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Hadamard Source Encoding Techniques Applied to Apollo Telemetry Links: An Evaluation

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ABSTRACT

The effort described in this paper is an investigation of the possible improvement in performance of the Apollo Unified S-Band Telemetry links due to the use of the Hadamard transform as a means of source encoding. Both rapidly and slowly varying telemetry signals were considered and three sizes of the Hadamard matrix were used. Results indicate that as much as 3-db improvement in system performance may be obtained in systems operating at a 2% RMS error level.

INTRODUCTION

In recent years, the practicality of space communications, both for manned space research and unmanned communications satellites, has become evident. The rapidly increasing amount of information to be transmitted is placing arduous demands on the system channel capacity. Also, the prospect of future manned deep space probes requires improved system signal-to-noise ratios. Data compression and channel encoding techniques have been developed to help meet these more rigid requirements [1], [2].

Recent developments of "fast" orthogonal transforms have provided new concepts of source encoding as a means of reducing bandwidth and carrier-power requirements for space communication systems. Some of the more promising transforms include the Fourier, Haar, Hadamard and Karhunen-Loeve transforms, [3], [4], [5].

The effort described in this paper is an investigation of the possible improvement of the Apollo Unified S-Band Telemetry Links due to the use of the Hadamard transform as a means of source encoding. Actual command and service module-ground station communication equipment with a simulated RF link was used in the test. These facilities are located at the Manned Spacecraft Center. Both slowly changing and rapidly changing digital signals were investigated. Hadamard matrices of orders 4, 16, and 64 were used in obtaining the transformations.

It is realized that the resulting system is not "optimal" in any manner, but the results do reflect the advantages of using the Hadamard technique without redesign of other system components. Areas where system changes could result in significant additional improvement in performance are also revealed.

Section II gives a brief summary of the Hadamard transform. The experimental study is described in Section III. The results of this study are shown in Section IV and conclusions are presented in Section V.

THE HADAMARD TRANSFORM

Given a one dimensional signal, f(t), its finite sampled transform can be represented as

\[ Y = HX \]  

(1)

where X is an N x 1 vector of signal samples of f(t), H is an N x N transformation matrix and Y is an N x 1 vector of transformed samples. One element of Y is

\[ Y(n) = \sum_{k=0}^{N-1} X(k) h_{kn} \]  

(2)

For orthogonal transforms, the signal vector elements are recovered by

\[ X(k) = \sum_{n=0}^{N-1} Y(n) h_{nk} \]  

(3)

In eqs. (2) and (3), h_{kn} is an element of the matrix H. If the element h_{kn} is defined as the inner product

\[ h_{kn} = \langle \text{Wal}(k), \delta(\xi - \frac{n}{N\Delta T}) \rangle \]  

(4)

then the resulting matrix is the Hadamard matrix and the transformation is known as
In eq. (4), the \( \text{Wal}(k,6) \) represent the set of Walsh functions, the first eight of which are shown in Figure 1 [3]. These are overlapping rectangular functions which are orthogonal in the normalized interval \( 0 < t < 1 \). Examples of three Hadamard matrices are shown in Figure 2. Matrices of higher order are easily constructed from lower order matrices. Each matrix is comprised of only the elements +1 and -1. Further, columns and rows are mutually orthogonal and any two rows or any two columns may be interchanged without affecting the orthogonality property.

Except for a constant factor equal to the rank of the matrix, the Hadamard matrix is its own inverse. Thus, to recover the original vector \( X \):

\[
X = H^{-1} \quad Y = \frac{1}{N} HY
\]

This expression is significant from the standpoint that implementation of the Hadamard transform technique requires development of only one piece of hardware for both the transform and inverse transform processes.

The most significant advantage of the Hadamard matrix, however, is its ability to effect a transform of a signal while requiring only the operations of additions and subtractions.

**EXPERIMENTAL STUDY**

The system diagram for this study is shown in Figure 3. One-hundred ninety-six signal samples were generated to correspond to the command and service module low bit rate telemetry format of 200 words which includes four frame synchronization words. Two types of signal samples were generated. The first type consisted of 196 samples evenly spaced over one cycle of a sine wave. This signal was chosen to represent slowly changing telemetry data such as battery voltage, vehicle velocity, etc. The second signal type was 196 data points chosen by sampling a random variable uniformly distributed over the interval \((0,1)\). This type of course represents rapidly or randomly changing telemetry signals.

The signals were processed in three phases. The first phase was accomplished on the Unicov 1108 Computing System and consisted of signal generation, Hadamard transformation of the signal if applicable, and pulse code modulation (PCM) of the transformed and untransformed signals. An eight bit uniform quantization method was used for the PCM. This was accomplished by generating 256 evenly spaced levels between the maximum and minimum values of input data and then assigning each data point to the closest level. Hence for each data point, an 8-bit binary number between 0 and 255 is generated. The sequence of 196 eight-bit numbers and four eight-bit sync words from one 1600 bit frame. This frame is supplied to the second phase of the system which is detailed in Figure 4.

Actual spacecraft-ground station communication equipment was used in phase two. The RF path from spacecraft to ground station was of course simulated but is accurately calibrated so that the simulation closely resembled actual mission conditions. Data was transmitted for three different system bit error rates (BER). Ten thousand frames (16 million bits) were examined for 10^-4 BER; 5000 frames for 10^-3 BER; and 1000 frames for 10^-2 BER. The PCM bit synchronizer output was recorded on magnetic tape for use in phase three.

The Univac 1108 computer was again utilized in the last step of the signal processing. Using the reference levels from phase I, the PCM signal recorded in phase II was decoded and the inverse transformation performed if necessary. The resulting estimates of the original signal were then compared with the actual values and the per cent RMS error rate per word was calculated.

For a standard of comparison, the signals were processed in the normal manner through the system and the RMS error calculated. The signals were then Hadamard transformed and processed in the same manner. Three sizes of the matrix \((N=4, N=16, N=64)\) were considered in each case. Hence, a total of eight different bit streams were transmitted for each of the three bit error rates.

**RESULTS**

Per cent RMS error vs bit error rate for slowly changing (sine wave) data is shown in Figure 5. Two cases are depicted; untransformed data and data transformed by the Hadamard technique with \( N=4 \). The data from runs with \( N=16 \) and \( N=64 \) were so close to the \( N=4 \) case as to be indistinguishable on a graph. Surprisingly, the \( N=16 \) and \( N=64 \) data were slightly worse than the \( N=4 \) case.

Figure 6 depicts the RMS error comparison for the fast changing (random) input signal. Again the \( N=16 \) and \( N=64 \) cases were very close, but slightly worse than for \( N=4 \).

Curves relating bit error rate and signal-to-noise ratio for PSK matched filter detection are well established [6].
using this known relation and the informa-
tion from Figures 5 and 6, one can con-
struct the comparison shown in Figure 7.
Improvement of system performance in db is
plotted as a function of % RMS error. For
example, if the system normally operates
at a 2% RMS error level and slowly varying
data is considered, then the required
system signal-to-noise ratio can be re-
duced 2.8 db by using the Hadamard trans-
form technique.

It is significant to note in Figures 5 and
6 that at low bit error rates (= 10^-4) the
conventional system outperforms the Hada-
mard system. This phenomenon can be
attributed to the quantization scheme used.
It is possible to show that the quantiza-
tion error for the Hadamard system is
equal to the quantization error of the con-
ventional system plus a positive quan-
tity that is a function of system para-
eters. Hence, for low bit errors when
quantization noise becomes significant
the performance of the Hadamard system
should be worse. However, as the bit
error rate increases and the channel
noise becomes the major contributor to
RMS error, the noise distributing proper-
ties of the Hadamard transform provides
significant improvement and the overall
performance of the Hadamard system as
compared to the conventional system
increases.

Also the RMS error of the Hadamard system
increases as the matrix size N increases
if there is no channel noise (SN = oo).
This may explain why the performance of
the N=16 and N=64 cases were slightly
worse than the N=4 case.

CONCLUSIONS

It has been shown experimentally that
Hadamard transform techniques can provide
significant improvement in Apollo Unified
S-Band System performance, especially if
the input data is changing slowly. How-
ever, for low bit error rates uniform
quantizing, the conventional system is
better. The study was made under the
constraint that no changes be made in the
existing Apollo System except the intro-
duction of the transform itself. It is
felt that significant additional improve-
ment could be obtained using other more
nearly optimum quantizing schemes.

As a sidelight, it was noted that a 4:1
data compression could be accomplished by
transmitting only transform values greater
than zero (approximately) and the position
of these values.

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\[
\begin{align*}
&\text{Wal}(0,0) \\
&\text{Wal}(1,0) \\
&\text{Wal}(2,0) \\
&\text{Wal}(3,0) \\
&\text{Wal}(4,0) \\
&\text{Wal}(5,0) \\
&\text{Wal}(6,0) \\
&\text{Wal}(7,0)
\end{align*}
\]

Fig. 1 First Eight Walsh Functions
Fig. 2 Examples of Hadamard Matrices

\[
N = 2 \quad H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
\]

\[
N = 4 \quad H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix}
\]

\[
N = 8 \quad H_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} H_4 & H_4 \\ H_4 & -H_4 \end{bmatrix}
\]

Fig. 3: System Diagram for Hadamard Transform Study

Fig. 4 Block Diagram of Phase Two Signal Processor
Fig. 5: % RMS Error vs. Channel BER for Slowly Varying Data

Fig. 6: % RMS Error vs. BER for Rapidly Varying Signal

Fig. 7: % RMS Error vs. DB Improvement