Microwave Atmospheric Refraction Correction Radiometer

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Range measuring systems are limited in accuracy by uncertainties in the atmospheric refraction correction. The effects of water vapor are difficult to evaluate and are responsible for most of the problem. The application of a microwave radiometer to measure the random noise radiation from water molecules along a transmission path can lead to a precise determination of the water vapor content of the atmosphere. The refraction correction can then be made to an accuracy of less than 1% as compared to present methods which give 3 - 5%.

I. Introduction

The accuracy of radar tracking systems has increased to the stage where one of the major limiting factors is tropospheric refraction, especially at low elevation angles [1]. Much effort has been expended to establish realistic error models and to correlate ground measurements with refraction over the ray path. Because of the variability, both in space and in time, of the refractive index of the atmosphere, these efforts have met with only limited success. Dr. A. C. Menius*, a consultant to the Pan American Technical Staff, has suggested the use of a radiometer in arriving at better refraction corrections. Proceeding from his preliminary calculations, the authors have developed a technique which shows considerable promise.

In particular, it is proposed to measure atmospheric noise as a function of elevation angle and then to relate this noise, or "antenna temperature," to the water vapor contribution to tropospheric refraction. Section II presents the effects of tropospheric refraction on electronic tracking signals in terms of integrals of atmospheric physical properties over the ray path from the tracking station to

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the tracked vehicle. One of these integrals is calculable with considerable certainty from meteorological measurements, but the other is not. In Section III, it is shown that this variable integral can be evaluated as a function of elevation angle if the integral of the total absorption coefficient at a suitable microwave frequency is known. The next section then shows that the integral of the total absorption coefficient along the ray path can be measured by a microwave radiometer, which is further discussed in Section V. Finally, conclusions are listed in Section VI.

II. Tropospheric Refraction and Radar Tracking

In radar tracking of vehicles either inside or outside the earth's atmosphere, the quantity which is actually measured is the transit time of a (generally microwave) electromagnetic signal between the vehicle and a ground station. If there were no atmosphere, then the true range would be obtained by multiplying the measured transit time, $\Delta t$, by the free space velocity of light, $c$. But $\Delta t$ is really the integral of $\frac{ds}{v}$ along the ray path, with $v$ the actual velocity of the signal. Using the fact that $v$ is the free space velocity divided by the index of refraction $n$, $\Delta t$ may be written

$$\Delta t = \int \frac{ds}{v} = \int \frac{ds}{c/n} = \frac{1}{c} \int n ds.$$ \hspace{1cm} (1)

Accordingly, the apparent range is

$$R_{ap} = c \Delta t = \int n ds$$ \hspace{1cm} (2)

The range correction which would then be necessary due to refraction effects is the difference between this apparent range and the true range $R$, or

$$\Delta R = R_{ap} - R = \int n ds - R \hspace{1cm} (3)$$

Because of the fact that the refractive index of the atmosphere varies both vertically and horizontally, the true ray path between tracking station and space vehicle will not be the same as that which would be followed in the absence of the atmosphere. However, the difference in path lengths is small for elevation angles above five degrees. If necessary, we could calculate the path difference
based on some atmospheric model (such as an exponentially varying refractive index) for elevation angles down to two degrees or so with sufficient accuracy as to leave negligible residual error from the same path assumption. We may then replace Equation (3) by

$$\Delta R = \int (n - 1) \, ds ,$$

(4)

with the integral taken along the actual ray path.

For convenience, it is common practice to introduce the quantity

$$N = 10^6 (n - 1) .$$

(5)

Thus, an accurate refraction correction requires a good estimate of the integral

$$\int N \, ds = 10^6 \Delta R .$$

(6)

For wavelengths in the microwave region, which is by far the most important region for missile tracking, the troposphere is the only portion of the atmosphere which contributes to refraction. For the correction of radar tracking data, the ideal solution would be through the independent measurement of \( \int n \, ds \) (or \( \int N \, ds \)) over the ray path simultaneous with the tracking measurement. Due to refractive index fluctuation, and the fact that the troposphere extends up to about 30 miles, it is, of course, not feasible to obtain refractive index measurements along a large set of ray paths which will be valid for an appreciable length of time.

Fortunately, the variability of the tropospheric refractive index is primarily due to only one of the constituents of the atmosphere, water vapor, and this is essentially confined to regions below 40,000 feet. It is thus possible for aircraft carrying refractometers to fly over actual ray paths in near real time. In this way, information can be obtained on the variability of the refractive index along typical ray paths; however, changes with time while a single ray path measurement is being made, the necessary time lag between measurements and missile flight, and the large number of flights which would be necessary to cover all ray paths preclude the use of aircraft refractometer data to follow all fluctuations in \( \int N \, ds \) as the ray path to a missile sweeps through the atmosphere.
To see how the fluctuating refractive index integral arises, let us now decompose \( \int N \, ds \) into integrals depending upon physical properties of the atmosphere along the ray path. From a combination of theory and experiment, it is found that the refractivity, \( N \), of the atmosphere can be expressed as

\[
N = \frac{A}{T} + \frac{B}{T^2},
\]

where \( T \) is the absolute temperature (°K), \( p \) is the total atmospheric pressure (millibars), \( e \) is the partial pressure of water vapor,

\[
A = 77.607 \pm 0.013 \text{ °K per millibar}
\]

and

\[
B = 3.73 \times 10^5 \pm 0.04 \times 10^5 \text{ (°K)}^2 \text{ per millibar}.
\]

Physically, the first term in Equation (7) is due to the induced polarization of all the air molecules, while the second term is associated with the permanent electric dipole moment of water molecules. A complete discussion of the theory has been given by Van Vleck [2], while the currently used values of the constants have been presented by Smith and Weintraub [3]. It may be noted that to accuracies of 1/10%, no frequency dependence (dispersion) has been found for Equation (7) in the microwave region [3, 4].

Since the ratio of pressure to temperature is proportional to density for any gas (except at high pressures and temperatures), we can also write the expression for refractivity as

\[
N = A \rho + B \rho_w T^{-1},
\]

where \( \rho_w T^{-1} = 216.7 \text{ e T}^{-2} \text{ grams per meter}^3 \text{ per °K} \), with \( \rho \) the total air density and \( \rho_w \) the density of water vapor. Integrating \( N \) along a ray path, one has

\[
\int N \, ds = A \int \rho \, ds + B \int \rho_w T^{-1} \, ds.
\]

As stated above, the integral of the total density along the ray path can be calculated with much more certainty at the low elevation angles than can the
integral of $\rho_w T^{-1}$. Particularly, the variation of $\int \rho_w T^{-1} \, ds$ with elevation angle is difficult to obtain accurately from either "standard" meteorological measurements or from aircraft refractometer flights. It is this type of information which should be provided through the microwave radiometer measurements.

III. Tropospheric Absorption of Microwaves

In Equation (9), we have expressed the effect of tropospheric refraction on range in terms of two integrals of atmospheric physical properties along the ray path from tracking site to missile. The first of these integrals is calculable with sufficient accuracy from ground level measurements and available knowledge of the variation of atmospheric density with altitude. The second integral, which depends on the amount of water vapor, is neither readily calculable nor directly measurable. In this section, we would like to express the latter integral in terms of two other integrals, one of which is calculable from a knowledge of the variation of atmospheric density with altitude, while the other is measurable by means of the microwave radiometer to be discussed in the following two sections.

That there should be a close relation between absorption measurements and refraction corrections is not really surprising, since the dispersion relations \[4\], derived on quite general physical grounds, show that if either the index of refraction or the absorption coefficient is known as a function of frequency, then the other can be calculated for all frequencies. However, we shall here be concerned with only a limited range of frequencies, all in the microwave region.

The absorption of the atmosphere in the centimeter region is due almost entirely to $O_2$ and water vapor. * For the latter the absorption of microwaves is a sum of contributions from a number of resonances, the most important of which is located at a wavelength of 1.35 cm. Van Vleck \[4\]

* Fog, clouds, and rain can also contribute significantly under some conditions. For most missile tracking situations they should be relatively unimportant.
has recommended an absorption formula derived from quantum mechanical perturbations calculations which gives good agreement with experiment if a semiempirical non-resonant background is added. With some trivial adjustments in notation the formula may be written

\[ \alpha_w = \nu^2 \rho_w T^{-1}(\Delta \nu) \left[ \frac{C}{(\nu - \nu_o)^2 + (\Delta \nu)^2} + \frac{C}{(\nu + \nu_o)^2 + (\Delta \nu)^2} + F \right] , \quad (10) \]

where \( \alpha_w \) is the absorption per unit length for an electromagnetic wave having the electric field

\[ E = E_0 e^{i 2 \pi \nu (t - nx/c)} e^{-\alpha_w x/2} \]

The \( T^{-1} \) dependence of \( \alpha_w \) is an approximation suitable for a narrow temperature range such as is encountered in the lower regions of the atmosphere.

In Equation (10), \( \rho_w \) is density in gram/m\(^3\), \( \nu \) is frequency in cps, \( \nu_o \) is resonant frequency (22.2 \( \times \) 10\(^9\) cps), and \( \Delta \nu \) is the "line breadth" in cps. The quantities \( C \) and \( F \) are independent of frequency, temperature and density, but \( \Delta \nu \) is proportional to the collision frequency. The values of \( C \) and \( F \) are

\[ C = 0.788 \times 10^{-14} \frac{\text{K sec}}{\text{g/m}^3} \]

\[ F = 1.25 \times 10^{-34} \frac{\text{K sec}^3}{\text{g/m}^3} \]

Since \( \Delta \nu \) depends upon collision frequency, it is a function of air density and temperature (approximately, \( \Delta \nu \propto \rho \sqrt{T} \)), either or both of which may vary over the path of the radar waves. However, we would like to extract from \( \int \alpha_w \, ds \) something which is proportional to \( \int \rho_w T^{-1} ds \). This will be strictly possible for an arbitrary distribution of \( \rho_w T^{-1} \) only if the product of \( \nu^2 \Delta \nu \) and the bracketed term in Equation (10) is independent of position along the ray path. Let us denote this function by \( g(\nu, \Delta \nu) \).

Considerable uncertainty exists as to the value of the constant \( F \). The value listed is such as to give approximate agreement with experiment.
Define $g'(y)$ as an "average" value of $g(y, A\nu)$ using the relation

$$\int a_w \, ds = \int \rho_w T^{-1} g(y, A\nu) \, ds = \bar{g}(y) \int \rho_w T^{-1} \, ds . \quad (13)$$

In this definition of $\bar{g}(y)$, we have only one parameter at our disposal, the frequency $\nu$, so one should select $\nu$ such that $g(y, A\nu)$ is as nearly constant as possible. On the assumption that $A\nu$ varies exponentially with altitude above the surface of the earth,

$$A\nu = (A\nu)_0 e^{-h/h_a} , \quad (14)$$

where $h_a$ is defined as the scale height for air. The function $g(y, A\nu)$ is shown graphically for several values of $\nu$ in Figure 1 for

$$(A\nu)_0 = 2.80 \text{ Gc/sec.} \quad (15)$$

$h_a = 9 \text{ km}$. as given by Van Vleck [4] and Bean and Thayer [5]. It is seen that on both sides of the resonance at 1.35 cm there is a region in which $g(y, A\nu)$ is approximately constant for the smaller values of $h$. The actual process of choosing the optimum frequency according to the above criterion amounts to setting

$$\frac{d}{d\nu} \left\{ \left[ \frac{1}{\bar{g}(\nu)} \int \rho_w T^{-1} g(\nu, A\nu) \, ds - \int \rho_w T^{-1} \, ds \right]^2 \right\} = 0 \quad (16)$$

Assuming $\rho_w T^{-1}$ to vary exponentially (as does its statistical mean) with a scale height of 3 km [5], it is found that the optimum frequencies are

$$\nu_1 = 20.8 \text{ Gc/sec.} \quad (17)$$

$$\nu_2 = 23.7 \text{ Gc/sec.} .$$
Substitution of $v_1$ into Equation (13) then gives

$$g(v) = 0.00119$$  \hspace{1cm} (18)

or

$$\int a_w \, ds = 0.00119 \int \rho_w \, T^{-1} \, ds$$  \hspace{1cm} (19)

with $\rho_w$ in grams/m$^3$, $T$ in degrees Kelvin, and path length in meters.

It should be noted that the above optimum frequency is near one of the major resonances of water vapor, and consequently the absorption cross section has a relatively large value. The other major absorber of microwaves in the atmosphere, molecular oxygen, does not have a resonance near 22.2 Gc/sec., so its absorption in this frequency region is much less than that of water vapor, though the greater abundance of $O_2$ along the ray path requires that it be taken into account.

Van Vleck [4] has given a formula for the absorption due to oxygen which can be rewritten as:

$$\alpha_x = 27.4 \times 10^{-16} \rho_x T^{-1} \nu^2 \left(\Delta \nu_x\right) \left\{ \frac{1}{(\nu - \nu_r)^2 + (\Delta \nu_x)^2} + \frac{1}{(\nu + \nu_r)^2 + (\Delta \nu_x)^2} + \frac{2}{\nu + (\Delta \nu_x)^2} \right\}$$  \hspace{1cm} (20)

where the notation is similar to that used earlier for water vapor. The resonant frequency $\nu_r$ is 60 Gc/sec. and $(\Delta \nu_x)_r = 0.6$ Gc/sec., so for $\nu = \nu_1 = 20.5$ Gc/sec. the $(\Delta \nu_x)^2$ term in the denominators can be neglected. Again the exponential height dependence for the line breadth can be assumed as approximately valid,

$$\Delta \nu_x = (\Delta \nu_x)_r e^{-h/h_a}$$  \hspace{1cm} (21)

since $\rho_x$ has the same dependence on height as $\Delta \nu_x$, $\alpha_x$ actually decreases with a scale height of $h_a/2$ for this frequency. The result for oxygen absorption is:

$$\int \alpha_x \, ds = (1.10 \times 10^{-6}) h_a (\rho_x)_r T^{-1} \csc \theta$$  \hspace{1cm} (22)
where $E$ is the elevation angle of the ray path. The magnitude of the integral can be estimated by substituting approximate values for the quantities on the right [5]:

$$h_a = 9,000 \text{ m}$$

$$\langle \rho_x \rangle_o = 279 \text{ gm/m}^3$$  \hspace{1cm} (23)

so that

$$\int \alpha_x \, ds = 0.00943 \csc E.$$  \hspace{1cm} (24)

The magnitude of the corresponding integral for water vapor can be estimated by using typical values for density, $\langle \rho_w \rangle_o = 15 \text{ g/m}^3$, and temperature, $T = 293 \degree \text{K}$, in Equation (19). We find

$$\frac{\rho_w}{T} = 0.051 \, e^{-3h/h_a} \text{ g/m}^3 \degree \text{K}$$  \hspace{1cm} (25)

and

$$\int \alpha_w \, ds = 0.182 \csc E.$$  \hspace{1cm} (26)

Consequently, for this case

$$\int \alpha_x \, ds = 0.05 \int \alpha_w \, ds$$  \hspace{1cm} (27)

We are thus fully justified in using the exponential atmosphere model for the computation of the integral $\int \alpha_x \, ds$, since the effective error will be reduced by about a factor of 20 when it is combined with the water absorption coefficient integral.

The total microwave absorption coefficient is the sum of that for oxygen and for water vapor,

$$\int \alpha \, ds = \int \alpha_x \, ds + \int \alpha_w \, ds$$  \hspace{1cm} (28)

and the refractivity can be separated similarly:

$$\int N \, ds = \int N_a \, ds + \int N_w \, ds$$

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Combining the above with Equations (9, 19, and 22) one may write the water vapor contribution to the refractivity as:

\[ \int \Nw \, ds = 1.45 \times 10^6 \int \alpha \, ds - 1.59 \, h_a (\rho_x)_0 \, T_o^{-1} \, \csc \, E \]  

(29)

IV. Theory of Radiometer Measurements

The above development has shown that the variable part of the refraction correction can be estimated if the integral of the absorption coefficient along the ray path can be measured. We shall now show that this integral can be obtained from microwave antenna "noise" measurements.

Consider a highly directional microwave antenna pointed into the atmosphere, connected to a receiver of bandwidth \( b \) and tuned to a frequency \( \nu \). Assuming no man-made signals at this frequency, it is necessary to consider three different natural sources: extraterrestrial sources, atmospheric "noise," and radiation from the surface of the earth itself. We will assume that the antenna beam pattern is such that the third source is negligible compared to the first two.

By definition, the antenna temperature \([6, 7, 8]\) is the temperature of a totally absorbing medium (black body) in which the antenna could be placed such that the antenna would collect the same power within its bandwidth \( b \) as it does in the experimental situation. Similarly, the temperature of the extraterrestrial source is defined as that of a black body which would radiate the same power in the same bandwidth. In passing through the atmosphere the radiation is reduced by absorption and is increased by spontaneous and induced emission. At any point in the atmosphere an equivalent black body temperature (which would be measured by the antenna) can be defined as that temperature which would produce the same flux density in the same bandwidth at that point. The brightness \( B(\nu) \) is the flux per unit solid angle subtended by the source, and for a black body is related to the temperature by the Rayleigh-Jeans Law:

\[ B(\nu) = \frac{2 \kappa T}{\lambda^2} \, \text{watts} \, \text{meter}^{-2} \, \text{steradian} \, \text{cps} \]  

(30)
where \( k \) is Boltzmann's constant. The variation of \( B(\nu) \) or \( T \) along a ray path depends simply on the difference between the equivalent black body temperature for the incident radiation accumulated from distant sources and the excitation temperature \( T_e \) of the local sources:

\[
\frac{dT}{dr} = \alpha(\nu) \left[ T - T_e \right]
\]  

(31)

where \( \alpha(\nu) \) is the absorption coefficient. Upon integration one finds for the temperature at the antenna:

\[
T_a = T_s e_0^{\int_0^l \alpha(r')dr'} + \int_0^l \alpha(r) T_e(r) dr e_0^{\int_0^r \alpha(r')dr'}
\]  

(32)

where \( T_s \) is the temperature outside the atmosphere due to extraterrestrial sources. For an intuitive derivation of the above result the reader should consult the paper of Dicke [6] and for a complete discussion he should refer to the book of Steinberg and Lequeux [7].

If \( T_e \) were independent of \( r \), the last integration could be readily performed. Since atmospheric temperatures vary with altitude by only about 10% for the regions of primary interest, let us define a mean temperature \( T_m \) by

\[
T_m = \frac{\int_0^l \alpha(r) T_e(r) e_0^{\int_0^r \alpha(r')dr'} dr}{\int_0^l \alpha(r) e_0^{\int_0^r \alpha(r')dr'} dr}
\]  

(33)

so that:

\[
\int_0^l \alpha(r) T_e(r) e_0^{\int_0^r \alpha(r')dr'} dr = T_m \left( 1 - e_0^{\int_0^l \alpha(r) dr} \right)
\]  

(34)
With this substitution, (32) becomes

\[
T_a = T_s e^\int_0^\ell \alpha(r) \, dr + T_m \left( 1 - e^\int_0^\ell \alpha(r) \, dr \right).
\]  

(35)

This may be solved for the integral \( \int_0^\ell \alpha(r) \, dr \) to give

\[
\int_0^\ell \alpha(r) \, dr = -\ell \ln \left( \frac{1 - T_a/T_m}{1 - T_s/T_m} \right).
\]  

(36)

The limit \( \ell \) on the integral here is assumed to extend through the atmosphere, so the brightness temperature \( T_s \) is the temperature of extraterrestrial sources. In the microwave region, however, this temperature is not more than a degree Kelvin \( [7] \), and can thus for all practical purposes be neglected except in the case of the sun or the moon.

V. Instrumentation Requirements

From Equation (36) one finds that temperature variations of 1/2° K could give changes in water vapor absorption or refractivity of about 1%. For quite unfavorable conditions (surface temperatures of 305° K and 100% humidity) radiometer temperatures of 0.5° K accuracy at elevation angles of \( \sim 6° \) lead to residual range errors of five to ten centimeters. Average weather conditions would produce residual range errors of only three centimeters. This would correspond to an accuracy of 1/2 N unit for the refractivity averaged over the entire path from antenna to tracked vehicle. Such an accuracy represents a significant improvement over present methods for evaluating the refraction correction. Though refractometers might obtain even higher accuracy at one point in the atmosphere, it is impractical to make sufficient refractometer measurements to give the path integral correction available from the radiometer. Thus, a reasonable design goal is a radiometer sensitivity of 0.5° K.
The sensitivity of a Dicke radiometer [8] is related to receiver noise temperature, bandwidth $b$, and integration time $\tau$. For a modern amplifier with semiconducting elements or a low noise travelling wave tube the following formula is a good working approximation:

\[
(\Delta T_a)(b\tau)^{1/2} \approx 4000^\circ K
\]  

(37)

Consideration of the arguments for frequency optimization dictates a maximum bandwidth of 1 Gc/sec., so the lower limit on the integration time is 0.064 sec. if $\Delta T_a = 0.5^\circ K$. To determine if this is appropriate for tracking purposes one must compare the antenna beamwidth with the angle swept by the antenna during two integration periods. The factor of two is necessary because the Dicke radiometer switches back and forth between the antenna and a reference load. For a sweep rate of 0.02 rd./sec. the angle would be 0.15 degrees. To obtain a "matching" beamwidth at 20.5 Gc/sec., a large antenna (7 m. diameter) would be required. Thus in a practical application the antenna size rather than the receiver sensitivity might be the limiting factor; i.e., the averaging over the main antenna lobe might be greater than the averaging due to the sweep.

This consideration is relevant since it is desirable to detect five structure inhomogeneities in the atmosphere in order to improve velocity data. Also the $\Delta T_a$ discussed earlier refers to relative measurements and not to an absolute temperature. A detailed study of the error budget leads one to the conclusion that even with somewhat relaxed requirements on the radiometer, $\Delta T_a = 1^\circ K$ for a beamwidth of $1^\circ$, a refraction correction with a residual error of much less than 1% of the total correction is possible. This is easily better than the estimated residual of 3 to 5% with currently used methods.

The most important limitations on the present method are:

1) The antenna cannot be pointed directly toward the missile since its exhaust presents an extraneous high temperature source.

2) The accuracy deteriorates if there is a high liquid water content in the atmosphere (rain or dense fog and cloud).
VI. Conclusions

Both refraction and absorption of microwave signals in the atmosphere depend upon known physical properties of the constituent molecules. All constituents contribute to refraction, but absorption is due primarily to water vapor and oxygen, the former producing the largest effect for centimeter wavelengths. In the preceding sections the relevant theory has been developed and it has been shown that the integrated refractive index due to atmospheric water vapor can be expressed in terms of the properties of oxygen and the integrated absorption, a measurable quantity. Thus, the effects of varying amounts of water vapor can be effectively eliminated through use of a microwave radiometer sensitive to the atmospheric noise background associated with the absorption. From Equations (29) and (36) the contribution of water vapor to the integral \( \int N \, ds \) may be written:

\[
\int N_w \, ds = -1.45 \times 10^6 \, \ell \, n \left( \frac{1-T_a/T_m}{1-T_s/T_m} \right) - 1.36 \times 10^4 \, \csc \, E \quad (38)
\]

where \( T_a \) is the temperature determined by the radiometer, \( T_m \) is a mean atmospheric temperature, \( T_s \) is an extraterrestrial background, and \( E \) is elevation angle. The optimum frequency for the system is 20.8 Gc/sec. and it should be capable of detecting 1.0° K changes in \( T_a \), though absolute accuracy may be limited to about 3° K with present antenna technology. The corresponding figures for range correction due to water vapor refraction would be 3 centimeter changes and 10 centimeter absolute inaccuracy. The antenna beamwidth should be less than 1° in order to adequately resolve atmospheric inhomogeneities.

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Figure 1. The function $g(\nu, \Delta \nu)/10^{11}$ C displayed above shows the response or sensitivity of the absorption to the water vapor density and temperature (Equation 13). The desired uniform response over varying altitude is most nearly attained for $\nu = 20.7$ Gc/sec. or 23.8 Gc/sec.