Global Coverage By Banked Aeroglide Atmospheric Entry

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Summary

"Global Coverage" is defined as the capability of an atmospheric entry vehicle to return to any point on the globe following descent from any orbit. Global Coverage is not possible with current aerospace vehicle systems, but advanced lifting systems offer this promise.

Spherical earth analytical results are presented showing a possible bank-speed schedule (the minor circle turn) and the necessary lift-drag ratio for Global Coverage as a function of initial atmospheric entry speed and other relevant parameters. Global Coverage is available for entry at circular speed for \( L/D \geq 3.56 \); for entry at parabolic entry speed for \( L/D \geq 2.34 \). Comparison with optimized bank programs give indication that this minor circle maneuver is close to "optimum" and will provide a practical guidance scheme for controlled atmospheric entry from space.

Notation

\( D \) Aerodynamic drag force
\( g \) Proportionality factor between weight and mass
\( h \) Altitude of entry vehicle above planet surface
\( L \) Aerodynamic lift force
\( L/D \) Aerodynamic lift-drag ratio
\( m \) Entry vehicle mass
\( Q \) Minor circle turn parameter-constant during the maneuver
\( r \) Radius to center of planet
\( V \) Speed
\( V_c \) Circular orbit speed, \( \sqrt{\frac{g}{r}} \) - 26,000 feet per second at the Earth
\( \beta \) Bank angle of entry vehicle-measured from wings level condition
\( \gamma \) Flight path angle - measured up from horizon
\( \phi \) Latitude angle - measured from the plane of the equator
\( \psi \) Heading angle - measured north from due east
\( \lambda \) Longitude angle - measured east from initial point of maneuver
\( \eta \) The square of the speed ratio, \( \frac{V^2}{g} \)

Cross range capability will permit considerable operational flexibility, such as return to more landing sites or quicker return to a specified site. Horizontal landing will be possible with the higher \( L/D \) values discussed here; thus touch-downs can be handled routinely, as opposed to the extraordinary task force required for present Mercury/Gemini type returns which have very little lateral capability.

Extra atmospheric propulsion (a rocket impulse before entering the atmosphere) is one way to provide lateral capability. Simple orbit plane rotation requires about 450 fps per degree, or 7 fps per nautical mile lateral displacement one quarter revolution later. Yawing the deboost retrofire impulse reduces this cost for small excursions of lateral range. But large lateral capability of several thousand miles requires prohibitive velocity impulse values; for example over 26,000 fps to land at the pole from an orbit initially in the plane of the equator.

The promising concepts of propulsion during entry (aerocruise and other techniques) have been presented previously. Further work in this direction is currently under USAF contract and will be reported later. Fey has studied the use of aerodynamic maneuvers during boost propulsion to attain an orbit plane which does not contain the launch site.

Minor Circle Aeroglide Solutions

In contrast, the present paper will discuss lateral maneuvering by aerodynamic atmospheric entry.

The aeroglide concept has been studied for some years. Early numerical and analytical studies utilized truncated equations of motion (for example, Slye) which can be identified as appropriate for a "cylindrical planet" model. Generally, these truncated equations gave lateral glide ranges which were too large. Jackson identified this difficulty and suggested an analytical solution which is just a minor circle on the planet surface. The equations of motion for a shallow glide in vertical equilibrium are given below. The terms underlined are those discarded for the truncated "cylindrical planet" solution. If the centrifugal force term (mg) of equation (1) is further abandoned, the model is of a "flat planet".

\[
L \cos \beta = mg (1-\eta) \quad \text{where} \quad \eta = \frac{V^2}{g} \tag{1}
\]

\[
L \sin \beta = mV^0 + mV^0 \sin \eta \tag{2}
\]

\[
D + mV^0 = 0 \tag{3}
\]
These three equations give the orthogonal force balances; the kinematics provide:

\[ \dot{h} = v \lambda \]  
\[ r \dot{\psi} = v \sin \psi \]  
\[ r \cos \lambda = v \cos \psi \]  

For a particular bank schedule with speed, an analytical solution of the full spherical set of equations was recognized as a minor circle on the planet, tangent to the original entry plane. For east entry in the equatorial plane, the minor circle longitude-latitude trace is given by

\[ \cos \lambda = \frac{Q \sin \psi}{Q \cos \psi} \]  

The heading angle is available as

\[ \sin \psi = Q \sin \lambda \]  

These expressions are considerable simplifications of equations 12 and 20 of reference 8. The requisite bank schedule is a simple function of speed and the minor circle parameter Q, given by the expression

\[ \tan \beta = \frac{Q}{1 - \eta} \]  

The parameter Q is a constant in any given entry. The minor circle trajectory is characterized by this parameter Q, which is just the ratio of lateral force \( L \sin \beta \) to the radial component of centrifugal force, \( mV^2 \). A zero value of Q (unbanked vehicle) leads to a maneuver in the original orbit plane which is thus a major circle. Increased values of Q provide lateral maneuvering. Only a value of Q = 1 permits a crossing of the pole from an entry originally in the equatorial plane. For values of Q greater than unity, the minor circle is totally on the original side of the pole.

The bank schedule is sketched in figure one as a function of speed and the constant parameter Q. Note that inverted flight is required at super circular speed. At circular speed, all values of the Q parameter indicate 90° bank (wings vertical). The wings become more level as the speed decreases in the glide.

Global Coverage requires a latitude capability of \( \phi = 90^\circ \), and thus a value of the minor circle parameter Q of unity. The vehicle initially entering in the plane of the equator can then aeroglide to either pole.

The "Cylindrical Planet" Solution

Application of the same constant Q bank-speed program to the truncated (cylindrical) equations (equations 1-6, neglecting underlined terms) yields a plane circle in the longitude-latitude plane

\[ \lambda^2 = \frac{2}{Q} \phi - \phi^2 \]  

\[ \sin \psi = Q \lambda \]  

This provides the only known analytical solution for direct comparison of the spherical and cylindrical sets of equations. These spherical and cylindrical solutions are sketched in figure two, for glides from circular speed at Q = 1, verifying and systematizing the previous numerical results. Note that the spherical and cylindrical latitudes for Q = 1 are nearly coincident functions of L/D, although the trajectories are entirely different. Thus latitude data alone may not be sufficient to evaluate the accuracy of a set of truncated entry equations.

Global Coverage Lift Drag Ratio

The minor circle turn bank schedule is superior to constant bank schedules for generation of lateral range, as large bank angle is used at near circular speed to generate initial heading change. While the bank schedule as a function of speed maintains the vehicle on the minor circle, it is the energy balance which specifies the trajectory length along the minor circle. Both high aerodynamic lift drag ratio (L/D) and high initial speed propagate the glide farther along the minor circle. Thus it is of interest to provide the trade-off of L/D required as a function of initial glide speed for a given lateral maneuver capability. For the case of Global Coverage, circular initial glide speed requires L/D \( \geq 3.56 \). For entry at parabolic speed, \( V = \sqrt{2} \ V_c \), Global Coverage is available for any L/D \( \geq 2.34 \). The general expression for latitude capability of a Q = 1 aeroglide from initial speed to negligible final speed is

\[ 2 \sin \phi_{\text{max}} = 1 - \cos \left( \frac{1}{2} \log_e \frac{N + 2n + 1}{\sqrt{2} - 1} \right) \]  

where \( n \) is the square of the initial speed ratio, Global Coverage requires \( \phi_{\text{max}} = 90^\circ \), and the requisite Lift Drag values are shown in figure three. The dotted line indicates the values required for a maximum lateral range of \( \phi_{\text{max}} = 45^\circ \).
Several investigators have searched for optimum bank control schedules. From a distillation of Dynasoar computer runs, Wallace and Gray\(^1\) present a bank schedule decreasing with the speed, as in the minor circle case. Even their numerical value of L/D = 3.6 required for Global Coverage is in excellent agreement with the 3.56 value found here with the Q = 1 minor circle glide. Wagner\(^1\) has presented optimized stepwise bank programs for maximum lateral range which call out steep bank early in the glide, followed by reduced bank at the slower speeds. General agreement with the minor circle schedule is observed. Bryson\(^2\) gave one numerical result of glide from a subcircular speed which is in general agreement with minor circle requirements.

Since Q = 1 is the only minor circle turn to develop ninety degrees lateral range, early interest was focused on this value. It has been shown that the maximum latitude is developed along a Q = 1 minor circle for any L/D value if the maneuver entry speed is circular. For a given value of L/D, increasing the entry speed will increase the minor circle latitude reached until the limiting value of \(\phi_{\text{max}} = \text{ARC Sin} \frac{Q}{1+Q}\). Further increase in \(1+Q\) initial speed results in lesser latitude, as the vehicle goes "over the top" for this given Q. Moving the value of Q toward unity permits utilization of additional entry speed (kinetic energy), with the terminal case of Q = 1 being reached for any L/D given high enough entry speed. At subcircular entry speeds, Q > 1 maneuvers are best; about Q = 1.25 for the example of reference 12. At supercircular speeds, optimum Q is usually one.

For aeroglide from circular speed, the maximum latitude reached along the Q = 1 maneuver is shown in figure four, using equation 12. This curve is also in good agreement with available numerical optimum trajectories\(^1\) and results of complex optimizations\(^1\).

**Conclusion**

Global Coverage in aeroglide atmospheric entry has been shown feasible for reasonable values of the vehicle Lift Drag ratio. The analytically simple minor circle solution provides a convenient bank schedule for further study; it also appears that this bank schedule provides near maximum lateral capability.

**References**

5. Slye, R.E., "An analytical method for studying the lateral motion of atmospheric entry vehicles", NASA TN D-325 (September 1960)
Fig. 1 Minor Circle Bank Schedule as a Function of Glide Velocity and Velocity Gain, Q.

Fig. 2 Minor Circle Aeroglide Trajectories for Spherical and Cylindrical Planet Models. Q = 1

Fig. 3 Required Lift Drag Ratio for Global Coverage as a Function of Speed. Q = 1

Fig. 4 Maximum Latitude Available as a Function of Lift Drag Ratio for Aeroglide from Circular Speed. Q = 1