Apr 1st, 8:00 AM

Atmospheric Noise in Interferometer Radars - Observations on GE Mod III Tracking System

John N. Strand
General Electric Company RADIO GUIDANCE OPERATION Syracuse, New York

Follow this and additional works at: http://commons.erau.edu/space-congress-proceedings

Scholarly Commons Citation
http://commons.erau.edu/space-congress-proceedings/proceedings-1964-1st/session-2d/1
Random Atmospheric Propagation Noise
in Range Rate Difference Data While
Tracking a Moving Target

John N. Strand
General Electric Company
RADIO GUIDANCE OPERATION
Syracuse, New York

Summary

Recent low elevation angle space missions have provided a means of detecting random atmospheric propagation noise in range rate difference radar data while tracking a moving target.

The range rate difference data noise increases as the elevation angle approaches the horizon. The noise increases as an approximate function of \( \frac{1}{\sin^2 \theta} \) down to about 4 degrees elevation and increases less rapidly below 4 degrees.
Introduction

Data noise is very significant while radar tracking a missile or satellite when it is necessary to make real time corrections. The data processing equations must be designed to make timely and accurate computations in spite of the noise on the data. Therefore, the characteristics of this noise must be understood so that the proper allowances can be made.

The General Electric Mod III radio guidance system tracked and guided the launch vehicles of the highly successful Mercury Program. During these and other low elevation angle missions, propagation effects were noticed in the range rate difference* data. An intensive investigation of these propagation effects is continuing. Some of the pertinent results of this investigation are included in this report.

This report presents some of the "raw" radar data noise, suggests a curve fitting method based upon mechanism considerations, and attempts to determine the significance and implications of this curve fitting.

Random Atmospheric Noise in Range Rate Differences

Any theory as to the method of reducing the effects or determining the cause of atmospheric propagation noise must take into consideration the observed characteristics of this noise. Much data is available on propagation effects while tracking a stationary target. However, data showing propagation effects while tracking a moving target is not widely published. Therefore, Figure 1 presents samples of propagation noise as observed in radar data obtained during four representative low elevation angle missile flights.

A better understanding of the information in this report is obtained by knowing and understanding the terms and concepts used. These are as follows: The noise on the radar signals is observed as noise in the radar data. Therefore, the noise analyzed in this report is radar data noise. The magnitude of the data noise is obtained by smoothing the raw radar data and then computing the standard deviation of the smoothing residuals, using a fixed number of residuals for each computation. Careful selection of filters ensures efficient removal of noise without erroneously smoothing out missile steering. The adequacy of these processes has been demonstrated by comparing powered flight and free flight noise estimates. Although more sophisticated noise determination methods may be developed, there is no reason to believe that these methods will produce results which significantly differ from those already obtained.

In Figure 1, the radar data noise has been plotted as a function of elevation angle. (Elevation angle is measured up from the horizon.) The decision to plot noise as a function of elevation angle results from the consistency with which this method can be used to compare the noise of different missile flights. Often weapon system missile flights exceed the radar range, altitude and flight time of the low elevation angle missions, without random propagation noise being observed in any significant amount. The noise has been normalized by dividing by a constant for the following reasons:

1) differing radar sampling rates, sampling duration and smoothing techniques change the apparent noise level,
2) the relative changes of noise are more significant than the absolute changes, and
3) to avoid security problems.

* Range rate differences (also called lateral rates) are defined as the difference between the line-of-sight range rate observed at one antenna and the line-of-sight range rate observed at another antenna. The second antenna is located at some base line distance from the first antenna.
The crux of understanding propagation noise while tracking a moving target is appreciation of the dependence of noise magnitude upon elevation angle. Determining this relationship and interpreting the significance of this relationship is the purpose of the remainder of this paper.

**Curve Fitting Propagation Noise Based on Mechanism Considerations**

To simplify the presentation of noise data, facilitate generalizations, permit extrapolations, and most significant, provide an insight into the noise mechanism, it is necessary to determine some analytical relationship between noise and elevation angle. Whereas the atmosphere is the cause of propagation noise, it is natural to determine various relationships of the radar beam in the atmosphere.

Figure 2 is representative of the efforts to determine which relationship between the radar beam and the atmosphere better describes the observed noise. (To facilitate comparisons, the data has been normalized to unity at $10^\circ$.) The small ovals of Figure 2 represent the actual noise values from a typical low elevation angle mission. Two attempts were made to fit this data.

The first attempt resulted from assuming that the noise is proportional to the length of the radar beam in the atmosphere. This radar beam length would be calculated to some effective atmospheric altitude ($A$). If a flat earth is assumed, this length of the radar beam is $A \csc E$. The normalized (to unity at $10^\circ$) $A \csc E$ curve is illustrated in Figure 2. The curve does not fit the actual noise values depicted in Figure 2, therefore, it seems that the length of the beam in the atmosphere is not the most significant propagation noise factor.

The second attempt results from assuming that the noise is proportional to the horizontal velocity of a moving radar beam along some effective atmospheric altitude $A$. If a flat earth is assumed, this horizontal velocity of the radar beam is $A \csc^2 E$ (see Figure 3 for the derivation). A normalized (to unity at $10^\circ$) $A \csc^2 E$ curve is illustrated in Figure 2, and is depicted in Figure 2 by a solid line. The curve does fit the actual noise values depicted in Figure 2, therefore, it seems that the horizontal velocity of the beam along some effective altitude is a significant propagation noise factor.

Figure 2 is a somewhat simplified presentation. A larger sample of data is available from each missile flight, and about a dozen applicable flights are available. Therefore, to better verify this data fitting technique another but more complete illustration has been prepared. This new illustration presents the data in a form which enables a simpler check on the validity of $E \csc^2 E$ as a noise model. If $E \csc^2 E$ represents the rate of noise change, then division of the raw noise by $E \csc^2 E$ should produce a scatter of points which are independent of elevation angle and, therefore, a noise model validity check. (For reasons previously mentioned, the raw noise is normalized by dividing by a constant “K.”)

Figure 4 presents the data noise statistics from two flights after division by $K E \csc^2 E$. The resultant scatter of points is fairly independent of elevation angle. Therefore, this indicates that the noise model is valid.

However, as Figures 5 and 6 indicate, not all low elevation angle missile flights can be fitted this well. The top half of Figure 5 presents the quotient of noise divided by $E \csc^2 E$. This quotient increases as $E$ decreases (until about 4 degrees). This indicates that a power of $\csc E$ higher than 2 might be used to produce a scatter of points fairly independent of elevation angles. Figure 6 presents a case where division by $E \csc^2 E$ seems to overcompensate the dependence on elevation angle and, therefore, the resultant decreases as $E$ decreases. This suggests a power of $\csc E$ less than 2 might be used to produce a scatter of points fairly independent of $E$. Fitting the data to a variable powered $\csc E$ term would produce a scatter of points independent of $E$ in most of the cases. However, it is more difficult to imagine a noise mechanism based upon a variable powered $\csc E$. Also, the discrepancy is not as bad as one might first assume. The noise model of $C$ has a spread of a factor of about 3 from minimum to maximum whereas the noise data has a much larger spread of a factor of about
Likewise, the noise model of D has a spread of a factor of 2, but the data noise has a greater spread of a factor of 4 from minimum to maximum. Thus, a considerable reduction in spread has been achieved by using $E \csc^2 E$, and a closer fit by using a variable power $E E E$ is not a necessity.

**Interpretation of the Curve Fit**

Generalizations and extrapolations can be made from the curves which are fit to the observed noise. However, these generalizations and extrapolations must be qualified. If $A E E E E$ does represent the horizontal beam velocity which, in turn, represents the noise mechanism, then it is necessary to determine the conditions under which $A E E E$ is valid.

If $E$ is zero, then the equation implies that no propagation noise is to be expected. However, extensive work by the National Bureau of Standards in tracking stationary targets indicates that this is not true. This apparent discrepancy is probably the result of the "stationary target" noise being insignificant in comparison with equipment noise and "moving target" noise.

The equation $A E \csc^2 E$ is derived for a flat earth and goes to infinity when $E$ goes to 0 degree. This equation is, therefore, inaccurate at low elevation angles and should be replaced by an equation derived for a spherical earth. The horizontal (arc) velocity of a radar beam along layers around a spherical earth is given by:

$$\text{horz. vel.} = \frac{\dot{E} (R + A)}{1 - \frac{R \sin E}{(R + A) \sqrt{1 - (\frac{R \cos E}{R + A})^2}}}$$

where:

- $R$ = earth radius
- $A$ = altitude of the effective atmosphere
- $E$ = elevation angle of target, up from horizon
- $\dot{E}$ = elevation angle rate

Figure 7 illustrates the divergence of flat earth effects (the $\csc^2 E$ curve) from spherical earth effects (the curves which are a function of $A$ and $E$). This effect of noise increasing less rapidly than $\csc^2 E$ (as $E$ decreases) has been observed. From Figure 5, comparisons can be made of the effects of assuming a flat earth with the effects of assuming a spherical earth. Below 4 degrees for a flat earth (in Figure 5), the $\csc^2 E$ increases much more rapidly than the magnitude of the data noise. By assuming a spherical earth and an effective atmospheric altitude of 21,500 feet (suggested by analysis of radiosonde information), the divergence of the values at the low elevation angles is significantly reduced.

**Conclusions**

Atmospheric phenomena cause noise in range rate difference radars which increases as a function of $E \csc^2 E$ down to 4 degrees elevation angle, and more slowly below 4 degrees.

The magnitude of phase noise in range rate difference radars can be estimated by knowing the missile trajectory.

Once atmospheric noise is detected and measured by a radar, its magnitude at still lower elevation angles can be estimated.
Figure 1

RELATIVE CHANGE IN RAW RANGE RATE DIFFERENCE NOISE DATA

ELEVATION ANGLE

NOISE AS A FUNCTION OF ELEVATION ANGLE
Figure 2

ATMOSPHERIC NOISE CHARACTERISTICS

ACTUAL NOISE VALUES FROM A TYPICAL MISSION

VALUES RELATIVE TO 10 DEGREES

ELEVATION ANGLE ~ DEGREES

\( \dot{E} A C_{sc}^2 E \)

\( A C_{sc} E \)
COMPUTATION OF THE HORIZONTAL BEAM VELOCITY (FLAT EARTH)

\[ x = A \cot E \]
\[ v = \frac{dx}{dt} = A \csc^2 E \frac{dE}{dt} \]
Figure 4

NOISE MODEL VALIDITY CHECKS
Figure 5

COMPARISON OF FLAT EARTH AND SPHERICAL EARTH MODELS
Figure 6

NOISE MODEL VALIDITY CHECK

ELEVATION ANGLE

NOISE $\div K E C_{sc}^2 E$
THEORETICAL INCREASE IN PROPAGATION NOISE (NORMALIZED TO UNITY AT 10°)

\[ \text{THEORETICAL EFFECTS OF ELEVATION ANGLE AND ALTITUDE UPON NOISE MAGNITUDE} \]