi = \text{abs(atan2}(By,Bx)) - \frac{90}{180}\pi$;
    end
end
$\phi_{\text{std}} = \left( \left( \frac{1}{Bx/(1+By^2/Bx^2))*\text{By}_{\text{std}}} \right)^2 +...\left( \left( \frac{-By/Bx^2/(1+By^2/Bx^2))*\text{By}_{\text{std}}} \right)^2 \right)^{(1/2)}$;

$P = \text{mean}(p)$;
$P_{\text{std}} = \text{std}(p)$;

$D = \text{Re}*(\left(\alpha*Beq^2/(2\mu_o*P)\right)^{(1/6)}$;
$D_{\text{std}} = \left( \left( -1/12*\text{Re}^2(5/6)/(\alpha^2*Beq^2/\mu_o/P)^{(5/6)}*... \right. \left. \alpha^2*Beq^2/\mu_o/P^2)*\text{P}_{\text{std}} \right)^2 \right)^{(1/2)}$;

$Y = \text{mean}(y)$;
$Y_{\text{std}} = \text{std}(y)$;
$Z = \text{mean}(z)$;
$Z_{\text{std}} = \text{std}(z)$;

$L = \text{sqrt}(Y^2+Z^2)$;
$L_{\text{std}} = \left( \left( 1/(Y^2+Z^2)^{(1/2)}*Y)*Y_{\text{std}} \right)^2 +...\left( 1/(Y^2+Z^2)^{(1/2)}*Z)*Z_{\text{std}} \right)^2 \right)^{(1/2)}$;

$Vx = \text{mean}(vx)$;
$Vx_{\text{std}} = \text{std}(vx)$;
$Vy = \text{mean}(vy)$;
$Vy_{\text{std}} = \text{std}(vy)$;
$Vz = \text{mean}(vz)$;
$Vz_{\text{std}} = \text{std}(vz)$;
\[
V = \sqrt{V_x^2 + V_y^2 + V_z^2};
\]
\[
V_{\text{std}} = \left( \left( \frac{1}{(V_x^2 + V_y^2 + V_z^2)^{1/2}} \right)^2 + \left( \frac{1}{(V_x^2 + V_y^2 + V_z^2)^{1/2}} \right)^2 + \left( \frac{1}{(V_x^2 + V_y^2 + V_z^2)^{1/2}} \right)^2 \right)^{1/2};
\]

if \( B_x < 0 \) && \( B_y < 0 \)
if \( Y < 0 \)
\[
S = L \tan(\phi);
\]
elseif \( Y > 0 \)
\[
S = 0;
\]
end
end

if \( B_x > 0 \) && \( B_y < 0 \)
if \( Y < 0 \)
\[
S = 0;
\]
elseif \( Y > 0 \)
\[
S = L \tan(\phi);
\]
end
end

if \( B_x < 0 \) && \( B_y > 0 \)
if \( Y < 0 \)
\[
S = 0;
\]
elseif \( Y > 0 \)
\[
S = L \tan(\phi);
\]
end
end

if \( B_x > 0 \) && \( B_y > 0 \)
if \( Y < 0 \)
\[
S = L \tan(\phi);
\]
elseif \( Y > 0 \)
\[
S = 0;
\]
end
end
if \( S == 0 \)
\[
S_{\text{std}} = 0;
\]
else
\[
S_{\text{std}} = \left( \left( \left( \tan(\phi) \right) L_{\text{std}} \right)^2 + \left( L \left( 1 + \tan(\phi) \right)^2 \right) \ldots \phi_{\text{std}} \right)^2 \right)^{1/2};
\]
end

\[ X = \text{mean}(x); \]
\[ X_{\text{std}} = \text{std}(x); \]

\[
\tau_{\text{u1}} = \left( X + S + \left( 7 \beta - 1 \right) D \right) / V / 60;
\]
\[
\tau_{\text{u1, std}} = \left( \left( 1 / V \right) X_{\text{std}} \right)^2 + \left( 1 / V \right) S_{\text{std}} \right)^2 + \left( \left( 7 \beta - 1 \right) / V \right) \ldots D_{\text{std}} \right)^2 + \left( - \left( X + S + \left( 7 \beta - 1 \right) D \right) / V^2 \right) V_{\text{std}} \right)^2 \right)^{1/2}/60;
\]

\[
\tau_{\text{u2}} = 18;
\]
\[
\tau_{\text{u2, std}} = 10;
\]

\[
\tau = \tau_{\text{u1}} + \tau_{\text{u2}}
\]
\[
\tau_{\text{std}} = \left( \left( 1 \tau_{\text{u1, std}} \right)^2 + \left( 1 \tau_{\text{u2, std}} \right)^2 \right)^{1/2}
\]
A.3 Support Functions

The following MATLAB codes were to perform more mundane tasks such as reading data files and interpolating to replace bad data points in the original data files.

**INTRP.m**

This function was designed for replacing bad data points for satellite data. Specifically used with ACE and WIND for the MFI, 3DP, and SWE instrumentations.

The input of the function is a time series sampled at regular intervals, containing bad data points and the corresponding time vector. Also the form of the bad data point (i.e. 9999, 1.00000x10^-31, etc...).

The output of the function is the same time series with the bad data points replaced. The method used for replacement of data points is linear interpolation for any points in the middle and linear extrapolation for the two end points of the time series.

```matlab
function [V] = INTRP(V,t,error)

% Finds index location of a bad data point
E = length(V);
m = 1;
I(1) = 0;
for n = 1:E
    if V(n) == error
        I(m) = n;
        m = m + 1;
    end
end

% Exits function if no bad data points are in the file
if I(1) == 0
    return
end
```
%Extrapolates the first data point if it is bad
if V(1) == error
  if V(2) ~= error && V(3) ~= error
    V(1) = V(2)-(V(3)-V(2))/(t(3)-t(2))*(t(2)-t(D));
  end
end

F = I(length(I));
a = 0;
ii = 1;
jj = 1;
if V(1) == error
  if V(2) == error
    while ii <= 50 && a == 0
      if V(2 + ii) ~= error
        a = 2 + ii;
        b = a + 1;
      end
      ii = ii + 1;
    end
  end
end

if V(b) == error
  while jj <= 50
    if V(b + jj) ~= error
      b = b + jj;
    end
    jj = jj + 1;
  end
end

V(1) = V(a)-(V(b)-V(a))/(t(b)-t(a))*(t(a)-t(D));
end

%Extrapolates the last data point if it is bad
if V(E) == error
  if V(E-1) ~= error && V(E-2) ~= error
    V(E) = (V(E-1)-V(E-2))/(t(E-1)-t(E-2))*(t(E)-t(E-1))+V(E-1);
  end
end
\begin{verbatim}
end

F = I(length(I));
a = 0;
ii = 1;
jj = 1;
if V(E) == error
    if V(E-1) == error
        while ii <= 50 && a == 0
            if V(F - 1 - ii) ~= error
                a = F - 1 - ii;
b = a - 1;
            end
            ii = ii + 1;
        end
    end

if V(b) == error
    while jj <= 50
        if V(b - jj) ~= error
            b = b - jj;
        end
        jj = jj + 1;
    end

V(E) = (V(a) - V(b)) / (t(a) - t(b)) * (t(E) - t(a)) + V(a);
end

% Recalculate the index location of bad data points
m = 1;
J(1) = 0;
for n = 1:E
    if V(n) == error
        J(m) = n;
m = m + 1;
    end
end
\end{verbatim}
\begin{verbatim}
J(length(J)+1)=0;

% Exits function if no bad data points are in the file
if J(1) == 0
    return
end

% Fix bad data points
for n = 1:(length(J)-1)
    % Replaces single data points not at the ends
    if V(J(n) + 1) ~= error
        V(J(n)) = (V(J(n)+1) - V(J(n)-1)) / (t(J(n)+1) - t(J(n)-1)) * ...
        ... (t(J(n)) - t(J(n)-1)) + V(J(n)-1);
    endif
    % Replaces groups of data points not at the ends
    elseif V(J(n) + 1) == error
        for nn = 1:150
            if V(J(n) + 1 + nn) ~= error
                V(J(n)) = (V(J(n)+1+nn) - V(J(n)-1)) / (t(J(n)+1+nn) - ...
                ... t(J(n)-1)) * (t(J(n)) - t(J(n)-1)) + V(J(n)-1);
                break
            end
        end
    end
end
end
\end{verbatim}
TIME.m

%This function will create the time vector for an imported text
%through textread in the format HH:MM:SS.SSS

%The required input is the time as a string of text in the above
%format

function [time] = TIME(time_string)

for uu = 1:length(time_string);
    hours(uu) = str2num(time_string{uu}(1,1:2));
    mins(uu) = str2num(time_string{uu}(1,4:5));
    secs(uu) = str2num(time_string{uu}(1,7:12));
end

time = hours' + mins'./60 + secs'./3600;
Bibliography


