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Orbit Determination and Prediction for Low or Intermediate Altitude Satellites

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Summary

A computer system is described which can process raw tracker data from low or intermediate altitude artificial satellites and produce a tracker drive tape for future passes. Alternately (once the future passes have been tracked) graphical printout of the errors between actual and predicted pointing angles and ranges can be obtained. The system is a descendant of those used for the intermediate altitude *TELESTAR*® satellites.

The use of the system with data from a low altitude satellite (perigee height \(110 \text{ miles} \), eccentricity \(0.02 \), inclination angle \(50^\circ \)) is documented. One particular result, obtained by processing 10 passes from 5 sites over a 3-1/2 day interval (estimating the drag parameter from data within this interval), and then making a one day prediction had typical geocentric angular errors of \(0.005^\circ \). These errors have the same magnitude as those obtained when reconstructing the data, and thus could be reduced significantly only by obtaining more accurate data.

This discussion is presented in the hope that more detailed descriptions of operational procedures than are usually found in the literature may prove beneficial in practical situations.

Introduction

This paper describes a computer system for the determination and prediction of low or intermediate altitude satellite orbits. This is a complete orbit determination and prediction system since it can process a raw data tape and produce a tracker drive tape for future passes. Thus it could be used in the field for satellite orbit refinement. However, it is believed to be compact, rapid, and modularized sufficiently to be useful for theoretical studies, as well. Later, we give some numerical results using real data to demonstrate its operational usefulness and several brief studies to indicate the effect of varying particular modules.

This system is a descendant of one which was developed for use with the intermediate altitude communications satellites of the *TELESTAR*® project. The rationale behind this earlier system and a description of its data processing and orbit prediction routines are given in Refs. 1 and 2. The earlier system had been written in *FORTAN* II for the IBM 7090 computer; this necessitated conversion to *FORTAN IV* for use on the GE 600-series computers now available. Since the system consists of about 50 programs and subroutines, the conversion was a lengthy procedure. However, the conversion allowed for the use of various new programming features now available. The resulting system requires less than 35K of core storage; thus it can all fit into the computer at once. Computation time has been reduced by a factor of perhaps 10. Operation is also considerably simplified.

One conceptual change has been made in the treatment of intermediate altitude orbits, namely the conversion to node-to-node operation. This can result in considerable savings in computer time and storage. (The present version has not made complete use of this option although it could be done fairly easily by replacing certain modules. See Ref. 3 for a discussion of the savings resulting when the orbit prediction method is converted.) The only resulting complication is that after new data has been processed, the new elements should be referred back to the node. This is easily implemented.

The major change has been the incorporation of the capability of treating low altitude orbits. This requires a consideration of the effects of atmospheric drag. (See Refs. 4 and 5.)

In the following section we give a general functional description of the system.

Functional Description

Initial estimates for the orbital elements of the satellite are obtained by fitting a Keplerian ellipse between two data points observed from a single site. These elements are then referenced to the nearest nodal crossing, either by the simple expedient of assuming them constant or by integrating numerically to the node. Whatever procedure is followed is performed only once for any satellite. The inverse covariance matrix associated with these initial estimates is set equal to zero so that they are given zero weight during the data processing. Such an automated starting procedure is convenient to have; moreover, a simple procedure such as the above is accurate enough for the data processing method.
All previously processed data are represented by a set of orbital elements and an associated covariance matrix. These quantities are processed with new data i.e., using sequential batch filtering, and a revised set of orbital elements and its covariance matrix are produced. A "fading memory" factor is applied to the inverse covariance matrix of the revised estimate to de-weight the effects of old data.

The types of data which can be handled at present are angles only or angles plus ranges from any number of sites. The data enter the data processing program on tape after certain pre-processing, such as rejection of faulty data points, re-arrangement of data, and refraction correction, has occurred. The production of such a tape is rather dependent upon the particular project being considered and thus the tape provides a useful conceptual interface between the real world and the system.

The data being processed are considered to come from an unperturbed ellipse. This is implemented by removing dynamic trends due to perturbations from the data either by an Encry-type perturbation method or a series expansion for short passes. This in-trap element updating is sufficiently accurate when only oblateness is considered; however, routines which include the drag perturbation are also available. The ephemeris production procedure (i.e., the production of drive tapes) utilizes the trend removal subroutines, thus reducing the size of the system.

The revised set of orbital elements (after processing of new data) may no longer be valid at the nodal crossing as was the original set. This effect is small; however, we account for it in the case of the satellite period, since long range prediction accuracy is sensitive to small errors in this element.

Inter-pass updating of the orbital elements is performed by an analytic perturbation method which includes the first and second order effects of the oblateness harmonic and the first order effects of the third and fourth harmonics plus luni-solar gravitation, if desired. First order drag effects can also be included. The luni-solar perturbations would be included (by setting certain logical variables) when considering intermediate altitude satellites such as the TELSTAR satellites; the drag perturbation would be included for low altitude satellites. The effects of oblateness are considered during the inter-pass updating of the elements of the covariance matrix.

The analytic perturbation method is also used to predict future passes. The latest estimates of the orbital elements are updated and used in a satellite visibility program which calculates visibility intervals for the satellite over a site or mutual visibility between a pair of sites.

The updated elements are used in the ephemeris production routine to produce pointing angles. These can simply be listed, or used to produce a tracker drive tape. Also a graph routine is available which will produce plots of the discrepancies between predicted pointing angles and ranges and the actual pointing angles and ranges when these become available.

Use of the System with Real Data

The use of this system on intermediate altitude satellites should essentially duplicate the results obtained with the system used successfully on the TELSTAR satellites. Here we will consider in detail the problem of orbit determination and prediction for a low altitude, high drag satellite.

Raw data were obtained on a satellite with perigee height 1,100 miles, eccentricity ~ 0.23, inclination ~ 20°, and nominal drag parameter (m/C_A) ~ 0.03 slugs/ft². Some of the passes used are briefly described in Table 1. The five sites, labeled A-E, are widely separated, spanning 55° in latitude (all in the Northern hemisphere) and 80° in longitude (all in the Western hemisphere). Refraction corrections were applied to the data by assuming reasonable values for temperature, humidity, and barometric pressure, since the actual values were not available. The passes listed in Table 1 are about 20% of those available for that time interval, so an excessive amount of data was not used. Typical pass lengths were 4.6 minutes, data rates were 1 point every four seconds.

Using initial estimates obtained by fitting an ellipse between two points, the data of pass 2 were processed, and predictions were made for pass 4. A nominal value of D (m/C_A = 2.0) was used. The errors in re-constructing the data of pass 2 using the elements obtained from the data are shown in Figs. 1-3, and the prediction errors for pass 4 are shown in Figs. 4-6.

Consideration of Figs. 1 and 2 indicates angular errors with a scatter of about ±0.004 radians. These are pointing angle (azimuth and elevation) errors. To gain some feeling for these note first that 0.004 radians is about 0.24°. The TELSTAR satellites were about 10 times higher in altitude. Thus typical errors for these satellites would be about 0.24°. This is still too high (i.e. from previous experience) by a factor of about 10. Thus the sigmas for these trackers are about 10 times higher than those for the Anodover horn. In processing the data, values of σ_R and σ_a were set at 0.003° and σ_R = 1000 ft. were used. The range values being taken deliberately large to minimize computational difficulties known to be possible otherwise (again from experience with the TELSTAR satellites).

We see from Figs. 1-2 that more correct values for σ_a and σ_R would be perhaps 0.003°. Our results are unchanged by multiplying all sigmas by a constant; thus we may consider our σ_R = 10,000 ft. This high value of σ_R results in the range values being de-weighted and the ranges seem to serve largely to ensure computational stability. (Perhaps

* These estimates are listed in Table 2 along with the correct values.
it is worth explicitly nothing that the ability
to converge to acceptable values of orbital
elements from one short pass (pass 2 is less than
4 minutes in length) is indicative of a quite
sturdy data processing method.) Lower range
errors, probably with some slight increase in
angular errors could be obtained by decreasing
the value of $C_R$ used; this was not done in the
present study. We should further note that the
geo-centric errors corresponding to our pointing
angle errors are obtained approximately by
multiplying by (100 miles/4000 miles) or $1/40$.
Thus .01° error in pointing angles corresponds
to about .006° of geocentric angular error.

Consider the predictions for pass 4 as
indicated in Figs. 4-6. The errors are con-
siderably larger but the results seem to be good
enough for acquisition. Combining passes 2 and 4,
still with $D = 3.025$, and predicting pass 6 results
in the reconstruction errors shown in Figs. 7-8
for pass 2 and Figs. 9-10 for pass 4, and the
prediction errors for pass 6 shown in Figs. 11-12.

The errors in reconstructing pass 2 based on
2 passes are comparable to the one pass results.
(Compare Figs. 7-8 with Figs. 1-2.) The errors in
reconstructing pass 4 (Figs. 9-10) are somewhat
larger. Consider the prediction errors for pass 6.
We first note (Fig. 11) the large errors in azimuth
around the center of the pass (about 50 seconds
before and after the point of maximum elevation).
This is a high elevation pass (see Table 1); refer-
ring to the data tape we see that the azimuth rate
at the center of the pass is over 6°/second. Since
this peak is noticed only on high passes with very
large azimuth rates we may attribute it to dynamic
lags in the trackers. The elevation error is more
interesting (see Fig. 12). It is fairly easy to
convince oneself that this shape error curve is due
to a time shift between the predicted and actual
elevations. Some reflection will show that the
"predicted satellite" is always leading the actual
case. A probable reason for this is an incorrect
value of the drag parameter.

A lead is caused by over-estimating the drag.
This means that $D$ (which equals $m/a_C^2$) is too small
and should be increased. Figs. 13-16 show the
result of implementing this hypothesis. Passes
2,4,6 were combined and used to predict pass 7,
using $D = 3.025$. The results are shown in Figs.
13-14. Typical scatters of errors of ±.004° are
again apparent. (The single point in Fig. 14
with $m = 13/1000$ apparently fell through the
data rejection process.)

Similarly, the results of combining passes
2-6, adding pass 7 and predicting pass 10, using
$D = 3.025$, are shown in Figs. 15-16. The errors
are again of the expected magnitude. We denote
combination of passes 2,4,6 and addition of 7 as
(2,4,6), 7 on the graph. This is a convenient
short-hand since the usual use of the method
consists of starting by combining the data from
several passes and then sequentially adding a pass
at a time.

The fact that the nominal value of $D$ had
to be changed by over 50% for optimal prediction
may seem alarming. However, we should realize
that what we are fitting is really some product
of the drag coefficient and a correction to the
mean density. We have represented the density
by an exponential fit to the 1962 U.S. Standard
Atmosphere. This is an average atmosphere; the
actual density can vary by a factor of three at
100 n.mi. over the extremes of the solar
cycle. Thus our choice of $D$ is a calibration
of our atmosphere as well as of $D$.

We have chosen $D$ so that it reproduces well
the data of the first day. This would be a
futile exercise if the same value could not be
used for future predictions. This can indeed
be done, as is shown by Figs. 17-18 where we
have processed the data from the first 3½ days,
i.e. passes (2,4,6), 7, 10, 16, 21, 25, 28, 36
and made a one day prediction using $D = 3.025$.
The usual scatter of ±.004° in pointing angles
(±.006° in geocentric angles) is seen again.
A similar result is seen in Figs. 19-20, where
we process in addition pass 5½ and predict 55,
still with $D = 3.025$.

Again in Fig. 19 we see large azimuth errors
at the center of the pass, and referring to
Table 1 we see the reason is as before, namely
a high elevation pass with large azimuth rates
leading to dynamic lags in the tracker. It is
possible that somewhat better results could be
obtained by adding a test on azimuth rate to
the data rejection process.

The results might also be improved by
estimating $D$ after each pass is processed by
optimizing the prediction of the next pass.
This could very well be automated. The method
of bias reduction without bias estimation
developed by Claus could be profitably used
here since we are not really interested in the
numerical value of $D$, but only in optimizing
our prediction accuracy.

Finally, let us note that since the scatter
in errors in prediction is comparable in mag-
nitude to the scatter of errors in reconstruc-
tion, significant increase in accuracy is not
possible without more accurate data.

Conclusion

The system is usable in its present form
for operational orbit determination and prediction.
The convenience of operation would be
enhanced by an automated treatment of the drag
magnitude.

Some Experimental Results

To illustrate the use of the system as a
theoretical tool we briefly describe several
studies which were performed:

1. Effect of no perturbations in the matrix
    updating: To test the effect of the

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* This may seem paradoxical. It is arrived at as
follows: Drag too high → decrease in period too
big → period too small → predicted satellite
arrives first.
omission of the oblateness perturbation* during the updating of the elements of the covariance matrix a special routine (MUST C = 0) was used which set these perturbations to zero. The results are shown in Figs. 21-22, which should be compared with Figs. 17-18. Some degradation is apparent, although the results might be improved somewhat by a new calibration of D.

2. Effect of simplified trend removal and ephemerides production: The run whose results are shown in Figs. 17-18 required .0214 hr (77 sec) of GE 635 computer time. This used an Encke-type perturbation method to calculate the intra-pass perturbations. Since the passes for low altitude satellites are short, a simplified method based on a Taylor series expansion around the center of the pass can be used. Using such a subroutine (NEW CMLR2) the results in Figs. 23-24 were obtained. The errors are comparable in magnitude to Figs. 17-18 and computing time was .0196 hr (70.5 sec), a reduction of 8.5%.

3. Effect of "fading memory": Based on TELSTAR experience, a factor, F = exp(-P/0.5), was applied to the elements of the inverse covariance matrix to de-weight the effects of past observations. (P is the number of periods elapsed; 0 is chosen so that P = .5 when P = 30 i.e. after 2 days.) The results of runs with and without fading memory are shown in Table 3. They are not significantly different. The covariance matrix associated with the fading memory results is probably more realistic, however. Longer runs would be needed to really test the necessity of the fading memory.

4. Effect of simplified perturbation analysis during iterations of the filter: When the initial estimates are updated to the center of the first pass the deltas in the elements are stored.1 Then, when the elements are changed after the first iteration of the data processing program, the stored deltas are used (to avoid recomputing the perturbations) to obtain the inputs for the second iteration. This is repeated before the third iteration. A test run was made in which the deltas were recomputed each time and the results are compared against the results of the usual simplified procedure in Table 2. As expected, the discrepancies are minor and the use of the simplified method is justified.

Acknowledgments

The raw data tape was obtained with the assistance of L. J. Vanden Bos of the Aerospace Corporation and F. W. Kruczak. F. T. Geyling suggested this study and provided periodic encouragement. I am grateful to A. J. Claus for many helpful discussions and to Mrs. N. Katz and Mrs. C. B. Wood who ably retraced the steps of an entire project group of programmers in producing this computer system. R. G. Schonbeck, I. T. Cundiff, and W. C. Ridgway, III were responsible for earlier versions of several programs and aided in their conversion.

Table 1

<table>
<thead>
<tr>
<th>No.</th>
<th>Site</th>
<th>Day</th>
<th>Maximum Elevation (degrees)</th>
<th>Time (seconds)</th>
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<tr>
<td>2</td>
<td>E</td>
<td>1</td>
<td>7.7°</td>
<td>29 748</td>
</tr>
<tr>
<td>4</td>
<td>E</td>
<td>2</td>
<td>22.5°</td>
<td>35 216</td>
</tr>
<tr>
<td>6</td>
<td>E</td>
<td>4</td>
<td>76.5°</td>
<td>40 712</td>
</tr>
<tr>
<td>7</td>
<td>B</td>
<td>5</td>
<td>33.4°</td>
<td>45 500</td>
</tr>
<tr>
<td>10</td>
<td>A</td>
<td>1</td>
<td>20.9°</td>
<td>56 780</td>
</tr>
<tr>
<td>16</td>
<td>A</td>
<td>3</td>
<td>31.8°</td>
<td>3 976</td>
</tr>
<tr>
<td>21</td>
<td>E</td>
<td>4</td>
<td>78.5°</td>
<td>41 528</td>
</tr>
<tr>
<td>23</td>
<td>C</td>
<td>5</td>
<td>17.0°</td>
<td>5 280</td>
</tr>
<tr>
<td>28</td>
<td>A</td>
<td>6</td>
<td>85.0°</td>
<td>52 868</td>
</tr>
<tr>
<td>36</td>
<td>D</td>
<td>7</td>
<td>76.8°</td>
<td>31 604</td>
</tr>
<tr>
<td>54</td>
<td>E</td>
<td>8</td>
<td>19.8°</td>
<td>32 832</td>
</tr>
<tr>
<td>55</td>
<td>E</td>
<td>9</td>
<td>74.5°</td>
<td>38 324</td>
</tr>
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</table>

Table 2

<table>
<thead>
<tr>
<th>Node Angle (radians)</th>
<th>Initial Estimate</th>
<th>With Updating</th>
<th>Without Updating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.773698</td>
<td>4.773532</td>
<td>4.773511</td>
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<tr>
<td>Inclination Angle (radians)</td>
<td>1.393268</td>
<td>1.395392</td>
<td>1.395390</td>
</tr>
<tr>
<td>Period (seconds)</td>
<td>5465.173</td>
<td>5455.556</td>
<td>5455.551</td>
</tr>
<tr>
<td>e cos ω *</td>
<td>-0.014344</td>
<td>-0.0145902</td>
<td>-0.0145672</td>
</tr>
<tr>
<td>e sin ω *</td>
<td>0.0153710</td>
<td>0.0142139</td>
<td>0.0142058</td>
</tr>
<tr>
<td>T (seconds)</td>
<td>-27.4700</td>
<td>-25.4965</td>
<td>-25.4975</td>
</tr>
</tbody>
</table>

* e is eccentricity, ω is argument of perigee, T is time of pseudonodal passage.

* The updating required even in Keplerian motion due to the change from one nodal passage to the next is of course retained.

† This is always done. However, the effect is greatest at the start since the initial estimates may be in error by a large amount and they may have large corrections applied to them.
Table 3
Effect on elements at pass 36 of fading memory
on inverse covariance matrix of elements

<table>
<thead>
<tr>
<th></th>
<th>With Pading Memory</th>
<th>Without Pading Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node Angle (radians)</td>
<td>4.696041</td>
<td>4.696045</td>
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<tr>
<td>Inclination Angle (radians)</td>
<td>1.395457</td>
<td>1.395444</td>
</tr>
<tr>
<td>Period (seconds)</td>
<td>5450.9327</td>
<td>5450.9409</td>
</tr>
<tr>
<td>$e \cos \omega$</td>
<td>-.0119827</td>
<td>-.0119344</td>
</tr>
<tr>
<td>$e \sin \omega$</td>
<td>.0163799</td>
<td>.0163840</td>
</tr>
<tr>
<td>$T$ (seconds)</td>
<td>-28.9998</td>
<td>-28.8661</td>
</tr>
</tbody>
</table>

References
Fig. 1 - Azimuth Delta - File 2
(2) D=2.0

Fig. 2 - Elevation Delta - File 2
(2) D=2.0
Fig. 3 - Range Delta - File 2
D=2.0

Fig. 4 - Azimuth Delta - File 4
D=2.0
Fig. 5 - Elevation Delta - File 4
(2) D=2.0

Fig. 6 - Range Delta - File 4
(2) D=2.0
Fig. 11 - Azimuth Delta - File 6
(2, 4)
D=2.0

Fig. 12 - Elevation Delta - File 6
(2, 4)
D=2.0
Fig. 13 - Azimuth Delta - File 7
(2, 4, 6)  D=3.025

Fig. 14 - Elevation Delta - File 7
(2, 4, 6)  D=3.025
Fig. 15 - Azimuth Delta - File 10
(2, 4, 6), 7  D=3.025

Fig. 16 - Elevation Delta - File 10
(2, 4, 6), 7  D=3.025
Fig. 17- Azimuth Delta - File 54
(2, 4, 6), 7, 10, 16, 21, 23, 28, 36  D=3.025

Fig. 18- Elevation Delta- File 54
(2, 4, 6), 7, 10, 16, 21, 23, 28, 36  D=3.025
Fig. 19 - Azimuth Delta - File 55
(2, 4, 5), 7, 10, 16, 21, 23, 28, 36  D=3.025

Fig. 20 - Elevation Delta - File 55
(2, 4, 6), 7, 10, 16, 21, 23, 28, 36  D=3.025
Fig. 21- Azimuth Delta - File 54
(2, 4, 6), 7, ..., 36  MUSTC=0  D=3.025

Fig. 22- Elevation Delta - File 54
(2, 4, 6), 7, ..., 36  MUSTC=0  D=3.025
Fig. 23- Azimuth Delta - File 54
(2, 4, 6), 7, ..., 36  NEWCORE  D=3.025

Fig. 24- Elevation Delta- File 54
(2, 4, 6), 7, ..., 36  NEWCORE  D=3.025