Apr 5th, 8:00 AM

Three-Degree-Of-Freedom Simulation of Gemini Reentry Guidance

Martin Rush
*International Business Machines Corporation*

Walter Vodges
*International Business Machines Corporation*

Follow this and additional works at: [http://commons.erau.edu/space-congress-proceedings](http://commons.erau.edu/space-congress-proceedings)

Scholarly Commons Citation
http://commons.erau.edu/space-congress-proceedings/proceedings-1965-2nd/session-9/2

This Event is brought to you for free and open access by the Conferences at ERAU Scholarly Commons. It has been accepted for inclusion in The Space Congress Proceedings by an authorized administrator of ERAU Scholarly Commons. For more information, please contact commons@erau.edu.
THE THREE-DEGREE-OF-FREEDOM SIMULATION OF GEMINI REENTRY GUIDANCE

Martin Rush and Walter Vogdes
International Business Machines Corporation
Bethesda, Maryland

The necessity of providing a real-time simulation of controlled reentries of the Gemini vehicle for filtering of tracking data, performance monitoring, and real-time impact prediction has led to the development of a computer program which simulates the Gemini vehicle's performance during computer-controlled reentries. The system characteristics incorporated in the simulation as well as a discussion of some typical Gemini reentry trajectories make up the body of this paper.

Introduction

The Gemini vehicle will be the first United States manned spacecraft capable of controlled reentry through the atmosphere. This capability is obtained by a center of gravity which is offset from the body center line of the vehicle. The asymmetric attitude which results gives rise to an aerodynamic lift force, which can be used to alter the reentry trajectory. By controlling the roll attitude of the capsule, the lift vector can be driven to a location anywhere in a plane normal to the wind vector. The time history of the orientation of the lift vector is then a controlling factor in descent trajectory characteristics and determines the impact point of the vehicle within its range capability profile or footprint. The control of the orientation of the lift vector or roll angle is obtained by the utilization of an inertial platform and a digital computer carried aboard the vehicle. The vehicle's orientation with respect to the platform provides a means of determining the attitude of the vehicle, while accelerometers mounted on the platform provide a means of sensing the aerodynamic and thrust forces on the vehicle. The computer monitors the accelerometer outputs and computes the position and velocity of the vehicle in the navigation section of its math flow. In the guidance phase of the computer math flow the computer generates attitude commands to control the roll orientation of the vehicle. These commands are transmitted to the attitude control system, which controls the action of jets mounted on the vehicle.

Description of Reentry Phases

Gemini reentry may be broken down into five phases of flight (Fig. 1):

A. Pre-retrofire
B. Retrofire
C. Descent
D. Maneuver
E. Parachute

Phase A is the phase in which the vehicle and systems are prepared for retrofire. Phase B is concerned with the period during retrofire. Phase C deals with the vehicle after completion of retrofire and prior to the time the Gemini computer initiates atmospheric maneuver of the vehicle to the target. During Phase D, the computer provides roll commands to the attitude control system (ACME) that will cause the vehicle to maneuver for impact at the target. It is in this phase of flight that the computer's generation of bank angle commands and the vehicle's response to these commands are simulated. Phase E, the final phase, is initiated with the release of the parachute and terminates upon impact with the earth's surface.
At the initiation of the pre-retrofire phase, the computer accepts the following parameters transmitted to the vehicle from the ground via the Digital Command System (DCS) or manually inserted in the computer by the astronaut crew via the Manual Data Insertion Unit (MDIU):

\[ T_R \] - Time to retrofire

\[ X_{IR}, Y_{IR}, Z_{IR} \] - Predicted position of the vehicle at \( T_R \) in a geocentric inertial coordinate system.

\[ \dot{X}_{IR}, \dot{Y}_{IR}, \dot{Z}_{IR} \] - Predicted inertial velocity of the vehicle at \( T_R \) in a geocentric inertial coordinate system.

\[ \Delta \lambda_{IR} \] - Inertial longitude of Greenwich at \( T_R \)

\[ \phi_T \] - Geocentric target latitude

\[ \lambda_T \] - Earth referenced target longitude

The astronaut controls the attitude of the vehicle to maintain a view of the earth's horizon and to allow for proper retrothrust, which will be initially applied 16 degrees below the horizontal and opposite the orbital velocity. The inertial platform is torqued to provide one platform axis oriented toward the center of the earth (opposite the geocentric radius vector), one axis in the orbital plane and normal to the radius vector, and one axis normal to the orbital plane forming an orthogonal triad.

The computer is in a standby mode during the pre-retrofire phase. The accelerometer outputs are not accumulated in the velocity registers, and the computer running light is off. Prior to retrofire the computer deactivates its coupling with the DCS, preventing the transmission of new data from the DCS to the computer. A Time Reference System is used to count down the time to retrofire.

The retrofire phase is initiated at \( T_R \) (time to retrofire) equal to zero. At this time the computer running light goes on, providing the crew with an indication that the computer is ready for retrofire. The computer will now begin to monitor the accelerometer outputs. The nominal vehicle position and velocity \( (X_{IR}, Y_{IR}, Z_{IR}, \dot{X}_{IR}, \dot{Y}_{IR}, \dot{Z}_{IR}) \) are used as initial conditions for the integration routine during this phase of flight. The accelerometer outputs are accumulated to provide velocity changes due to retrofire along the three platform axes, which are now non-rotating with respect to inertial space. These velocity changes are transmitted to the astronaut display for retrofire monitoring. The specific forces are transformed to geocentric inertial coordinates and combined with the gravitational force generated within the computer to provide the total force on the vehicle. The geocentric components of these forces are integrated asynchronously to provide inertial velocity and position of the vehicle during retrofire. This operation continues for 60 seconds after retrofire initiation.

The descent phase of the flight is initiated 60 seconds after nominal retrofire begins and terminates at an altitude of 400,000 feet. During this phase of flight the forces on the vehicle used to compute velocity and position of the vehicle are generated within the computer. The computer generates the gravitational and drag forces on the vehicle and applies them to a set of equations of motion. The equations of motion are integrated with a fixed \( \Delta t \) of 16 seconds to provide the vehicle's position and velocity in a geocentric inertial coordinate system. A transformation is utilized to provide computed latitude, longitude, velocity, flight path angle, and heading angle in a rotating earth coordinate system.

Below an altitude of 400,000 feet, the vehicle enters the maneuver phase of flight. During this phase of flight the vehicle's roll attitude is controlled by commands from the computer.
When the computed altitude reaches 400,000 feet, the computer commands an attitude of zero degrees bank angle. This angle is maintained until the sensed specific force, smoothed over four computer computation cycles, exceeds 0.4 ft/sec\(^2\). At this level of acceleration the reentering capsule encounters sufficient dynamic pressure to allow for trajectory control.

The computer now generates bank angle commands to alter the trajectory so that the vehicle will impact at the target. Fig. 2 illustrates the information flow of the guidance system during the maneuver phase. The inertial platform provides the Gemini computer with the specific forces sensed by the accelerometers mounted on its inner gimbal, and with the roll, pitch, and yaw of the vehicle by means of transducers mounted on the corresponding gimbals. As shown in the figure, the computer reads the specific forces that are in platform coordinates and transforms them to a geocentric inertial coordinate system. A gravity model, operating on the computed vehicle geocentric distance and geocentric latitude, computes the gravitational forces on the vehicle in geocentric inertial coordinates. The computer then sums the forces on the vehicle and applies them to a set of equations of motion. By integration, the vehicle's inertial position and velocity are obtained.

The computer now transforms the vehicle's velocity and position to earth-referenced coordinates which are utilized by the reentry guidance section of the math flow. The reentry guidance math flow is utilized to provide bank error commands to the attitude control system (ACME). It computes these commands as a function of the vehicle's position relative to that of the target. This portion of the computer math flow was incorporated in the simulation. The output of the computer is a bank error command that is converted to an analog signal and transmitted to the ACME. The operation of the computer is asynchronous with a total computation cycle time of approximately 1.2 seconds.

**Reentry Guidance in Maneuver Phase**

Reentry guidance is initiated when the smoothed sensed acceleration reaches 0.4 ft/sec\(^2\). At this level the dynamic pressure on the vehicle is such that impact may be controlled by controlling bank angle. This critical point is generally reached in the altitude range of 305,000 to 310,000 feet. Above this point the bank angle is continuously driven to zero, and the major portion of the guidance logic is not entered.

Fig. 3 illustrates the math flow instrumented by the simulation. The capability of guiding the vehicle to the target is obtained by controlling the bank angle of the vehicle. The function of the reentry guidance math flow is to provide bank angle commands to ensure that the vehicle will arrive at the target's location on the earth. The vehicle's position is continuously updated by the navigation section of the computer math flow. This, together with the target position that is inserted during the pre-retrofire phase, is used to compute the great circle distance to the target. This computation is then resolved into down-range and cross-range distances "to go."

In Fig. 4 the vehicle's position (V) is shown at a relative longitude and latitude of \(\lambda_E\) and \(\phi\), respectively. The target's relative longitude (\(\lambda_T\)) and latitude (\(\phi_T\)) are shown defining the target's position (T) on the earth. By means of the spherical triangle VPT,

\[
\begin{align*}
B & = \sin \tilde{R}_T \sin \psi_T = \cos \phi_T \sin (\lambda_T - \lambda_E) \\
E & = \sin \tilde{R}_T \cos \psi_T = \sin \phi_T \cos \phi - \cos \phi_T \sin \phi \cos (\lambda_T - \lambda_E)
\end{align*}
\]

where \(\tilde{R}_T\) is the great circle arc from the vehicle to the target and \(\psi_T\) the heading to the target.
Thus, from the expressions above can be obtained:

\[
R_T = 60 \sin^{-1}\left[\frac{B^2 + E^2}{2}\right]^{1/2} \quad \text{(n.m.)}
\]

\[
\psi_T = \tan^{-1}\left[\frac{B}{E}\right]
\]

where \(R_T\) is the distance on a circular earth in nautical miles corresponding to the arc \(\hat{R}_T\).

The down-range arc (\(R_n\)) is defined by the projection of arc \(\hat{R}_T\) onto a plane defined by the relative velocity vector of the vehicle and the center of the earth. Cross-range arc (\(R_c\)) is now defined as the projection of the arc \(\hat{R}_T\) onto a plane normal to the down-range plane. While these range components are, in reality, great circle arcs forming a spherical triangle, a flat earth approximation is instrumented in the guidance computer. It is to be noted that as the vehicle is guided to the target, this approximation becomes more valid.

Fig. 5 shows the down-range distance to go and cross-range distance to go as:

\[
R_n = R_T \cos(\psi_E - \psi_T) \quad \text{n.m.}
\]

\[
R_c = R_T \sin(\psi_E - \psi_T) \quad \text{n.m.}
\]

where

\[\psi_E \equiv \text{relative heading of the vehicle.}\]

A quantity \(D\), which is characteristic of the altitude of the vehicle, is calculated from the expression

\[
D = \log_{10}\left[\frac{V_E^2}{a}\right]
\]

where

\[V_E \equiv \text{relative velocity}\]

\[a \equiv \text{smoothed sensed acceleration.}\]

The ballistic range capability, \(R_p\), is then predicted as:

\[
R_p = R_0(D) + F_1(D) \gamma_E + F_2(D) V_E \quad \text{(n.m.)}
\]

where

\[F_0(D), \quad F_1(D), \quad F_2(D) \equiv \text{functions of } D\]

\[\gamma_E \equiv \text{relative flight path angle.}\]

This expression is obtained by Taylor series expansion for spinning descent from a maximum lift trajectory. The coefficients are obtained from computer simulation. The \(F_0(D)\) term
includes a bias that causes the vehicle to spin early, preventing the possibility of overshooting the target. As the vehicle descends to the target, this bias is removed.

The quantity D also serves the function of ending the guidance regime. When D passes below a value of 4.675, the roll angle is driven to zero in preparation for deployment of the parachute. The operation is effected by setting the commanded bank angle to zero and letting the commanded roll rate equal the negative of the actual bank angle. This termination of guidance occurs at an altitude of 80,000 to 85,000 feet.

The guidance technique used is based upon a comparison of the down-range distance to go (Rn) and the predicted ballistic range (Rp). A controlled bank angle mode is used if Rp is less than Rn, and a spin mode is applied if Rp is equal to or greater than Rn.

In the controlled mode the computer commands a bank angle to reduce the cross-range error. The calculation of commanded bank angle is based upon the plane triangle of Fig. 5. The tangent of \( \alpha \) is computed as:

\[
B_{cf} = \frac{R_c}{R_n - R_p + KC_1}
\]

where \( KC_1 \) is a constant used to remove any bias placed in Rp for protection against overshooting the target.

If \( B_{cf} \) is greater than 1 (that is, the angle \( \alpha \) is greater than 45 degrees), the commanded bank angle is set equal to plus or minus 90 degrees depending on the sign of \( R_c \). If \( B_{cf} \) is less than 1, the commanded bank angle is a function of \( B_{cf} \). Specifically,

\[
B_c = \left[ |B_{cf}|^3 K_{R1} + |B_{cf}|^2 K_{R2} + |B_{cf}| K_{R3} \right] \frac{|B_{cf}|}{B_{cf}}
\]

where \( K_{R1}, K_{R2}, \) and \( K_{R3} \) are constants stored in the computer.

When the commanded bank angle has been determined the bank error, \( B_E \), is given by

\[
B_E = B_C - B_A
\]

where \( B_A \) is the actual bank angle. The commanded roll rate, \( \Delta \phi_b \), is set equal to the bank error except when the vehicle has been spinning. In this case a logical decision is made as to whether to continue to roll in the same direction or to reverse the direction of spin to attain \( B_C \). The direction of spin is changed only if the vehicle must rotate more than 230 degrees in its present direction of rotation to reach the commanded bank angle. In this manner the commanded bank rate is determined for the controlled mode.

The spin mode is used whenever the ballistic range is greater than the down-range to the target (\( Rp \geq Rn \)). In this case the computer has predicted that if the controlled mode is maintained the vehicle will overshoot the target. If the cross-range is less than 1 mile, the vehicle is commanded to spin by setting the commanded roll rate to plus or minus 15 deg/sec, where the direction is determined by either the previous commanded rate or the actual bank angle.

During the spin mode a continuous check is made upon cross-range. Should the cross-range exceed 1 n.m., spin will cease and the vehicle will be banked 90 degrees in a direction
that will reduce the cross-range distance to the target. This maneuver has the advantage that it will not affect the down-range distance. When the magnitude of cross-range has been reduced to a value less than 1 n.m., the vehicle will once again be commanded to spin.

Vehicle Response to Bank Angle Commands

Fig. 6 represents the computer and attitude control system as it pertains to the vehicle’s response to bank angle commands. The left-hand portion of the figure represents a simplification of the math flow within the computer. The vehicle’s actual bank angle \( \beta_A \) is sampled 1.0 second after the beginning of a computation cycle by means of the gimbal angle subroutine in the reentry guidance section of the math flow. This function is shown by the sampler.

The bank angle error \( \beta_E \), which is computed 1.2 seconds after the initiation of a computation cycle, is obtained in a different manner for each mode of operation. In the controlled bank angle mode the bank angle error is computed as the difference between the commanded and actual bank angles, while in the spin mode \( \beta_E \) is set equal to the nominal spin rate of \( \pm 15 \, \text{deg/sec} \). The bank error is then limited to 15 deg/sec and converted to an analog signal transmitted to the attitude control system (ACME). The output of the computer is \( \Delta \phi_h \), the bank error command, which is treated by the ACME as a roll rate command. The ACME then compares the commanded roll rate with the actual roll rate measured by a rate gyro. A roll rate error, \( P_E \), is then generated, and jets are applied in a direction that will reduce the roll rate error.

As shown in the figure, the jet actuators have a dead band, \( K_{DB} \). The function of the dead band is to prevent limit cycles and, therefore, unnecessary fuel consumption in the roll jets. \( K_{DB} \) has a value of 2 deg/sec. The effect of the dead band is to cause the roll attitude jets to remain off unless the roll rate error is greater in magnitude than \( K_{DB} \). The action of the jets is such as to produce, to a first-order approximation, a constant roll acceleration, \( \vec{B}_A \). With the above assumption, the magnitude of the roll acceleration is the ratio of the torque generated by the jets to the roll moment of inertia of the vehicle. The crew has the capability of selecting the number of jets used in torquing. For the Gemini missions investigated, the approximate angular acceleration obtained with one ring of jets is 5 deg/sec\(^2\), and with two rings of jets it is 10 deg/sec\(^2\). The actual roll rate and bank angle are obtained from the acceleration.

To understand the vehicle’s response to a spin command, assume that the previous commanded bank angle, actual bank angle, and roll rate were zero. The roll rate command \( \Delta \phi_h \) will be a constant value of 15 deg/sec, since the bank error \( \beta_E \) will be maintained at that value for as long as a spin is commanded. The roll rate error \( P_E \) initially will be 15 deg/sec because the initial roll rate was zero. Since this value is larger than \( K_{DB} \), the jets will go on. Assuming a single ring of jets is being used, there will be a roll acceleration of 5 deg/sec\(^2\). The vehicle will continue to accelerate about the roll axis until it has obtained a roll rate of 13 deg/sec. This rate will be reached 2.6 seconds after the jets are turned on. At this time the roll rate error \( P_E \) has dropped to 2 deg/sec, and owing to the jet dead band the jets will go off. Hence the steady-state roll rate will be 13 deg/sec rather than the commanded rate of 15 deg/sec. Any variation of the actual roll rate between 17 deg/sec and 13 deg/sec will not actuate the jets because of the dead band.

In the case of a commanded bank angle, the performance is more complex. Fig. 7 illustrates the response of the system to a commanded bank angle of \( \pm 60 \) degrees with the vehicle initially at zero bank angle and zero roll rate. The lower figure shows the commanded roll rate \( \Delta \phi_h \), which is generated by the computer and transmitted to the ACME. As described previously, this rate is equal to the bank angle error limited to a magnitude of 15 deg/sec. Note that a roll rate command is not generated until 1.2 seconds or a full computation cycle after the initiation of control.
The value of $\Delta \phi_b$ at the end of a computation cycle is obtained from the guidance section of the math flow. The center figure presents the time history of the jets resulting from the comparison of commanded roll rate and actual roll rate. When the jets are "+ on" a positive angular acceleration of 5 deg/sec$^2$ is obtained, and when the jets are "- on" a negative angular acceleration of 5 deg/sec$^2$ is produced. The jets are off for those time intervals in which the roll rate error is within the dead band of 2 deg/sec.

The upper figure shows the actual bank angle as a function of time. When the jets are on, the bank angle has a parabolic time history. When the jets are off, the actual bank angle rate is a constant, and thus there are straight line segments for the actual bank angle. The figure shows that for a step command of +60 degrees the actual bank angle will overshoot the desired value, reaching a maximum angle of approximately 77.6 degrees, and will take over 40 seconds to damp to the commanded value. This characteristic loose damping is further illustrated in the typical closed-loop results presented below.

Fig. 8 represents the program logic that simulates the vehicle’s response to bank angle commands. Since the jets are assumed to produce constant roll accelerations, integration to obtain roll rate and bank angle is not necessary. The time period for which the jets will be on is calculated algebraically, and the bank angles throughout the next interval are then computed.

Simulation Results

The implementation of the computer math flow and vehicle attitude response discussed above has led to the development of the Gemini Reentry Integration Program (GRIP).

Typical controlled reentry trajectories will now be illustrated. Retrofire from a constant altitude orbit of 161 n.m. was used to obtain the trajectory parameters during the descent phase. Targets were obtained from Fig. 9, which shows the location of impact points obtained by maintaining a constant bank angle for a specified time and then spinning until impact. The trajectory parameters 1448.4 seconds after retro burnout are shown on the figure and represent a point on the descent trajectory before a value of smoothed sensed acceleration equal to 0.4 ft/sec$^2$ is reached. The figure provides the maneuver capability of the vehicle as the locus of "no spin" impact points. This is the locus of impact points obtained by holding the indicated bank angle until impact.

For the simulation study, four targets were selected from the figure. The first target selected was the +60 deg/400 sec impact point. This point is obtained by holding a bank angle of +60 degrees for 400 seconds and then spinning to the ground. The time to spin is referenced to the initial conditions shown in Fig. 8 and represents a target near the southern edge of the footprint. The second target used was the 0 deg/300 sec target. Here maximum lift (zero degree bank angle) is maintained for 300 seconds after entry, and then spin is initiated. Two other targets, -30 deg/250 sec and -60 deg/100 sec, are presented in order to demonstrate the specific characteristics of the resulting bank angle time histories during entry.

Utilizing GRIP, computer-controlled trajectories were generated for the above targets and the resultant impact points are shown in Table 1. Detailed results of the simulation runs follow.

Figures 10 through 13 show the bank angle time histories of the vehicle during the maneuver phase of flight. For entry to the +60 deg/400 sec target, the bank angle time history obtained has a somewhat different response from the specified angle and time to spin because of inaccuracies in the ballistic range predictor and the flat earth approximations instrumented in the computer.
In Fig. 10 the computer initially commands a bank angle of approximately +70 degrees. Here the computer is in the controlled mode. It remains in this mode until an altitude of 82,000 feet is reached, at which point the bank angle is driven to zero in preparation for parachute deployment.

The bank angle time history for the 0 deg/300 sec target is shown in Fig. 11. Again, owing to guidance approximations, the angle initially commanded is not zero but 15 degrees. The actual angle slowly decreases as the computed cross-range error is driven through zero. At 1718 seconds after retro burnout the predicted ballistic range capability becomes greater than the computed down-range to the target and the spin mode is entered. At this time the computed cross-range is less than 1 n.m., and the vehicle spins. At 1731 seconds, approximately 300 seconds after entry, the ballistic range predicted becomes less than the down-range distance to the target, and the commanded angle mode is reentered. From 1731 seconds until 1776 seconds the commanded angle slowly changes from 16.6 degrees to -13.6 degrees while the actual bank angle oscillates about the command. At 1776 seconds the vehicle once again enters the spin mode. The spin mode is continued until 1815 seconds, when the predicted ballistic range once again drops below the down-range to the target. The guidance system now commands a small angle and the vehicle continues to spin toward the command, finally oscillating about a negative commanded angle of approximately -10 degrees. At an altitude of 82,000 feet, guidance is terminated and the vehicle is driven to a zero-degree bank angle. For this trajectory the guidance system passes back and forth between the controlled and the spin mode, even though the programmed bank angle time history used to obtain the switch of mode.

The simulation for the -30 deg/250 sec target provides a typical entry, where the actual bank angle response is a negatively increasing angle that varies widely from the nominal -30 degrees. At 1676 seconds after retro burnout, the spin mode is entered. At this time the computed cross-range error is greater than 1 n.m., and a -90 degree angle is commanded. This command is followed by a period of switching between the controlled mode and the spin mode. Finally, at an altitude of 82,000 feet the vehicle is rolled to a zero-degree bank angle.

The resulting bank angle for a -60 deg/100 sec target illustrates the sensitivity of the guidance system operation to range prediction. At initiation of control the vehicle immediately enters the spin mode. This action is due to the combined effect of the bias in the range prediction equation and approximations in the guidance laws. The initial computed cross-range error is -7.5 n.m.; the vehicle rolls to +90 degrees, actually moving away from the target. At 1538 seconds, the computed cross-range error drops below 1 n.m. and the vehicle spins. During most of the descent from this point, the guidance switches from spin to control mode and back to spin. In the control mode, which occurs for short durations, the cross-range computed is positive and the vehicle reverses direction of spin in an attempt to reach these
commands. The vehicle finally enters a controlled mode phase, correcting cross-range error and then rolls to zero bank angle.

Despite the deviations shown by these runs from the programmed bank angle time histories used to obtain the targets, the miss distances as displayed in Table 1 show good performance.

Summary

The simulation program described has been incorporated into the ground-based Gemini data processing system to generate inputs for tracking and communication systems test. It also finds application in the filtering of tracking data by the method of differential corrections.
Figure 1. Typical Gemini Reentry Trajectory Phases
Specific Forces \( (F_x, F_y, F_z) \)

Transmormed Specific Forces

Integration of Equations of Motion

Vehicle Position, Velocity and Heading

Entry Guidance Math Flow

Inertial Platform

Transformation to Geocentric Inertial Coordinates

Gravitational Forces

Gravity Model

Transform to Earth Ref. Coords.

Attitude \((\phi, \psi, \theta)\)

Bank Error Command

Attitude Control System (ACME)

Figure 2. Information Flow During the Maneuver Phase of Reentry

\[
E = \sin \theta \cos \phi - \cos \theta \sin \phi \cos \lambda
\]

Figure 3. Entry Guidance Computer Math Flow

499
Figure 4. Geometry Used in the Computation of Distance and Heading to the Target

Figure 5. Geometry Used in the Computation of Downrange and Crossrange Distances to the Target
Computer Limiter

\[ \text{Out} \]

\[ \text{In} \]

\[ -15 \]

\[ 15 \]

Attitude Control System (Acme) Vehicle

Deadband

\[ \text{In} \]

\[ \text{Out} \]

\[ -\Delta \theta_B \]

\[ \sum \]

\[ K_{\text{DB}} \]

\[ \text{On} \]

\[ \text{On} \]

Sampler

Figure 6. Conversion of Command to Actual Bank Angle

Figure 7. Performance of ACME to a Commanded Bank Angle of +60 Degrees
\[
\Delta \phi_B = B_E \\
\text{Limit } |\Delta \phi_B| \leq 15 \\
PE = \Delta \phi_B - B_{Att} \\
\]

\[
\begin{align*}
\tau_0 &= \min(PE - 2) \\
\end{align*}
\]

\[
\begin{align*}
\phi &= 5 \cdot \frac{\text{PE} - 2}{\text{PE}} \\
\tau_1 &= \min(\frac{\text{PE} - 2}{5}, 1.2) \\
\tau_2 &= \min(\tau_2, 1.0) \\
\tau_\theta &= \min(\tau_1, 0.6) \\
\end{align*}
\]

\[
\begin{align*}
\phi_6 &= \phi + 0.6 \cdot \bar{B}_{Att} + 0.5 \bar{B}_A \tau_\theta + \bar{B}_A \tau_0 (1-\tau_0) \\
\phi_11 &= \phi_12 + \bar{B}_{Att} + 0.5 \bar{B}_A \tau_1 + \bar{B}_A \tau_1 (1-\tau_1) \\
\phi_12 &= \phi_12 + 1.2 \cdot \bar{B}_{Att} + 0.5 \bar{B}_A \tau_2 + \bar{B}_A \tau_2 (1-\tau_2) \\
\bar{B}_{Att} &= \bar{B}_{Att} + \tau_\theta \bar{B}_A \\
\end{align*}
\]

Figure 8. Math Flow of Simulated Attitude Control System

Program
Initial Conditions
\[
\begin{align*}
T_0 &= 1448.4 \text{ Sec.} \\
\lambda_B &= 242.37661 \text{ Deg.} \\
\phi &= 23.91279 \text{ Deg.} \\
H &= 310,896.96 \text{ Ft.} \\
V_E &= 24,496.761 \text{ Ft.} / \text{Sec.} \\
\phi_E &= 88.429053 \text{ Deg.} \\
\gamma_E &= -1.4210693 \text{ Deg.} \\
\end{align*}
\]

Figure 9. Location of Impact on Earth's Surface for Commanded Bank Angle and Time to Initiate Roll for Reentry From Circular Orbit
Figure 10. Roll Angle vs Time Run Number +60/400 Gemini Reentry

Figure 11. Roll Angle vs Time Run Number +00/300 Gemini Reentry
Figure 12. Roll Angle vs Time Run Number -30/250 Gemini Reentry

Figure 13. Roll Angle vs Time Run Number -60/100