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ADAPTATION OF SELF-PHASING ANTENNA SYSTEMS TO SPACE MISSIONS

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Abstract

The ability of the self-phasing antennas to not only provide high gain but also to minimize the requirement of satellite attitude control has rendered them very attractive for use in satellite communication systems. The potential advantages and versatility afforded by self-phasing antennas are analyzed in this paper with discussions of results obtained for two specific applications.

The paper begins with a description of two types of self-phasing antennas, namely, the Van Atta array and the so-called phase conjugation array; similarities and differences in operation of the two types of arrays of various geometrical configurations such as planar, cylindrical, and spherical, are then discussed. Problem areas which are common to both types are indicated and techniques such as array scaling and frequency translation which are useful in dealing with some of these problems are also included in the discussion.

Examples of two specific applications of self-phasing antennas are used to demonstrate their attractive features. The first involves the use of self-phasing arrays on a communication satellite in a critically inclined elliptic orbit. The second deals with the mutual acquisition problem between two vehicles employing self-phasing antennas for space to space communication. In the latter application, the finite Markov chain technique is used to analyze the problem and the results are compared with those of the corresponding problem wherein the satellites employ narrow beam antennas pointing at each other to accomplish acquisition. The acquisition condition which would render a self-phasing system more attractive than a narrow beam system are indicated.
I. Introduction

The desirability of employing space-borne high directivity antenna systems for satellite communications has already initiated a great deal of research and development effort in adapting phase steered array antennas for such applications. The advantages afforded by such a steerable array are at least twofold. Firstly, its high directivity provides the needed enhancement of a signal to noise ratio for satellite to earth communication; secondly, the use of an electronically steerable antenna beam can provide a reduction in complexity within the satellite due to relaxation of the satellite attitude control requirement. The type of phased array antenna which can best satisfy these two conditions simultaneously is the so-called self-phasing antenna, due to its ability to automatically transmit a wave which propagates toward the direction of an incident wave. However, the usefulness of the self-phasing antennas need not be restricted to satellite communication alone; in fact, instead of employing a more conventional type of antenna, they can be used for other missions such as communication between satellites or space vehicles.

This paper begins with an exposition on the operational principles of two different types of self-phasing antennas, namely, the Van Atta array and the phase conjugation array. The similarities and differences between the two types are discussed, primarily in terms of how arrays of various geometrical configurations such as planar, cylindrical, and spherical can be synthesized on a satellite in conformity with a given mode of satellite attitude control. In general, the array configuration and the mode of satellite attitude control are intimately related to each other. Attention is also paid to the problem areas which are common to both types of self-phasing antennas, and techniques for alleviating some of these problems are discussed.

The potential versatility and advantages in the use of self-phasing antennas are exemplified with discussions of their applications to specific situations. The first application pertains to a spin stabilized communication satellite placed in a critically inclined elliptic orbit with its spin-axis perpendicular to the earth's equatorial plane. The second example pertains to the mutual acquisition problem between two space vehicles or satellites in order to establish a communication link between them, with both vehicles employing self-phasing antennas for acquisition and communication. In the second case, the finite Markov chain technique is used to analyze the acquisition problem, and the results are compared with those of the corresponding acquisition problem which employs two narrow antenna beams pointing at each other. The acquisition conditions which render the self-phasing antenna system more attractive than the conventional approach are also discussed.
II. Self-Phasing Antennas

For a given array of antenna elements, if every element reradiates the signal received with a phase conjugate to that of the received signal, then the wave is reradiated exactly toward the direction of the incident wave with the directivity equal to that given by the physical aperture of the array. The required phase conjugation can be achieved in at least two different ways, and for this reason the self-phasing antennas to be considered in the present work can be broadly classified into two types.

The well-known Van Atta array\(^1\) will be considered as the first type. If every pair of the conjugate antenna elements of a regular planar array, shown in figure 1 are connected together with transmission lines of equal electrical length, the phase conjugation requirement is automatically satisfied to make the array retrodirective. A Van Atta array can be operated either in a passive or an active manner. As a passive device, it behaves like a corner reflector. When identical bilateral amplifiers are inserted into every interconnecting transmission line, it becomes an active device.

The second type may appropriately be referred to as a phase conjugation array, and its required phase conjugation is achieved through the use of a coherent lower sideband after heterodyne action. The conjugation process is more explicitly shown in figure 2. The array geometrical configurations can be linear, planar, cylindrical or spherical. However, the choice of the configuration is often closely tied in with the particular mode of attitude control employed for the satellite, or vice versa. Thus, for the self-phasing antennas themselves, the merits of the two types of arrays can also be compared in terms of their adaptiveness towards a given mode of satellite attitude control.

In the phase conjugation array, each antenna element and its associated RF components operate primarily as a self-sufficient single unit and the entire array can be synthesized freely on surfaces of any dimensions. Such a synthesis procedure means obviously that the phase conjugation array can be conveniently adapted on satellites possessing any given mode of attitude control. For a Van Atta array, the interconnections between the conjugate pairs of antenna elements limit the array structure more or less to a symmetrical form in order to preserve the phase coherence for maintaining the retrodirectivity. Thus, when a planar Van Atta array is used on a satellite, some attitude control, either spin stabilization or gravity gradient stabilization, would be desirable in order to maintain an acceptably constant antenna aperture. For a non-oriented satellite, the antenna elements of a phase conjugation array may be distributed over the entire satellite surface. As the satellite tumbles freely, a constant antenna aperture can be maintained but the elements which make up the aperture are only those illuminated by the incident wave.
The specific geometrical configuration of an array is, of course, dependent on the satellite configuration and its mode of attitude control. For a non-oriented satellite, it is quite advantageous to have the antenna elements of the phase conjugation array disposed regularly on a spherical satellite to maintain a constant aperture independent of satellite orientation. The orientation of individual antenna elements with respect to one another may be arbitrary, as it can be shown that the phase conjugation process nullifies the differences in phases of the antenna excitation function caused by arbitrary orientations.

When a satellite is spin stabilized with its spin-axis perpendicular to the orbital plane, a regular circular array of either phase conjugation or Van Atta types can be adapted to a cylindrically shaped satellite. The phase conjugation array might be placed either completely around the satellite body or extended from the top or the bottom of the satellite. When the array is placed away from the satellite, the full array aperture is utilized at all times as the satellite spins. A circular Van Atta array, as shown in figure 3, can be quite easily synthesized by interconnecting all the pairs of elements placed at opposite ends of the diameters with transmission lines of equal electrical length. The Van Atta array should be placed away from the satellite body so that it is completely illuminated at all times by an incident wave.

For a gravity gradient stabilized satellite where one surface of the satellite always faces the earth, either the phase conjugation or the Van Atta array type is readily employable. For an array placed on the bottom surface of the satellite, either a rectangular or a circular form might be used. In fact, the antenna elements of a phase conjugation array need not be spaced in a regular manner on the plane.

III. Common Operational Problems

In employing self-phasing antennas for various missions such as communication and telemetering, it is often desirable to make the transmit frequency different from the receive frequency. The frequency separation provides a desirable isolation between the transmit and receive signals. However, the frequency translation does introduce an undesirable effect in the array operation. For a Van Atta array, the main lobe of the array for transmission is shifted slightly in angular position from that of the received wave. The shift in the main lobes is related to the frequency translation quite simply in the following manner:

\[ f_r \sin \theta_r = f_t \sin \theta_t \]  \hspace{1cm} (3-1)

where the subscripts \( r \) and \( t \) refer to the conditions of reception and transmission and the angle \( \theta \) is measured from the axis normal to the array. Equation (3-1) also shows that the amount of frequency separation depends to a first approximation on the allowed amount of beam shift.
The beam shift can be avoided if separate arrays for transmission and reception are employed and the element spacings of the two arrays are properly scaled. In a regular rectangular Van Atta array, the correct scaling requires the element spacing $d_r$ and $d_t$ of the two arrays to obey the following relationship

$$\frac{d_r}{\lambda_r} = \frac{d_t}{\lambda_t} \quad (3-2)$$

The scaling can also be performed for uniform circular Van Atta arrays so that both main lobes would point exactly in the same direction. The scaling relationship is found to be

$$\frac{R_r}{\lambda_r} = \frac{R_t}{\lambda_t} \quad (3-3)$$

where $R$ denotes the array radius, and the arrays can usually be placed in a concentric manner.

The effect produced by the frequency translation on a spherical or cylindrical phase conjugation array is different from that described in the last paragraph for the Van Atta arrays. For the present case, the frequency translation introduced symmetrical errors in the phase front of the excitation function appearing across the illuminated antenna aperture for the transmit signal, and this symmetrical phase error results in more of a decrease in its antenna gain than there would ordinarily be if the phase errors were absent. In fact, the amount of frequency translation allowable for a spherical or cylindrical phase conjugation array is governed by the tolerable amount of phase error appearing across the array aperture.

There are also other operational problems which are common to both types of self-phasing arrays. As the phase of the aperture excitation function plays an important part in directing the antenna beam of the transmit signal, certain important constraints are necessarily imposed on some critical passive and active components associated with the array. The RF transmission lines feeding the antenna elements should be identical in their electrical characteristics and so should the lines connecting the oscillators and mixers. The gain and phase stabilities of RF amplifiers and mixers must all fall within tolerable ranges so that degradations in antenna gain due to such instabilities are minimized. Highly phase-coherent oscillator signals are essential, and it is desirable also that the frequency translation can be performed through a converter circuit having high efficiency. The major problems mentioned in this section are some of the more obvious ones, and they become more acute when the number of elements in the array increases. It would not be uncommon to use an array having a few hundred elements, especially for long-range space communication in the microwave frequency range in order to obtain the required antenna gain.
IV. Critically Inclined Elliptical Orbit
Communication Satellite System

Consider a system of communication satellites randomly deployed in critically inclined elliptic orbits (inclination angle $i = 63^\circ 26'$). The apogee and the perigee of the orbit are assumed to be 12,000 nautical miles and 1,000 nautical miles, and the argument of the perigee is 270° from the ascending node. In practice, the desirable characteristics of such an orbit are that the satellite is visible in the northern hemisphere for a relatively large fraction of its orbital period and that the apogee will remain at the same latitude. Thus, when a coverage of the northern part of the world is required, communication satellites launched into these orbits can provide efficient coverage. The usable portion of the orbit would generally include the apogee and the orbital portions along which the satellite approaches and recedes from the apogee.

The selection of this communication system as an example for discussion was motivated by two interesting facts. First of all, the orbital configuration is such that gravity gradient stabilization is not feasible but spin stabilization with the spin axis normal to the earth's equatorial plane can be accomplished. Secondly, the orbital configuration and the feasible stabilization scheme are such that the implementation of conventional satellite transmitting antenna with reasonable gain in the direction of earth poses a rather difficult problem.

It is perhaps obvious that a spherical phase conjugation array disposed on a spherical satellite offers one good solution since the satellite would then require no attitude control and the array transmits to the direction of earth upon interrogation. In addition, the satellite is usable over the entire portion of the elliptic orbit as long as it is visible to the ground stations.

However, as a demonstration of the flexibility of self-phasing antennas, let it be assumed that a spherical array cannot be synthesized without making undue sacrifices in available surface areas for accommodating solar cell panels. With the spin axis perpendicular to the earth's equatorial plane, it is possible to use a cylindrically shaped satellite whose axis coincides with the spin axis, and a self-phasing antenna array placed on the bottom (i.e., facing South) surface of the satellite. As the satellite travels through the usable portion of the orbit, the array beam will be continuously steered toward the ground station which interrogates the satellite.

As to the choice of the type of self-phasing antenna, either a planar phase conjugation array or a Van Atta array could be used. For instance, two circular Van Atta arrays operating at different frequencies, one for transmission and the other for receiving, can
be selected and the radii of the two arrays are scaled according to equation (3-3). To obtain circular polarization, turnstile antennas would be used as basic array elements.

To relate the scan angle of the array beam with the position of the satellite, define an attitude angle $\Psi_s$, which is formed by the satellite spin axis and the line passing through the centers of the earth and the array. Let $\theta_s$ and $\phi_s$ represent the scan angle coordinates measured in the polar and azimuthal planes and the array normal corresponds to the Z-axis; it is obvious that $\Psi_s$ and $\theta_s$ are equivalent. For any array structure, its effective aperture will show some degradation as the scan angle increases. If the array uses circularly polarized elements, the degradation of the polarization ratio is also a function of the scan angle and the larger the scan angle, the larger the degradation. When the satellite is approaching the apogee, the loss of RF power due to the range, being inversely proportional to the square of the range, increases but degradations in the polarization ratio and effective array aperture decrease since the scan angle $\theta_s$ decreases also. When the satellite is receding from the apogee, the RF power loss due to the range would decrease but the loss due to degradations in polarization ratio and effective array aperture would increase. As the satellite travels through the usable portion of the orbit, a continuous trade-off between RF power losses incurred due to these different phenomena takes place so that the variation in the effective power received at the ground could be kept within a few decibels.

V. Mutual Acquisitions Between Space Vehicles

In this section, a somewhat idealized acquisition problem between satellites or space vehicles employing self-phasing antennas will be examined. The results of the corresponding problem dealing with narrow beam antennas pointing at each other are available, and the results of the two separate systems will be compared directly in order to show more explicitly the conditions under which the self-phasing antennas can operate with advantages. The assumptions which are needed to idealize the acquisition problem apply identically to both systems. They can be summarized stated in the following manner: (1) each acquisition receiver is assumed to have a priori knowledge of the frequency and the type of signal to be received; (2) during the acquisition phase, the satellites are assumed stationary; (3) the signal propagation time is much shorter than the time required for each search step of acquisition; and (4) the transmitting and receiving antennas point at the same direction. It should be noted that the fourth assumption is required to simplify the analysis for a narrow beam acquisition system but unnecessary for a self-phasing system.

The acquisition operations of two satellites employing self-phasing arrays can be described in terms of a simple sequence. Satellites A and B possess, respectively, pilot signals of given frequencies, namely, $f_A$ and $f_B$ where $f_A \neq f_B$. For instance, B
wishes to acquire A and in turn be acquired by A. First, B will send its pilot signal to search for A. When A detects B through A's self-phasing array, A will send its pilot signal at \( f_A \) to acknowledge B and also acquire B through the self-phasing array of B. In fact, the same pilot signals used for acquisition can be conveniently used for establishing communication as well. In other words, the pilot signal is used to interrogate for both acquisition and communication. To maintain a continuous link of communication, it will be further assumed that A and B remain locked on to each other once acquisition has been achieved.

The various states associated with the acquisition operation can be defined as follows: \( S_1 \) states that A and B have no knowledge of each other; \( S_2 \) states that B is detected by A only; and \( S_4 \) states that A and B acknowledge each other. It should be noted that \( S_3 \), the state of A being detected by B only, does not exist. The absence of state \( S_3 \) is solely due to the use of self-phasing arrays by A and B since state \( S_3 \) would imply that A must have already detected B. \( S_4 \) is the desired final state. However, for a narrow beam acquisition system \( S_3 \) must be included in the aggregate of states and there will be four states altogether.

As mentioned in the introduction section, the finite Markov chain technique will be used to analyze the acquisition problem. The symbols used here for the analysis are identical to those defined in the text written by Kemeny and Snell\(^5\), and in order to facilitate direct comparisons between results obtained here for the self-phasing system with those of the narrow beam system, the definitions of various probability terms used here will also be similar to those given in reference \((4)\). These definitions are given in appendix A. The essential results and their interpretations will be presented in the present section, but the derivations are given in appendix B.

The transition matrix \( P \) of the Markov chain is given as follows:

\[
(P) = \begin{bmatrix}
S_1 & S_2 & S_4 \\
S_1 & P_{11} & P_{12} & P_{24} \\
S_2 & 0 & P_{22} & P_{24} \\
S_4 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{align*}
S_4 & \quad 1 & 0 & 0 \\
S_1 & \quad P_{14} & P_{11} & P_{12} \\
S_2 & \quad P_{24} & 0 & P_{22}
\end{align*}
\]

where \( S_i \)'s denote the various states and \( p_{ij} \) denotes the transitional probability of going from the \( i \)th to the \( j \)th state. To determine the expected number of steps in which the process is in a transient state when the starting state is the \( i \)th one, a column matrix \( M_i(t) \) with its elements \( \tau_i \)'s is used. For the acquisition system employing self-phasing arrays, it can be shown that
As discussed previously, the narrow beam system consists of altogether four states; furthermore, for \( S_2 \) and \( S_3 \) it is also implied that the antennas of A and B are pointing in desired directions but the signal is detected by one of them only. Its column matrix \( M_1(t) \) was shown to be:

\[
M_1(t) = \begin{pmatrix}
\tau_1 \\
\tau_2 \\
\tau_3
\end{pmatrix} = \begin{pmatrix}
\frac{P_{12} + P_{24}}{P_{24}(P_{12} + P_{14})} \\
\frac{1}{P_{24}}
\end{pmatrix}
\]  \hspace{1cm} (5-2)

In the column matrix \( M_1(t) \) for both acquisition systems, the element \( \tau_1 \) denotes the expected number of steps in which the process is in a transient state when it started from state \( S_1 \), and \( \tau_1 \) is the basic parameter through which the behaviors of the two systems will be compared.

\( \tau_1 \) can be expressed also in terms of probabilities containing various system parameters (see appendix A), and it can be shown that for the self-phasing system

\[
\tau_1 = \frac{1}{P_a} + \frac{1}{P_b P_a} - 1 \]  \hspace{1cm} (5-4)

and for the narrow beam system

\[
\tau_1 = \frac{1 + P_a P_b}{P_b P_a (P_a + P_b - P_a P_b)} (P_a + P_b) \]  \hspace{1cm} (5-5)

If A and B are perfectly symmetrical, that is, \( P_a = P_b \) and \( P_a = P_b \), equations (5-4) and (5-5) are reduced respectively to

\[
\tau_1 = \frac{1}{P_b} (1 + \frac{1}{P_b}) - 1 \]  \hspace{1cm} (5-6)

106
and

$$\tau_1 = \frac{1 + \frac{2 p_a p_b (1 - p_h)}{p_a^2 (2p_b - p_b^2)}}{p_b}$$  \hspace{1cm} (5-7)$$

An overall comparison of the two acquisition systems can be performed by plotting $\tau_1$, given by equations (5-6) and (5-7), versus the antenna beam pointing probability using the probability of detection as a parameter. The plot is shown in figure 4. The detection probability values were chosen arbitrarily to show the performance of the two systems under more or less extreme conditions; however, a more realistic comparison utilizing system variables such as antenna gains, transmitter RF power for communication and acquisition and receiver bandwidth can be performed also. Figure 4 indicates that, for low detection probability, the expected number of steps $\tau_1$ increases more rapidly for the narrow beam system than for the self-phasing system when the antenna beam pointing probabilities decrease. In practice, it would not be unusual to acquire under high detection probability but low antenna beam pointing probability. For instance, if the complete azimuth must be searched for acquisition, the probability of an antenna beam pointing in the desired direction is proportional to ratio of the azimuthal beamwidth to $2\pi$ which can be quite low. The acquisition conditions which would render the self-phasing system more attractive than the corresponding narrow beam system are that of high detection and low antenna beam pointing probabilities, and that of low detection and low antenna beam pointing probabilities.

VI. Conclusions

The ability of self-phasing antennas to respond to the interrogator renders them very attractive as a part of electronic subsystems for application to space communications. Some of these attractive features were demonstrated in the present work by results obtained for two specific examples of applications.

In the first application, the self-phasing antennas of circular Van Atta array type were employed on the communication satellite launched into a critically inclined elliptic orbit, and the satellite was spin stabilized with its spin axis perpendicular to the earth's equatorial plane. In this case, the self-phasing antenna would allow its main beam with high directivity to be steered by the interrogator on the ground, whereas the chosen orbital configuration made the implementation of more conventional type of satellite antenna with reasonable gain a rather difficult problem.

In the second application, the mutual acquisition problem between two satellites employing self-phasing antennas was analyzed through the use of a finite Markov chain technique. The results were compared with those obtained for the corresponding problem wherein the satellites employed narrow beam antennas pointing at each other. The comparisons showed that the self-phasing system appears somewhat superior especially under the acquisition condition of low antenna beam pointing probability and low detection probability.
Appendix A

The definitions of various probability terms will be given first for the self-phasing acquisition problem under the condition that B wishes to acquire A and in turn be acquired by A.

\[ p_{12} = \text{Probability that the signal is detected at A and not at B when the antennas of A and B are pointing at desired directions.} \]

\[ p_{14} = \text{Probability that the signal is detected at both A and B.} \]

\[ p_{24} = \text{Probability that the signal is detected at B under the condition that A has detected B.} \]

\[ p_A = \text{Probability of the antenna of A pointing in the desired direction.} \]

\[ p_B = \text{Probability of the antenna of B pointing in the desired direction.} \]

\[ p_a = \text{Probability of A detecting the desired signal.} \]

\[ p_b = \text{Probability of B detecting the desired signal.} \]

\[ p(A,B) = \text{Probability of the antenna of A and B pointing at each other.} \]

\[ p(A/B) = \text{Conditional probability of the antenna of A pointing at B under the condition that the antenna of B is pointing at A.} \]

\[ p(A/B) = \frac{p_A p_a}{p_B} = p_a \text{ where } p_A = 1 \text{ since A employs a self-phasing antenna.} \]

\[ p(A,B) = p(A/B)p_B = p_a p_B \]

\[ p_{12} = p(A,B) (1 - p_b) = p_B p_a (1 - p_b) \]

\[ p_{14} = p(A,B) p_b = p_B p_a p_b \]

\[ p_{24} = p_b \]

\[ p_{11} + p_{12} + p_{14} = 1 \]

\[ p_{22} + p_{24} = 1 \]
For the narrow beam acquisition problem, the definitions and meanings of transitional probability elements are self-explanatory.

\[ P_{12} = p_a p_b (1 - p_b) \]
\[ P_{13} = p_a p_b (1 - p_a) \]
\[ P_{14} = p_a p_b p_a p_b \]
\[ P_{24} = p_b p_b \]
\[ P_{34} = p_b p_a \]
\[ P_{11} = 1 - (P_{12} + P_{13} + P_{14}) \]
\[ P_{22} = 1 - P_{24} \]
\[ P_{33} = 1 - P_{34} \]
Appendix B

Appendix B indicates the abbreviated steps through which the column matrix $M_i(t)$ is finally obtained. The definitions and terminologies employed in the present discussion are identical to those given in reference 5.

The transition matrix $P$ of a finite Markov chain can be partitioned into four submatrices and is represented in the following manner:

$$
P = \begin{pmatrix}
S & 0 \\
R & Q
\end{pmatrix}
$$

The submatrix $Q$ concerns the process of transient states. The submatrix $R$ concerns the transition from transient to ergodic states. The submatrix $S$ deals with the process when it has reached the ergodic states. The elements of the submatrix $O$ are all zeroes. A matrix $\{M_i[N_j]\}$ was shown$^5$ to be equal to

$$\{M_i[N_j]\} = (I - Q)^{-1}
$$

where $I$ is an identity matrix and the exponent $-1$ indicates the inverse of a matrix. The interpretation of the matrix $\{M_i[N_j]\}$ is that it yields the expected number of times the process is in the state $S_j$ when it started in $S_i$. $M_i(t)$ is then found to be

$$M_i(t) = (N)(g)
$$

where $(g)$ is a column matrix all of whose elements equal to unity.
References


Figure 1. Van Atta Arrays

(a) Linear Van Atta Array

\[ \phi = \frac{2\pi d}{\lambda} \sin \theta \]

(b) Planar Van Atta Array (conjugate elements bear identical numerals)

Figure 1. Van Atta Arrays
Figure 2. Phase Conjugation Array
Conjugate elements bear identical numerals and are interconnected.

Figure 3. Circular Van Atta Array
Figure 4. Expected Number of Steps for Acquisition Versus Antenna Beam Pointing Probability