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A LOW-POWER MAGNETIC TORQUER FOR SATELLITE ATTITUDE CONTROL

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ABSTRACT

This paper analyzes a magnetic torquer for satellite attitude control which requires very little energy, or average power, to control its magnetic moment. The magnetic torquer consists of a solenoid with a hard magnetic-material core that has a large residual intensity of magnetization. Discharging capacitors through the solenoid winding generates current pulses to switch the polarity of the residual intensity of magnetization in order to control the magnetic moment of the magnetic torquer. A mathematical model and design equations are presented for the magnetic torquer.

INTRODUCTION

In recent years investigations have been made into the applications of magnetic torquers for attitude control of earth-orbiting satellites. A magnetic torquer is simply a device, such as a current-carrying coil, an electromagnet, or a permanent magnet, which generates a magnetic moment. The interaction of the magnetic moment of the magnetic torquer with the earth's magnetic field produces a torque on the magnetic torquer that may be transferred to an earth orbiting satellite if the magnetic torquer is mounted to the structure of the satellite. By controlling the resultant magnetic moment of a system of magnetic torquers while measuring the magnitude and direction of the earth's magnetic field, it is possible to control the resultant torque on a satellite for attitude control of the satellite or desaturation of momentum storage devices used for satellite attitude control.

Much of the work to date has been devoted to the development of three basic types of magnetic torquers(1)-(2). One type is the air core coil, which is a planar current-carrying coil. With a fixed enclosed area, the magnetic moment of the coil is controlled by controlling the current in the coil. A second type of magnetic torquer is the electromagnet, which consists of a solenoid with a soft magnetic-material core. By controlling the current in the solenoid the induced magnetic moment of the core can be controlled(2). A third type of magnetic torquer is the permanent magnet, which requires no energy to maintain its magnetic moment but affords no convenient way of controlling it. It is possible to control the direction of the magnetic moment by means of motor-driven gimbals in order to control the torque on the permanent magnet(1). The disadvantage of the magnetic torquers described is that a large amount of power or weight is normally required to produce even small torques because of the weak magnetic field of the earth(3).

Table 1 gives a power and weight comparison of typical magnetic torquer designs to generate 0.05 N-m of torque at an orbital altitude of 1000 km.

<table>
<thead>
<tr>
<th>Magnetic Torquer</th>
<th>Power(W)</th>
<th>Weight(N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Core Coil</td>
<td>100</td>
<td>250</td>
</tr>
<tr>
<td>Electromagnet</td>
<td>4</td>
<td>250</td>
</tr>
<tr>
<td>Permanent Magnet</td>
<td>0</td>
<td>250</td>
</tr>
</tbody>
</table>

There exists still another type of magnetic torquer, one which has received little attention(4). This fourth type of magnetic torquer consists of a solenoid with a hard magnetic-material core possessing a large residual intensity of magnetization. Current pulses through the winding of the solenoid are generated to switch the polarity of the residual intensity of magnetization and thereby control the magnetic moment of the core. This paper describes a magnetic torquer of this type and presents a mathematical model and design formulas for it. In most applications this magnetic torquer requires less average power than the electromagnet but affords more control of its magnetic moment than is possible with the permanent magnet.

Background

The torque about an arbitrary point experienced by a magnetic dipole in a uniform magnetic field is given as(5)

\[ T = \mathbf{m} \times \mathbf{B} \]  

where

\[ \mathbf{m} = \text{magnetic dipole moment, amperes-meters}^2 \]
\[ \mathbf{B} = \text{flux density of the magnetic field, weber/meter}^2 \]
\[ T = \text{torque experienced by the magnetic dipole, newton-meters} \]

Let \( \mathbf{m} \) represent the equivalent magnetic dipole moment of a magnetic torquer. On the assumption that the earth's magnetic field is essentially uniform in this region, Equation (1) becomes the expression for the torque \( T \) on the magnetic torquer where \( \mathbf{B} \)
represents the flux density vector of the earth's magnetic field, assumed to be the only source.

**Statement of the Problem**

Figure 1 shows the magnetic torquer to be considered. The magnetic torquer consists of a uniformly wrapped solenoid in which its diameter is small compared with its length. It is assumed that the turns of the solenoid are closely wrapped and the cross-sectional area of the solenoid is uniform. The core of the solenoid is a cylindrical bar of a homogeneous, hard magnetic material. It will be assumed that over the median cross section of the cylindrical bar, the hard magnetic material exhibits a major hysteresis loop as shown in Figure 2. In Figure 2,

\[ \langle M \rangle_{ave} \] = mean axial intensity of magnetization of the magnetic material over the median cross section of the cylinder, ampere/meter

\[ \langle H \rangle_{ave} \] = mean axial magnetic field intensity of the magnetic material over the median cross section of the cylinder, ampere/meter

\[ M_s \] = saturation intensity of magnetization, ampere/meter

\[ H_c \] = coercivity, ampere/meter.

Along the paths (a-b) and (d-e)

\[ \frac{d}{dt} \langle M \rangle_{ave} = 0, \]

and along the paths (b-c) and (e-f)

\[ \frac{d}{dt} \langle H \rangle_{ave} = K. \]

The statement of the problem of this paper is as follows. Synthesize and analyze a device that will switch the intensity of magnetization of the magnetic torquer core to either +M_s or -M_s to control the resultant torque on the magnetic torquer while dissipating very little energy, or average power.

Figure 3 is a diagram of the magnetic torquer and the device to switch the intensity of magnetization of the magnetic torquer core. For the relays in position a, capacitor C_1 is discharging to produce a current pulse through the solenoid to change the intensity of magnetization of the core from +M_s to -M_s; capacitor C_2 is charging. For the relays in position b, the reverse is happening. The procedure will be first to write the equations for the magnetic torquer magnetic moment, voltage, and current, then to synthesize the basic magnetic torquer/capacitor discharge circuit, and finally to analyze the complete system.

**ANALYSIS OF THE MAGNETIC TORQUER**

For the magnetic torquer of Figure 1, it is shown in Reference 5 that if

\[ m = M e \]

then

\[ M = \frac{\pi D^2}{4} \left( \langle M \rangle_{ave} \right)_{mcs} \left( LR_s + N_i s \right) \]

where

\[ \langle M \rangle_{ave} \] = mean axial intensity of magnetization of the magnetic material over the median cross section of the cylinder, ampere/meter

\[ L \] = length of the cylinder, meters

\[ D \] = diameter of the solenoid, meters

\[ R_s \] = shortening ratio of the cylinder and is a constant for a given length-to-diameter ratio

\[ N \] = number of turns of the solenoid

\[ i_s \] = current in the solenoid, amperes.

A plot of R_s versus the length-to-diameter ratio of the cylinder, L/D, is shown in Figure 4(6). It is also shown in Reference 5 that the voltage drop across the magnetic torquer may be written as

\[ v_s = \frac{\pi D^2}{4} \rho \left( 1 + \frac{d}{dt} \langle H \rangle_{ave} \right) \]

\[ \frac{d}{dt} \langle M \rangle_{ave} \]

and the current in the solenoid may be given as

\[ i_s = \frac{L}{N} \left( \langle H \rangle_{ave} + N_B \langle M \rangle_{ave} \right) \]

where

\[ \rho \] = resistivity of the solenoid wire, ohm-meters

\[ a \] = cross-sectional area of the solenoid wire, meters^2

\[ \mu_0 \] = permeability of free space

\[ N_B \] = ballistic demagnetization factor.

The ballistic demagnetization factor N_B is a function of L/D for the magnetic torquer core as well as the susceptibility, \( \chi_m \), of the magnetic material at the median plane, which must be assumed uniform at all points in the plane. Plots of N_B versus L/D of the cylinder for various values of \( \chi_m \) are shown in Figure 5(7).

**ANALYSIS OF THE BASIC MAGNETIC TORQUER/CAPACITOR DISCHARGE CIRCUIT**

Figure 6 is a diagram of the basic magnetic torquer/capacitor discharge circuit. The capacitor C discharges to produce a current pulse to change the intensity of magnetization of the core of the
magnetic torquer from $-M_s$ to $+M_s$. The resistor $R$ is a design variable for control of the damping in the current transient response. It follows from Kirchhoff's second law that

$$-v_c + v_R + v_s = 0.$$  

But

$$v_c = \frac{q_c}{C},$$

$$v_R = R i_s,$$

and

$$i_s = \frac{-dq_c}{dt}.$$  

From Equations (4)-(9), the differential equations for the basic magnetic torquer/capacitor discharge circuit of Figure 6 may be written as

$$\frac{dq_c}{dt} = -\frac{L}{N} <H_{ave}>_{mcs} + \frac{L}{N} B <M_{ave}>_{mcs}.$$  

and

$$\frac{d<H_{ave}>_{mcs}}{dt} = \frac{4 q_c}{N^2 D^2 R_s u_o \left(1 + \frac{d<H_{ave}>_{mcs}}{d<H_{ave}>_{mcs}}\right) C} - \frac{4L \left(R + \frac{\pi D u_o}{a}\right) <H_{ave}>_{mcs}}{N^2 D^2 R_s u_o \left(1 + \frac{d<H_{ave}>_{mcs}}{d<H_{ave}>_{mcs}}\right)} - \frac{4L N_B \left(R + \frac{\pi D u_o}{a}\right) <M_{ave}>_{mcs}}{N^2 D^2 R_s u_o \left(1 + \frac{d<H_{ave}>_{mcs}}{d<H_{ave}>_{mcs}}\right)}.$$  

The design of the basic magnetic torquer/capacitor discharge circuit has two parts. The first part is to design the magnetic torquer core to produce a magnetic moment under equilibrium conditions specified by the desired maximum torque on the core for a given magnitude of the earth's magnetic-field flux density. The second part is to design the solenoid of the magnetic torquer and the remainder of the capacitor discharge circuit to insure proper switching of the intensity of magnetization of the core.

The required magnetic moment $M$ of the magnetic torquer core under equilibrium conditions to produce a desired maximum torque $T_{MAX}$ on the core, for a given magnitude of the earth's magnetic-field flux density $B$, follows from Equation (1) and is

$$M = \frac{T_{MAX}}{B}.$$  

At equilibrium the current $i_s$ in the solenoid is zero, and the intensity of magnetization $<M_{ave}>_{mcs}$ is assumed to equal either $+M_s$ or $-M_s$, which is the design goal of the low-power magnetic torquer. For $i_s$ equal to zero and $<M_{ave}>_{mcs}$ equal to $+M_s$, it follows from Equation (3) that

$$D^2 M_{LR} = \frac{4}{\pi} M$$

or

$$D^2 M_{LR} = \frac{4}{\pi} \frac{T_{MAX}}{B}.$$  

Hence, Equation (13) or Equation (14) represents a design formula for the core of the magnetic torquer.

The procedure for the design of the solenoid of the magnetic torquer and the remainder of the capacitor discharge circuit is to:

1. Determine the equilibrium points for the system,
2. Linearize the system differential equations about the equilibrium points,
3. Design the system for responses near the equilibrium points by means of the linearized system differential equations.

The equilibrium points for the system are determined by setting the time derivatives of $q_c$ and $<H_{ave}>_{mcs}$ equal to zero in Equations (10) and (11), respectively. This yields

$$q_c = 0$$

and

$$<H_{ave}>_{mcs} = -N_B <M_{ave}>_{mcs}.$$  

Assuming that in the neighborhood of the equilibrium points

$$\frac{d<M_{ave}>_{mcs}}{dt} = 0$$

and

$$N_B = \lim_{\lambda \to \lambda_m} N_B,$$

then in the neighborhood of the equilibrium points Equations (10) and (11) can be written as

$$\begin{bmatrix}
\frac{dq_c}{dt} \\
\frac{d<H_{ave}>_{mcs}}{dt}
\end{bmatrix} = \begin{bmatrix}
A & 0 \\
0 & A
\end{bmatrix} \begin{bmatrix}
q_c \\
<H_{ave}>_{mcs}
\end{bmatrix}$$  

7-23
where

\[
A = \begin{bmatrix}
0 & -\frac{L}{N} \\
\frac{4}{N^2 + D^2 R a C} & \frac{4L R + \pi D N p}{N^2 + D^2 R a C}
\end{bmatrix},
\]

\[q_c* = q_c^* \quad (20)\]

and

\[\langle H \rangle_{ave}^{mcs} = \langle H \rangle_{ave}^{mcs} - \lim_{N \to \infty} \frac{N B \langle M \rangle_{ave}^{mcs}}{N} \quad (21)\]

Design of the linearized system differential equations of Equation (19) can be achieved by means of the characteristic polynomial, which is the \(\text{det}(SI-A)\) where \(S\) is the Laplace transform variable, \(I\) is the identity matrix, and \(A\) is the system matrix. Solving for the \(\text{det}(SI-A)\) yields

\[
\text{det}(SI-A) = S^2 + \frac{4L (R + \pi D N p) a}{N^2 + D^2 R a C} S + \frac{4L}{N^2 + D^2 R a C} \quad (22)
\]

which is a second-order characteristic polynomial and has the form

\[
\text{det}(SI-A) = S^2 + 2\xi \omega_n S + \omega_n^2 \quad (23)
\]

Equating the right-hand sides of Equations (22) and (23) and manipulating,

\[
\omega_n^2 = \frac{4L}{\pi D^2 R a C} \quad (24)
\]

and

\[
R = \frac{\pi D^2 R a C}{2L} - \frac{\pi D N p a}{N^2 + D^2 R a C} \quad (25)
\]

which is physically realizable for \(R \geq 0\).

**ANALYSIS OF THE COMPLETE LOW-POWER MAGNETIC TORQUER SYSTEM**

A diagram of the complete low-power magnetic torquer system is shown in Figure 3. By employing a procedure similar to that used to derive the differential equations for the basic magnetic torquer/capacitor discharge circuit, the differential equations for the complete low-power magnetic torquer system may be written as

\[
\frac{dq_{c1}}{dt} = \frac{1}{R_{CH} C_1} q_{c1} + \frac{1}{R_{CH}} E \quad ,
\]

\[
\frac{dq_{c2}}{dt} = \frac{4 q_{c2}}{N^2 + D^2 R a C} \left( 1 + \frac{d <M_{ave}^{mcs}}{d <H_{ave}^{mcs}} \right) C_2 \quad (27)
\]

and

\[
\frac{d <H_{ave}^{mcs}}{dt} = \frac{4 q_{c1}}{N^2 + D^2 R a C} \left( 1 + \frac{d <M_{ave}^{mcs}}{d <H_{ave}^{mcs}} \right) C_1 \quad (28)
\]

when the relays are in position a, and

\[
\frac{dq_{c1}}{dt} = \frac{1}{R_{CH} C_1} q_{c1} \quad (29)
\]

\[
\frac{dq_{c2}}{dt} = \frac{1}{N} \frac{<H_{ave}^{mcs} >}{<M_{ave}^{mcs} >} \quad (30)
\]

and

\[
\frac{d <H_{ave}^{mcs}}{dt} = \frac{4 q_{c2}}{N^2 + D^2 R a C} \left( 1 + \frac{d <M_{ave}^{mcs}}{d <H_{ave}^{mcs}} \right) C_2 \quad (31)
\]

for the relays in position b. The magnetic dipole moment \(M\) of the magnetic torquer is determined from Equations (3) and (5) and is

\[
M = \frac{\pi D^2}{4} <H_{ave}^{mcs} > + \frac{\pi D^2}{4} (R_a + N_b) <M_{ave}^{mcs} > \quad .
\]
A digital computer program was constructed to solve numerically the differential equations for the complete low-power magnetic torquer system to simulate the operation of the system and verify that its performance is satisfactory for a typical system design. The modified Euler method for numerical solution of the differential equations was used. At time zero the relays of Figure 3 were assumed to be in position b, and hence Equations (29)-(31) were solved.

At 0.04 seconds the relays were assumed to be switched to position a so that Equations (26)-(28) were solved thereafter. The ballistic demagnetization factor $N_B$ was assumed to be a constant and equal to $\lim_{\chi \to \infty} N_B$ for worst case conditions. The values of the system parameters for the simulation were chosen with these assumptions. The magnetic torquer is assumed to operate at an orbital altitude of 1000 km where

$$B = 0.25 \times 10^{-4} \text{ weber/meter}^2.$$  

The desired maximum torque on the core is

$$T_{\text{MAX}} = 0.05 \text{ newton-meter}.$$  

The core of the magnetic torquer consists of the magnetic-material Remendur 38 and the solenoid consists of No. 14 AWG Standard Annealed Copper wire which has a current capacity of 15 amperes (9). Equations (12)-(14), (24), and (25) were utilized in selecting the system parameters for

$$\omega_n = 1220 \text{ radians/second}$$

and

$$\zeta = 1.$$  

For these conditions, a typical system design is

$$M = 1,430,000 \text{ amperes/meter}$$

$H = 3200 \text{ amperes/meter}$

$K = 10,000$

$L/D = 50$

$R_B = 0.75$

$N_B = 0.001$

$D = 0.0365 \text{ meter}$

$L = 1.82 \text{ meter}$

$a = 2.18 \times 10^{-6} \text{ meter}^2$

$\rho = 1.724 \times 10^{-8} \text{ ohm-meter}$

$N = 1120$

$C_1 = C_2 = 0.001 \text{ farad}$

$R_1 = R_2 = 0.64 \text{ ohm}$

$R_{CH} = 5 \text{ ohms}$

$E = 200 \text{ volts}$

where the value for $E$ was determined by simulation using the trial and error method. The system responses for the initial conditions

$q_{c1}(0) = 0.0$

$q_{c2}(0) = 0.2 \text{ coulomb}$

$$\langle M_{\text{ave}} \rangle (0) = -1,430,000 \text{ amperes/meter}$$

$$\langle H_{\text{ave}} \rangle (0) = 1430 \text{ amperes/meter}$$

are shown in Figures 7-11. The responses show that the intensity of magnetization $\langle M_{\text{ave}} \rangle$ was properly switched from $-M_B$ to $+M_B$ and then back to $-M_B$. The energy dissipated in the system each time the relays were switched was computed to be 40 joules.

**CONCLUSIONS**

For the generation of torques for attitude control of earth-orbiting satellites, it is concluded that control of the magnetic dipole moment of a hard magnetic-material bar by means of the solenoid and capacitor discharge networks in Figure 3 appears feasible and practical. However, the results as shown in Figures 7-11 should be compared with the experimental results of an actual model to determine how closely the idealized model of the magnetic torquer derived approaches the actual device. Two possible sources of any discrepancy between the analytical results and experimental data could be the idealized model for the hysteresis loop of Figure 2 and the simplifying assumption that the ballistic demagnetization factor $N_B$ is a constant in the simulations.

The results showed that 40 joules of energy are dissipated in changing the magnetic dipole moment of the bar from $-2000 \text{ amperes-meter}^2$ to $+2000 \text{ amperes-meter}^2$, or vice versa, for the sample system design investigated. A typical design of a magnetic torquer with a soft magnetic-material core requires about 4 watts of power to generate a magnetic dipole moment of 2000 amperes-meter$^2$ (see Table 1). Consider the case of an earth-orbiting satellite with a magnetic torquer for control. Assume that the earth’s magnetic field varies relative to the magnetic torquer such that every quarter of an orbit it is necessary to change the polarity of the magnetic dipole moment of the magnetic torquer. Assuming that the orbital period is 5400 seconds and that the desired magnitude of the magnetic dipole moment is 2000 amperes-meter$^2$, then the energy dissipated per orbit by a magnetic torquer with a soft magnetic-material core is 21,600 joules, whereas the energy dissipated per orbit by the magnetic torquer of this paper is 160 joules. In this example a large energy savings is realized by the magnetic torquer presented in this paper. On the other hand, in the case of a spinning satellite, the magnetic torquer of this paper may be less efficient. Consider a spinning satellite with a magnetic torquer for control. Assume that the satellite spin rate is one revolution per 10 seconds and every one-half revolution it is necessary to change the polarity of the magnetic dipole moment of the magnetic torquer. Assuming that the orbital period is 5400 seconds and that the desired magnitude of the dipole moment is 2000 amperes-meter$^2$, then the energy dissipated per orbit by a magnetic torquer with a soft magnetic-material core is 21,600 joules as before.
whereas the energy dissipated per orbit by the magnetic torquer of this paper is 43,200 joules. Thus, the application determines whether or not the magnetic torquer of Figure 3 is indeed more efficient. It would seem that the magnetic torquer with the hard magnetic-material core might be more efficient for desaturating momentum storage devices in an inertially stabilized satellite, rating damping of a gravity gradient stabilized satellite, or attitude control of an earth synchronous satellite; it would probably be less efficient for controlling a spin stabilized satellite. Regardless of the application it seems desirable, if not necessary, to introduce hysteresis in the relays of the magnetic torquer of Figure 3 to prevent the possibility of a large energy drain that would result if a high frequency limit cycle existed when the variable or variables being controlled approached their desired values. Still another factor to be weighed in comparing the magnetic torquer of this paper against the magnetic torquer with a soft magnetic-material core is that the magnetic dipole moment of the former can only assume two values at equilibrium, whereas the magnetic dipole moment of the latter can assume any value, theoretically, between its saturation limits. The effects of this consideration are really dependent upon the accuracy demanded of the magnetic torquer system. In summary, the magnetic torquer presented in this paper appears most applicable for satellites where energy savings are more important than accuracy.

Some areas of further study are: (1) to conduct experimental tests to compare the results in this paper of an idealized analytical model against an actual one with the same design, (2) to develop design formulas to choose the characteristics of the solenoid and core of the magnetic torquer in this paper for lower energy consumption, (3) to develop methods to determine the initial charge on the capacitor necessary to properly switch the intensity of magnetization of the core, other than by trial and error, and (4) to develop control policies to switch the relays in the magnetic torquer system and test the control policies by simulation considering vehicle dynamics, orbit mechanics, the earth's magnetic field, and environmental disturbances.

**NOMENCLATURE**

- A - system matrix.
- a - cross sectional area of the solenoid wire, meters$^2$.
- B - magnitude of $\mathbf{B}$, weber/meter$^2$.
- $<\mathrm{B}_{\mathrm{ave}}>$ - average flux density over the median plane of the cylinder, weber/meter$^2$.
- $C_1$, $C_2$, $C_3$ - capacitators, farads.
- D - diameter of the solenoid, meters.
- $\mathbf{e}$ - electromotive force, volts.
- e - unit vector, dimensionless.
- $<\mathbf{H}_{\mathrm{ave}}>$ - actual magnetic field intensity in the core averaged over the median plane, amperes/meter.
- $<\mathbf{H}_{\mathrm{ave}}>$ - defined in Equation (21), ampere/meter.
- $H_c$ - coercivity, ampere/meter.
- I - identity matrix, dimensionless.
- $I_s$ - current in the solenoid, amperes.

**REFERENCES**

Figure 1. Magnetic Torquer

Figure 2. Major Hysteresis Loop Over the Median Cross Section of the Magnetic Torquer Core

Figure 3. Magnetic Torquer and Controller
Figure 4. Shortening Ratios for Homogeneous Magnetic-Material Cylinders

Figure 5. \( N_B \) vs L/D

Figure 6. Basic Magnetic Torquer/Capacitor Discharge Circuit

Figure 7. \( q_{cl} \) vs Time
Figure 8. $q_{c2}$ vs Time

Figure 9. $<H_{ave}_{mcs}>$ vs Time
Figure 10. $\langle M_{\text{ave}} \rangle_{\text{mcs}}$ vs Time

Figure 11. $M$ vs Time